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STRUCTURAL AND PRICING DECISIONS IN MANUFACTURING/REMANUFACTURING SYSTEMS WITH VERTICALLY DIFFERENTIATED PRODUCTS

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ABSTRACT

This research encompasses three related papers to address some of the influencing factors in structural and pricing decisions in supply chains with manufacturing and remanufacturing We consider new and remanufactured products that are vertically differentiated, that is, the consumers perceive the remanufactured product as of a lower quality and thus they are not willing to pay for them as much as they would for the new product Examples of such products are seen in computer systems, automotive parts and office equipment

In the first paper, we consider a closed loop supply chain that includes a manufacturer, a remanufacturer and a retailer We investigate the pricing decisions for the new and remanufactured products under different coordination structures between members of the chain while taking into account the consumers' perception of the remanufactured product versus new and the quality of returns as two major parameters In addition, we find which coordination structure is a better option for the closed loop supply chain members Particularly, we find that although a lower price is charged for the new product when the retailer and the remanufacturer are coordinated (RREMC) compared to the completely decentralized (CD) structure, a higher number of new products are sold in the completely decentralized structure A similar result is found for the remanufactured product when comparing the CD structure with the one in which the retailer and manufacturer are coordinated (MRC) We also find that MRC results in the highest total profit while RREMC results in the lowest

In the second paper, we analyze the pricing decisions for a firm that produces both new and remanufactured products and also collects the used product returns (known as cores, which are used in remanufacturing) The firm needs to define the core acquisition price as well as the selling prices for both new and remanufactured products In our models, we capture the quality of returns (by assuming a stochastic collection yield rate) and the competition between new and remanufactured products, and show how they influence the optimal expected prices and profit of the firm We provide managerial insight on how varying the optimal prices could help the firm optimally accommodate for different conditions (i e with respect to changes in the consumers' perceptions of the products, the yield rate, and the salvage value of the cores) For example, we find that when the firm sells low margin products, a small change in the consumers' perception of the remanufactured products versus new could increase the firm's expected profit by more than 10%

Finally, in the third paper, we consider two core collection structures for a firm that produces both new and remanufactured products In the first structure (known as the centralized channel), the firm collects the cores directly from the consumers, while in the second structure (known as the decentralized channel), the firm uses a third-party collector to take care of the core acquisition We assume that the demands for new and remanufactured products are influenced by the product prices and also by a stochastic component We jointly find the optimal prices and lot sizes for each product and investigate the impact of the competition between products (i e consumers' perception of the remanufactured product versus new), the quality of returns (i e the collection yield rate) and the demand uncertainties on the optimal solution in each channel Furthermore, we compare the channels on the amount of change in their optimal values and expected profits with respect to changes in the parameters We also provide managerial insight on how the firm should change the optimal prices and lot sizes in each channel considering possible changes in the consumers' perception of the products, the collection yield rate and the demand uncertainties For example, we find that when the demand uncertainties for the new and remanufactured products are higher, the reduction in the firm's profit is about 2-3% less in the centralized channel compared to the decentralized one

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CHAPTER 1

INTRODUCTION

Remanufacturing is the process of bringing a used product to like-new condition through replacing and rebuilding component parts (Haynsworth and Lyons, 1987) The remanufactured products are usually attractive to the consumers that are interested in the brand, but are not willing to pay a price as high as the one for the new products Remanufacturing is considered as one of the common processes in Closed-Loop Supply Chain (CLSC) management, while the others are product acquisition, reverse logistics, testing, sorting and disposition, and distribution and marketing (Guide and Van Wassenhove, 2003) CLSC management is defined as the design, control and operation of a system to maximize value creation over the entire life-cycle of a product with dynamic recovery of value from different types and volumes of returns over time (Guide and Van Wassenhove, 2009) A CLSC consists of forward and reverse supply chains and, as a result, owns a higher complexity than the more traditional (forward only) supply chains

Remanufacturing has received a growing attention in practice and also in academia in recent years Companies may have several drivers for supporting remanufacturing and being involved in a CLSC In some industries, government legislations require the manufacturers to take responsibility for the take-back of their end-of-use/end-of-life products (also known as core acquisition) Some examples of such legislations are the Waste Electrical and Electronic Equipment (WEEE) and the End-of-Life Vehicle (EOLV) directives set by the European Union Although WEEE and EOLV directives do not impose remanufacturing, the companies may obtain further benefits when extracting additional values from the collected cores through remanufacturing In addition, from a strategic marketing perspective, remanufacturing practices send a message to consumers that the firm is more environmentally responsible and, consequently, create a competitive advantage when dealing with the increasing number of more environmentally conscious consumers As a result, some companies get involved in recovery processes to use them as a marketing lever As Lebreton (2007) mentions, Fujitsu-Siemens Computers and the BMW Group run their own recovery centers in Paderborn and Munich respectively, however, these centers are too small to impact on a firm's operating results and exist mostly for marketing reasons and to underline the environmental goodwill of these companies

Furthermore, remanufacturing, on its own, can be a profitable business Some manufacturers of complex products have set up reverse supply chains and successfully recovered value from their returned products The recovery activities of copier (Oce, Xerox), electrical equipment (OMRON, see Kuik et al 2005) or tire manufacturers (Michelin) are closely linked to their forward supply chain and are of crucial importance for their operating profits (Lebreton, 2007) This type of activity can also be seen in the automotive parts industry companies like Fenco and Cardone produce both new and remanufactured automotive parts It has been reported that the cost of remanufacturing is typically 40-60% of the cost of manufacturing a new product with only 20% of the effort (Dowlatshahi 2000, Mitra 2007) In the U S , there are over 70,000 remanufacturing firms with total sales of $53 billion (USD) (Guide and Van Wassenhove 2001)

We can classify the CLSCs with remanufacturing into different groups based on three different factors first, the product's life-cycle (i e long life-cycle versus short), second, the type of returns (i e end of use/life versus warranty returns or any other type of returns), and third, distinguishable versus indistinguishable new and remanufactured products Our focus in this research is on the CLSCs with long life-cycle products (such as automotive parts, office equipment, etc) that collect end of use/life returns to use in remanufacturing In these CLSCs, distinguishable new and remanufactured products are produced Based on our observations in the automotive parts industry, the new and remanufactured products could be produced by separate firms (e g Hitachi that produces new products and Champion that produces remanufactured products) or by the same firm (e g Fenco and Cardone who produce both new and remanufactured products) The former is captured in our first research paper (i e in chapter 3) and the latter in the second and third research papers (i e chapters 4 and 5 respectively) Making structural and pricing decisions is an important part of managing these CLSCs In this context, the pricing problem is that of determining the right prices for the new and remanufactured products as well as the core acquisition prices that companies might need to pay to the consumers We consider new and remanufactured products as two different product types although there is no difference between them in terms of the product features and functionality Thus, poor pricing strategies will result in capturing less total CLSC profits and also in distorting the market for at least one of the product types, either new or remanufactured Structural decisions can include finding the optimal coordination structure between CLSC members or determining the best core acquisition channel

There are several factors to be considered when setting prices in a CLSC with remanufacturing The knowledge about the consumers' demand and their behavior - how much they are willing to pay for each product type and under what circumstances they agree to return the products - is perhaps the most important factor in defining the prices for the end products and the core acquisition In addition, a good estimate of all effective cost parameters that determine the cost of the goods sold, the relationship between CLSC members, the level of information sharing and coordination, and the CLSC structures are some of the other key factors Our research addresses many of these factors in structural and pricing decisions in three related research papers as follows

I) In the first paper, our research is motivated by real-life applications where the product life-cycle is long enough that the new and remanufactured products coexist in the market Examples of such products are automotive parts, mainframe computer systems and office equipment (Ferrer, 1997, Ayres et al 1997, Ferrer and Swaminathan, 2010) We consider a CLSC in which a retail store sells both new and remanufactured (also known as rebuilt) products We concentrate our analyses for cases with two versions of a single product (i e new and remanufactured), taking into account scenarios in which the manufacturer and the remanufacturer are two separate firms We also consider a supplier who provides both the manufacturer and the remanufacturer with new parts By definition, the supplier is a member of the CLSC, however, in order to streamline our analysis, in this paper, we separate the supplier from the rest of the CLSC members This assumption implies that the supplier does not have much of an impact on decisions made by the CLSC members However, from a supply chain management point of view, we are also interested in investigating the impact of the decisions made by CLSC members on higher tier suppliers Having this supplier, as a representative for all second tier suppliers, helps us capture the impact of decisions made by CLSC members on the supplier side Note that there are also cases in which the manufacturer takes care of the remanufacturing as well We address such cases in the second and third papers in our research

More specifically, in this paper, we find the optimal prices for new and remanufactured products made by the wholesalers (i e , the manufacturer and the remanufacturer) and by the retailer In addition, we investigate the pricing decisions under different CLSC decision-making structures, defined by different coordination scenarios between members That is, we consider a completely decentralized (CD) channel, a channel where the retailer and manufacturer are coordinated (MRC), and a channel in which the retailer and the remanufacturer are coordinated (RREMC) Concurrently, we examine the impact of the quality of returns and the consumers' perception of the remanufactured product versus the new on the optimal profits In our models, we capture the competition between the manufacturer and the remanufacturer as two separate entities as well as the competition (substitution) between new and remanufactured products at the retailer Our aim is to answer the following research questions

- How do the optimal prices and quantities compare with each other under different CLSC coordination structures?
- What is the impact of the quality of returns on the optimal CLSC profits?

- What is the impact of the consumers' perception of the remanufactured products versus the new products on the optimal CLSC profits?
- What CLSC coordination structure is preferred by the CLSC members under each level of quality of returns and consumers' perception?

II) In the second paper, our research is again motivated by real-life applications where the product life-cycle is long enough that the new and remanufactured products coexist in the market As mentioned before, examples of such products are automotive parts, mainframe computer systems and office equipment (Ferrer, 1997, Ayres et al 1997, Ferrer and Swaminathan, 2010) We analyze the pricing decisions for a firm that collects the end of life/use product returns (known as cores) from consumers, and uses them in remanufacturing, while she manufactures a new product as well The firm needs to define the optimal core acquisition price and the selling prices for the new and remanufactured products at the same time

The existing academic literature has defined the acquisition price and the selling prices for the remanufactured products from a remanufacturer's perspective (see for example Guide et al , 2003, Bakal and Akcalı, 2006, and Karakayalı et al , 2007), but they have not considered the impact of having the new product in the market and its competition (substitution) with the remanufactured product on the optimal prices for the new and remanufactured products and the core acquisition We show how the expected optimal prices and the firm's expected profit change when there is a competition between new and remanufactured products We capture the competition between new and remanufactured products by the relative willingness to pay of the consumers for the remanufactured product versus the new This shows the consumers' perception of the remanufactured product versus new We model the quality of returns by assuming a stochastic core acquisition yield rate, while we take into account two cases of high and low profit margin products We analyze how the firm optimally changes the prices when the quality of returns and its level of uncertainty vary, and also when the salvage value of the collected cores changes We compare the optimal changes that the firm makes to the prices and quantities across high and low margin cases with respect to the model parameters In addition, we investigate how the firm's expected profit changes (in both cases of high and low margin products) with respect to the model parameters In summary, we aim to address the following research questions

- What is the impact of the competition between new and remanufactured products on the expected optimal prices and quantities (for the new and remanufactured products and the core acquisition) for high and low margin products?
- What is the impact of the competition between new and remanufactured products on the firm's expected profit for high and low margin products?
- What is the impact of the core acquisition yield rate and its uncertainty on the expected optimal prices and quantities for high and low margin products?
- What is the impact of the core acquisition yield rate and its uncertainty on the firm's expected profit for high and low margin products?

III) Finally, in the third paper, we consider two core collection structures for a firm that produces both new and remanufactured products In the first structure (known as the centralized channel) the firm collects the cores directly from the consumers, but in the second structure (known as the decentralized channel), the firm uses a third-party collector to take care of the core acquisition Considering centralized and decentralized collection channels has been addressed by some authors in the literature However, to our knowledge, we are the first to jointly determine the optimal prices and lot sizes for differentiated new and remanufactured products under different reverse channel choices We assume that the demands for new and remanufactured products are influenced by the product prices and also by a stochastic component Furthermore, we jointly find the optimal prices and lot sizes for each product in a single-period setting and investigate the impact of the competition between products (i e consumers' perception of the remanufactured product versus new), the quality of returns (i e the collection yield rate) and the demand uncertainties on the optimal solution in each channel Furthermore, we compare the channels on the amount of change in their optimal values and expected profits with respect to changes in the parameters above We also provide managerial insight on how the firm should change the optimal prices and lot sizes in each channel considering possible changes in the consumers' perception of the products, the collection yield rate and the demand uncertainties In summary, we aim to answer the following research questions

- What is the impact of the competition between new and remanufactured products and the collection yield rate on the optimal prices and lot sizes of the new and remanufactured products?
- What is the impact of the competition between new and remanufactured products and the collection yield rate on the optimal core acquisition price and quantity in each channel and which channel leads to a higher number of cores to be collected?
- What is the impact of the competition between new and remanufactured products and the collection yield rate on the firm's expected profit in each channel?
- What is the impact of the demand uncertainties for the new and remanufactured products on the optimal prices, lot sizes and profits in the channels?
- How do the centralized and decentralized channels compare with each other with respect to their optimal prices, lot sizes and profits under different conditions (i e different consumers' perceptions of the remanufactured product versus new, quality of returns, and demand uncertainties)?

In the next chapter, we review the relevant literature Chapters 3, 4 and 5 include the models and analysis results for the first, second and third papers (as mentioned above) respectively Finally, a summary of all conclusions and future research directions is presented in Chapter 6

CHAPTER 2

LITERATURE REVIEW

There ıs a consıderable amount of lıterature on CLSCs General overvıews of product recovery and remanufacturıng can be found ın Thıerry et al (1995), Fleıschmann et al (1997), and Guıde (2000) In the book edıted by Guıde and Van Wassenhove (2003), some of the busıness aspects of CLSCs are addressed In addıtıon, ın the book edıted by Dekker et al (2004), further dıscussıons of CLSC problems are covered

The lıterature that ıncludes structural and prıcıng decısıons ın CLSCs ıs dırectly related to our research We look at the lıterature from three dıfferent perspectıves Fırst, from a modelıng perspectıve, we take ınto account the number of tıme perıods that the models ınclude, and we dıvıde the lıterature ınto three groups sıngle-perıod, two-perıod, and multı-perıod/ınfınıte horızon/contınuous models In addıtıon, ın each group, we ınvestıgate ıf any of the papers have consıdered dıstınguıshable new and remanufactured products to show the level of competıtıon (substıtutıon) between them Consequently, we look ınto the type of prıces determıned ın each paper, that ıs, the prıces for new and remanufactured products, and the core acquısıtıon Second, as the qualıty of returns ıs one of the factors that we consıder ın thıs research, we concentrate on several of the aforementıoned papers that consıder the qualıty of returns, as well as other papers ın the lıterature (ı e the ones wıthout prıcıng decısıons) that capture the qualıty of returns ın theır models Fınally, sınce we consıder coordınatıon and structural decısıons ın CLSCs ın our research, we revıew some of the papers capturıng thıs ın theır models

Consıderıng the fırst perspectıve above, ın the sıngle-perıod models, Savaskan et al (2004), and Savaskan and Van Wassenhove (2006) consıder ındıstınguıshable new and remanufactured products, and as a result, determine the price for new products only They do not investigate any acquisition price in their models Savaskan et al (2004) assume fixed unit acquisition price in their models However, Ray et al (2005) determine a trade-in rebate as an acquisition price while considering indistinguishable new and remanufactured products Vadde et al (2007) find the optimal price for remanufactured components at a product recovery facility They assume a fixed acquisition price and their models do not capture any possible competition between new and remanufactured products The literature also considers a remanufacturing firm who determines the acquisition price and the price for the remanufactured products without taking into account the impact of having new products in the market (Bakal and Akcalı, 2006, Guide et al , 2003, Karakayalı et al , 2007) In terms of the number of time periods for modeling, our research falls under this group However, we find prices for distinguishable new and remanufactured products as well as the core acquisition price Note that the core acquisition price is considered in chapters 4 and 5

In the two-period models, we are not aware of any papers that consider the core acquisition price as a decision variable Ferguson and Toktay (2006), as an example in this group, consider distinguishable new and remanufactured products in their models and determine the optimal prices for each type of product, but they do not deal with the core acquisition price in their models Most of the literature in this group, however, assume that the new and remanufactured products are not distinguishable and simply define the optimal price for new products (Majumder and Groenevelt, 2001, Ferrer and Swaminathan, 2006)

In the third group of papers, similar to the second group above, Ferrer and Swaminathan (2006) consider indistinguishable new and remanufactured products in multi-period and infinite-horizon scenarios, while Vorasayan and Ryan (2006) and Debo et al (2005) take into account distinguishable new and remanufactured products and define two distinct prices for them None of these papers capture the decision-making for the core acquisition prices However, Liang et al (2009) determine the core acquisition price having it linked to the sale price of the remanufactured products They assume that the sale price of the remanufactured product follows a geometric Brownian motion, which is extensively used in the option pricing literature, and from there, they define the acquisition price However, they do not determine the price for the remanufactured product as a decision variable In addition, their models do not consider any possible competition of the new product in the market

It is evident from the summary above that there is a research void regarding decision-making structures where the prices for new and remanufactured products and the core acquisition price are treated as decision variables concurrently We determine these three pricing decisions in two of our research papers (i e papers 2 and 3 included in chapters 4 and 5 respectively) for cases in which the manufacturer is also involved in remanufacturing This problem, as mentioned earlier, has not been addressed in the literature

With regards to the quality of returns, there are quite a few papers that consider it in their models Many of these papers deal with the production planning issues in a CLSC with remanufacturing, however, some of them consider pricing decisions as we have referred to them earlier We can divide these papers into two groups the ones that consider a single period and the ones with multi-period models In the single-period models, some authors assume the same cost of remanufacturing for all reusable cores (Ferrer, 2003, Bakal and Akcalı, 2006, Zikopoulos and Tagaras, 2007), while others consider different costs of remanufacturing for cores with different levels of quality (Guide et al , 2003, Aras et al , 2004, Ray et al , 2005, Galbreth and Blackburn, 2006 and 2010, Karakayalı et al , 2007, Vadde et al , 2007) Our research falls under this group and we assume that the cost of remanufacturing is the same for all remanufacturable cores Note that in all our models, the total cost of remanufacturing (which includes the cost of materials and core collection) decreases if the quality of returns is higher In the first and third papers (i e chapters 3 and 5), we consider a deterministic quality of returns But, in the second paper (i e chapter 4), we model a stochastic quality of returns by considering a random yield rate for the core collection In the papers with multi-period models, some assume deterministic quality levels for cores (Golany et al , 2001), while others model the quality levels using stochastic approaches such as Markov Chain and random outcomes (Decroix, 2006, Ferrer and Ketzenberg, 2004, Inderfurth et al , 2001, Toktay et al , 2000, Van der Laan et al , 1999, Denizel et al , 2008)

With respect to structural decisions and coordination in CLSCs, Debo et al (2004) review some of the topics on the supply chain coordination literature, and summarize the papers which examine reverse logistics problems as part of the total supply chain structure with an emphasis on pricing, incentive alignment, and information sharing Savaskan and Van Wassenhove (2006) consider a manufacturer who sells new products through two competing retailers The manufacturer has the option to collect the end-of-use products directly or through the retailers to use in the production of the new products As a result, their new and remanufactured products are not distinguishable They compare different collection channels under different coordination scenarios The models are for a single period, and the only prices that might be determined (i e depending on the channel structure) are the wholesale price and the retail price at each retailer

Karakayali et al (2007) analyze how the decentralized channels can be coordinated to attain the end-of-life product collection rate that can be achieved in the centralized channel They consider a single-period model, and investigate how the pricing behaviors of the collector and remanufacturer impact the used product collection rates in decentralized channels They determine the optimal acquisition price of end-of-life products and the selling price of the remanufactured products Their models do not include any competition between new and remanufactured products, while we capture this competition as well as the competition between the manufacturer and the remanufacturer in our research Bhattacharya et al (2006) address four different CLSC structures Their focus is mainly on determining the optimal order quantities Our research in the first paper is similar to their work in terms of considering several decision-making CLSC structures However, their models do not include a supplier In addition, they do not consider the impact of quality of returns on the decision variables, and they have exogenous prices in their models, while we investigate the impact of different levels of quality of returns on the optimal (endogenously determined) prices Bhattacharya et al (2006) also assume that the new and remanufactured products are not distinguishable We, on the other hand, consider vertically differentiated new and remanufactured products

Savaskan et al (2004) consider three reverse channels for collecting the used products (cores) from customers (1) directly from the customer, (2) through the retailer who collects the cores for a suitable incentive, (3) subcontracting the core collection to a third party However, they do not consider distinguishable new and remanufactured products and do not address a joint pricing and lot sizing problem, while we consider distinguishable products and in our third paper (in chapter 5), we jointly determine the optimal prices and lot sizes for the new and remanufactured products In addition, they use deterministic demand functions, we assume deterministic demands in both the first and second papers and stochastic demands in the third paper Kaya (2010) considers centralized and decentralized channels where the new and remanufactured products are partial substitutes (distinguishable) with stochastic demands But, they only address the optimal production quantities, while we jointly determine the optimal prices and lot sizes in our third paper Moreover, they do not consider the consumers' willingness to pay for each product and the quality of returns (or the collection yield rate) However, as mentioned earlier, we capture these real-life characteristics in our models

As mentioned earlier, in the first paper (i e chapter 3), we investigate the impact of different coordination structures on prices, quantities and profits in a CLSC under different settings from the existing literature, and also consider the impact of some parameters such as the quality of returns and the consumers' perceptions of remanufactured products versus new on the optimal values Next, in the second paper (i e chapter 4), we consider a problem in which the prices for new and remanufactured products as well as the core acquisition price are determined by a manufacturer who is also in charge of the remanufacturing activities We show how the competition between new and remanufactured products influences the optimal pricing decisions Finally, in the third paper (i e chapter 5), we consider centralized and decentralized channels with respect to core collection We jointly find the optimal prices and lot sizes for differentiated (distinguishable) new and remanufactured products as well as the optimal core acquisition prices while assuming a stochastic demand for each product In addition, we investigate how the channels compare with each other with respect to the optimal decisions In the next chapter, we further discuss the first paper

CHAPTER 3

PAPER 1:

THE IMPACT OF COORDINATION STRUCTURES ON CLOSED-LOOP SUPPLY CHAIN DECISIONS

3.1. Model Description and Assumptions

In this paper, we model a CLSC that includes a retailer who sells new and remanufactured products, a manufacturer who only produces new products, and a remanufacturer who collects the returned products and uses the reusable parts for remanufacturing We also consider a supplier who provides the manufacturer and the remanufacturer with new parts This is a rather general setting representative of our potential applications

We consider three CLSC decision-making structures that show different coordination options between the retailer and her first tier suppliers The first structure is a completely decentralized (CD) one in which each member makes their own pricing decisions independently In the second structure, the manufacturer and the retailer are fully coordinated and they make pricing decisions as one coordinated unit (MRC) Basically, in this structure, we investigate the impact on the optimal prices and the total CLSC profit if the retailer develops a very close relationship with the manufacturer defined as the full coordination Finally, in the third structure, the retailer and the remanufacturer act as one fully coordinated unit in making pricing decisions (RREMC) We do not consider the coordination with the supplier, but we investigate the impact of each decision-making structure on the supplier's profit Figure 1 shows the CLSC structures

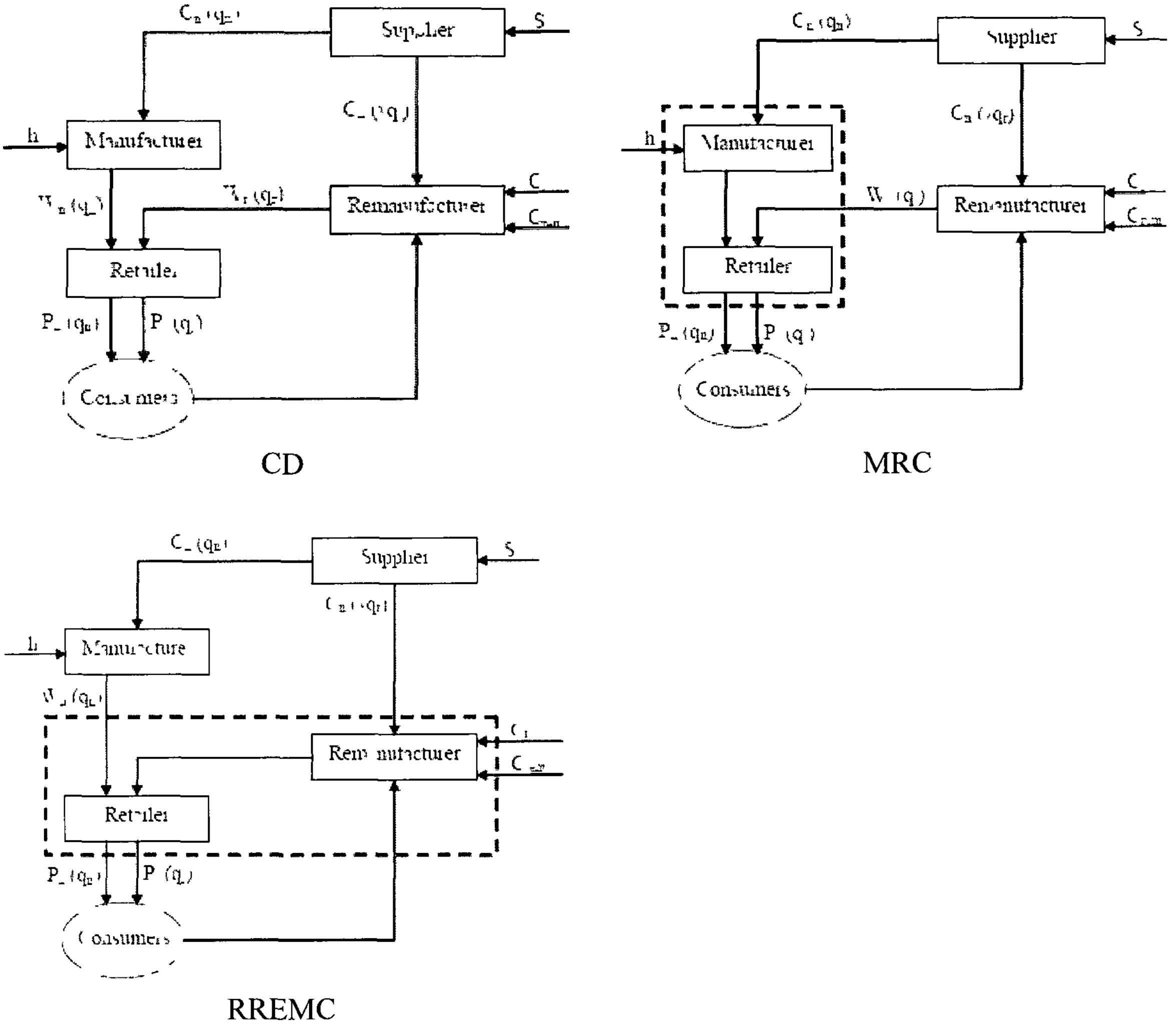


Figure 3 1 CLSC decision-making structures

In figure 1, the values in parentheses show the quantities of the parts or products shipped from one member to another The rest of the notation in the figure show the prices charged and costs incurred by the CLSC members Table 1 describes the notation used in this paper in more detail

q_i = Quantity (= demand) for product type i ($i = n, r$)

q_i^k = Optimal quantity of product type i in structure k ($i = n, r,\ k = MRC, RREMC, CD$)

P_i = Price for product type i ($i = n, r$)

P_i^k = Optimal price for product type i in structure k ($i = n, r,\ k = MRC, RREMC, CD$)

W_i = Wholesale price for product type i ($i = n, r$)

C_n = Total cost of new parts in one unit of product (if 100% new parts are used If not, a fraction of this cost will be taken into account)

C_r = Supply cost of reusable parts, incurred by the remanufacturer (cost of providing reusable parts out of returned products It can also account for the acquisition costs for the returns)

h = Cost of manufacturing the new product per unit

C_{rem} = Cost of remanufacturing per unit

S = Supply cost of new parts, incurred by the supplier

γ The portion of parts in a remanufactured product that needs to be replaced by new parts

δ The ratio of consumers' willingness-to-pay for remanufactured products to their willingness-to-pay for new products, $\delta \in [0,1]$

Π_R = Profit for the retailer

Π_M = Profit for the manufacturer

Π_{rem} = Profit for the remanufacturer

Π_S = Profit for the supplier

Π_k^* = Optimal CLSC profit for structure k ($k = MRC, RREMC, CD$)

Table 3 1 Notation

Because our research is motivated by real-life applications (such as automotive parts, mainframe computer systems, office equipment and any aftermarket service parts manufacturing and remanufacturing) and the existing academic literature, our models capture key properties of new and remanufactured products in the industry, while holding some of the useful modeling assumptions of similar models from the literature As with most industry practices mentioned above, new and remanufactured products are distinguishable, and each consumer's willingness-to-pay for a remanufactured product can be defined as a fraction (δ) of their willingness-to-pay for the new product A similar approach is used in Ferguson and Toktay (2006) The product life-cycle is long enough to allow both new and remanufactured versions of the same product to be present in the market at the same time, as one can find both on the shelves of the retail stores As mentioned earlier, some examples are automotive starters, alternators and water-pumps The relationships between the manufacturer, remanufacturer, and retailer under study are such that they share almost the same amount of power in the CLSC to determine prices As a result, it is reasonable to use Differentiated Bertrand models to capture the relationships between the members of the CLSC and to explain their (simultaneous) decision-making processes

Regarding the assumptions in our models, there is no capacity constraint either for new parts available from the supplier or for the number of returns available for remanufacturing While the number of cores available for remanufacturing could be limited in some cases, having this assumption in place helps us focus on the main research questions without the impact of the capacity constraint In addition, this assumption makes the models more tractable (Guide et al 2003) We also assume that return rates are independent of sales rates, that is, the market is mature enough and there are enough new products sent to the market in previous periods of time (Guide et al 2003)

In our models, no fixed cost is considered for manufacturing or remanufacturing Having fixed costs for manufacturing and remanufacturing would just shift the optimal values and has no significant impact on the results and insights of this research Our models are for a single period as we assume the previous existence of the product in the market Consumer demand is assumed to be price sensitive and deterministic Similar assumptions are considered by Savaskan et al (2004) and Guide et al (2003)

The core acquisition cost is assumed to be negligible Similar to Bhattacharya et al (2006), the core acquisition cost and related decisions are not the focus of this research We concentrate on the implications of the quality of the collected cores on the optimal profits in different CLSC structures In addition, due to the nature of the products under study, the cores could be remanufactured multiple times This helps us not to be constrained by the number of remanufactured products to be less than the number of new products sold at the retailer Moreover, the consumer willingness-to-pay is heterogeneous and uniformly distributed in the interval [0,1] and the market size is normalized to 1 As we show in Appendix A, expressions (3 1) and (3 2) hold among prices and quantities

$$P_n = 1 - q_n - \delta q_r \tag{3 1}$$

$$P_r = \delta(1 - q_n - q_r) \tag{3 2}$$

In our models, all supply chain members have access to the same information when making decisions This assumption allows us to control for the impact of information asymmetry and focus our attention on the quality of returns and the consumers' perception of the remanufactured products versus new in different CLSC structures A similar assumption is seen in Savaskan et al (2004) In the next section, we introduce the models and discuss the optimal values in more detail

3.2. Model Formulations and Analysis

In this section, we introduce our models and derive the optimal prices and quantities for new and remanufactured products in each CLSC structure We also compare the prices as well as the supplier's and the total optimal CLSC profits across the structures Furthermore, we analyze the impact of the quality of returns and the consumers' perceptions on the optimal profits

3 2 1 Models and optimal values for CLSC structures

Here we present the profit functions for each CLSC member and the supplier, as well as the optimal prices and quantities for new and remanufactured products

In the CD structure, the profit functions are defined as follows

$$\Pi_R = q_n(P_n - W_n) + q_r(P_r - W_r) \qquad (3\ 3)$$

$$\Pi_M = q_n(W_n - C_n - h) \qquad (3\ 4)$$

$$\Pi_S = (q_n + \gamma q_r)(C_n - S) \qquad (3\ 5)$$

$$\Pi_{rcm} = q_r(W_r - B) \qquad (3\ 6)$$

where $B = \gamma C_n + (1-\gamma)C_r + C_{rcm}$ represents the total cost of remanufacturing per unit and we have $q_n \geq 0$ and $q_r \geq 0$

In the other structures, any time that a member is coordinated with another, the profit of the coordinated unit is given by the summation of the profit functions for those members as presented above The business and decision-making responsibilities are similar in all structures However, in any case in which there is a coordinated joint unit in the CLSC, the unit makes pricing decisions on belalf of all members included in it For example, in the MRC structure, the manufacturer and the retailer jointly set the retail price for the new product This pricing decision making, which is prevalent in the supply chain literature, gives us the optimal values as if those members were fully coordinated In the following, we use the CD structure as a representative for all structures to show how we calculate the optimal values for the retail prices and quantities of new and remanufactured products The derivations are similar for the other structures given their own characteristics

The retailer sells both new and remanufactured products So her profit function (Π_R) consists of the profit from new products as well as the one from remanufactured products The remanufacturer collects the end-of-life products, tests and cleans them, and uses the reusable parts from them in the remanufacturing He replaces worn-out parts with new parts in order to obtain remanufactured products with acceptable quality So, the average cost of parts used in one unit of the remanufactured product is the summation of the cost of new parts and the cost of reusable parts, that is, $\gamma C_n + (1-\gamma)C_r$ Here γ shows the proportion of the parts in a particular remanufactured product that are new parts Since not all parts are identical in a finished product, γ is defined as the average dollar value of the parts that need to be replaced divided by the total dollar value of all parts in the product As a result, a higher γ stands for a lower average quality in the end-of-life returns, as more used parts would need to be replaced by new parts with higher total dollar value The total cost of remanufacturing per unit is defined as $B = \gamma C_n + (1-\gamma)C_r + C_{rem}$

In the CD structure, the retailer sets the prices for new and remanufactured products to maximize her own profit based on the wholesale prices that she receives from the manufacturer and remanufacturer Because the retail prices affect the demand for new and remanufactured products, and retailer's order quantities change accordingly, they have an indirect impact on the profits of the manufacturer and remanufacturer Knowing this, the manufacturer and remanufacturer set their wholesale prices in a way to maximize their own profits If they set high wholesale prices, the retailer will have to set higher retail prices, which in turn decreases the demand for new and remanufactured products and the order quantities from the retailer to the manufacturer and remanufacturer Wholesale prices, then, are determined by finding the best response to the best response of the retailer

The analysis starts by finding the values of q_n and q_r in terms of P_n and P_r from expressions (3 1) and (3 2) In order to find the optimal retail prices P_n and P_r and quantities q_n and q_r in terms of the wholesale prices W_n and W_r, we consider the Karush-Kuhn-Tucker (KKT) conditions and use the followig equation for the retailer

$$L_R = q_n(P_n - W_n) + q_r(P_r - W_r) + \mu_n q_n + \mu_r q_r \qquad (3\ 7)$$

This expression incorporates constraints $q_n \geq 0$ and $q_r \geq 0$ Note that the optimal conditions to be satisfied include $\partial L_R / \partial P_n = 0$, $\partial L_R / \partial P_r = 0$, $\mu_n q_n = 0$ and $\mu_r q_r = 0$ It is straightforward to show that the profit function for the retailer is a concave function and we have a convex set of constraints Hence, the solution to the KKT conditions is a unique optimal solution Next, we substitute these values for prices and quantities (in terms of wholesale prices) in expressions (3 4) and (3 6) and find the optimal wholesale prices set by the manufacturer and the remanufacturer through solving the best response equations of each to another Finally, we substitute these optimal wholesale prices in the optimal retail prices P_n and P_r and quantities q_n and q_r that we found earlier to define them in terms of the model parameters

Solving the KKT conditions leads us to consider four different cases as follows

Case 1 $q_n > 0$, $q_r > 0$, $\mu_n = 0$ and $\mu_r = 0$

Case 2 $q_n > 0$, $q_r = 0$, $\mu_n = 0$ and $\mu_r \geq 0$

Case 3 $q_n = 0$, $q_r > 0$, $\mu_n \geq 0$ and $\mu_r = 0$

Case 4 $q_n = 0$, $q_r = 0$, $\mu_n \geq 0$ and $\mu_r \geq 0$

The analysis of these cases is included in Appendix B It is intuitive to say that case 4 is not feasible, because none of the players will exist if the conditions in this case are in place Figure 2 shows the summary of our analysis for the rest of the cases Calculations for Figure 2 are also included in Appendix B We find that depending on how unit costs of manufacturing and remanufacturing compare with each other, a different case may be feasible If the unit cost of manufacturing is high enough as we have in region 2 of Figure 2, when the retailer coordinates with the remanufacturer, it is optimal for her (or in other words for the whole CLSC) not to sell any new products The new product becomes less desirable if the unit cost of manufacturing goes even higher than a certain amount as we have in region 3 In that case, no matter which coordination structure is in place, the CLSC will be better off if the retailer does not sell any new product

On the other hand, if the unit cost of remanufacturing becomes higher than a certain amount, as we have in region 4, the retailer will decide not to sell any remanufactured products if she is coordinated with the manufacturer The remanufactured product becomes less desirable if the unit cost of remanufacturing goes even higher In that case, as we have in region 5, no matter which coordination structure is in place, the CLSC will be better off if the retailer does not sell any remanufactured products

However, if the unit costs of manufacturing and remanufacturing vary in a certain range compared to each other, as we have in region 1, both players will exist in the market and the retailer will sell both new and remanufactured products Since the manufacturer and the remanufacturer in our study exist in the market, that is, they are producing positive quantities of products and selling them through the retailer, it is safe to assume that the model that explains the current situation the best is associated with the first case Having this in mind, we focus the rest of our analysis on case 1 Table 2 shows the optimal values for the retail prices and quantities of new and remanufactured products for all structures in case 1

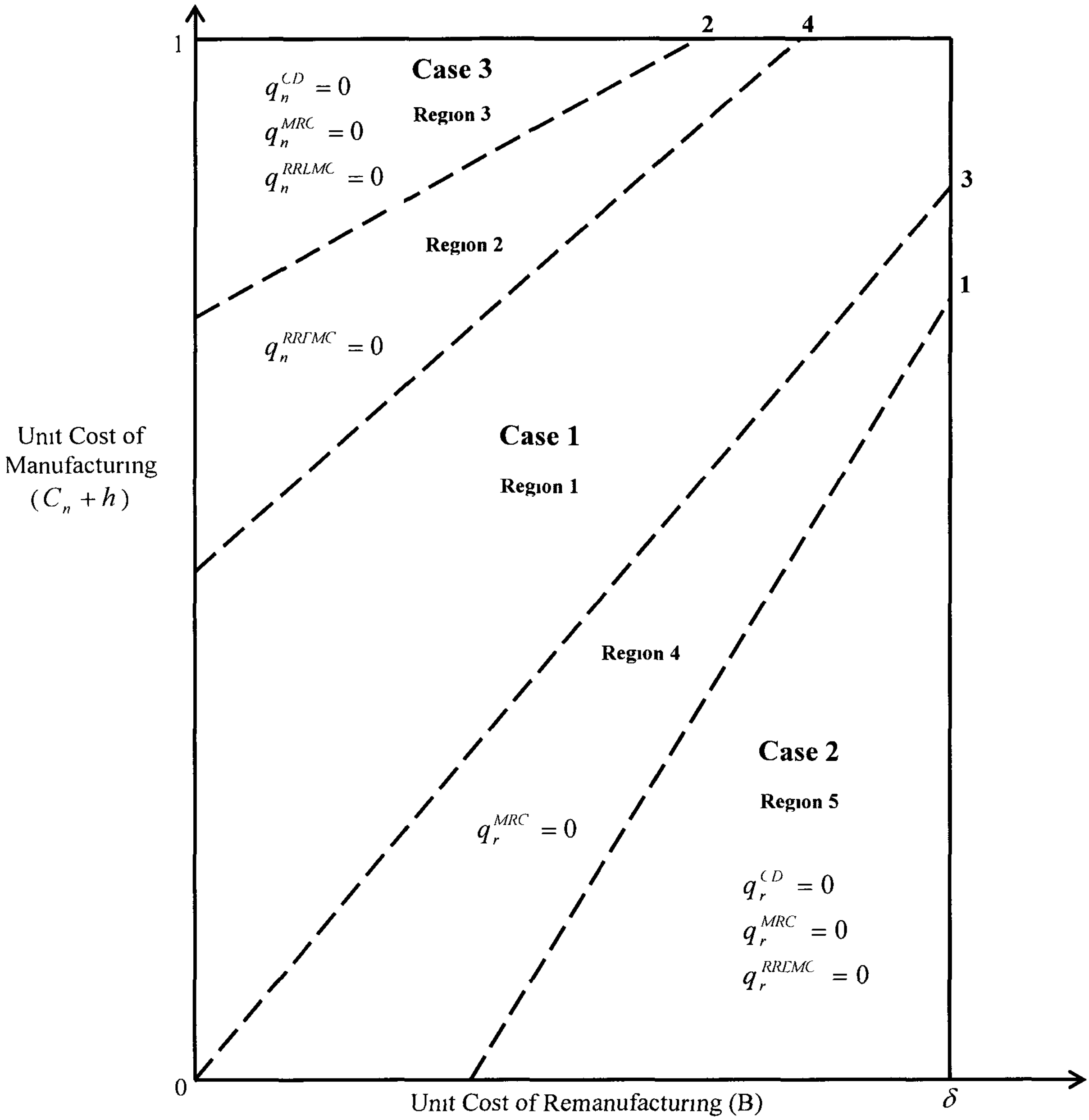


Figure 3 2 Analysis of the cases for KKT conditions based on the unit costs of manufacturing and remanufacturing

	CD	MRC	RREMC
P_n	$\frac{1}{2}+\frac{2(C_n+h)+2(1-\delta)+B}{2(4-\delta)}$	$\frac{1+C_n+h}{2}$	$\frac{3-\delta+C_n+h+B}{4}$
P_r	$\frac{\delta}{2}+\frac{\delta(1-\delta)+\delta(C_n+h)+2B}{2(4-\delta)}$	$\frac{\delta(C_n+h)+B+2\delta}{4}$	$\frac{\delta+B}{2}$
q_n	$\frac{2(1-\delta)-(2-\delta)(C_n+h)+B}{2(1-\delta)(4-\delta)}$	$\frac{1}{2}+\frac{B-(2-\delta)(C_n+h)}{4(1-\delta)}$	$\frac{1}{4}-\frac{C_n+h-B}{4(1-\delta)}$
q_r	$\frac{\delta(1-\delta)+\delta(C_n+h)-(2-\delta)B}{2\delta(1-\delta)(4-\delta)}$	$\frac{\delta(C_n+h)-B}{4\delta(1-\delta)}$	$\frac{1}{4}+\frac{\delta(C_n+h)-(2-\delta)B}{4\delta(1-\delta)}$

Table 3 2 Optimal retail prices and quantities for new and remanufactured products

3 2 2 Comparison of prices and quantities across structures

In this section, we present our analysis with respect to the optimal prices and quantities across structures We first explain the underlying conditions for the analysis and then discuss the results in the following sub-sections

3 2 2 1 The underlying conditions

We need to determine the conditions that the model parameters should hold in order for case 1 to be feasible As explained in Appendix B, we can summarize these conditions as follows

Condition 1 $\delta(C_n+h) > B$ (3 8)

Condition 2 $C_n+h < B+(1-\delta)$ (3 9)

where $B=\gamma C_n+(1-\gamma)C_r+C_{rcm}$

Condition 1, represented by expression (3 8), implies that the unit cost of remanufacturing should be low enough relative to the unit cost of manufacturing in order for remanufacturing to become a feasible option Condition 2, represented by expression (3 9), implies that the unit cost of manufacturing should not be too high relative to the unit cost of remanufacturing, otherwise, manufacturing will not be feasible

3 2 2 2 Comparison of prices

In Appendix C, we show analytically how the optimal prices for the new products compare with each other across different CLSC structures Similarly, we are able to show the comparison of the prices for the remanufactured product The following summarizes the results for new product prices as well as prices for remanufactured products across different structures

$$P_n^{MRC} < P_n^{RREMC} < P_n^{CD} \quad (3\ 10)$$

$$P_r^{RRLMC} < P_r^{MRC} < P_r^{CD} \quad (3\ 11)$$

In pricing the new products, the MRC structure can set a lower price since the double marginalization for new products does not exist in this structure In the RREMC and CD structures, the retailer is not coordinating with the manufacturer and, as a result, there exists a double marginalization due to the manufacturer's wholesale price Therefore, the prices are expected to be higher than the ones in the MRC structure

In the RREMC structure, however, the retailer, who is coordinating with the remanufacturer, is able to set a low price for the remanufactured products If she sets the price for new products as high as the one in the CD structure, new products could not compete with the remanufactured products in the market and, as a result, the retailer would lose too much of the demand for new products, which would not be desirable for her In this case, she sets a price lower than the one in the CD stucture but still higher than the lowest price in the MRC structure due to the existing double marginalization in RREMC

The same type of reasoning explains how the prices for remanufactured products compare with each other across structures In the RREMC structure, a lower retail price is charged for the remanufactured products since there is no double marginalization In the MRC structure, there is double marginalization for the remanufactured product, so a higher price is set compared to the RREMC structure However, due to the low price for new products in this structure, the price for the remanufactured product is set low enough so that it can compete with the new product in the market As a result, a lower price is charged compared to the CD structure

3 2 2 3 Comparison of quantities

In Appendix C, we also analytically prove how the optimal quantities compare with each other across CLSC structures The comparison of optimal quantities across CLSC structures is summarized as follows

$$q_n^{RREMC} < q_n^{CD} < q_n^{MRC} \quad (3\ 12)$$

$$q_r^{MRC} < q_r^{CD} < q_r^{RREMC} \quad (3\ 13)$$

Looking at the results above, one may think at first that the prices for the new and remanufactured products are deriving the quantities sold at the retailer While it is true for the number of new products sold in the MRC structure, by taking a closer look at the CD and RREMC structures in expression (3 12), we see that a higher number of the new product is sold at the CD structure while a higher price is charged compared to the RREMC structure A similar observation holds for the remanufactured product in the CD and MRC structures As a result, we can conclude that if the retailer coordinates with the manufacturer, she will sell a higher number of new products and a lower number of remanufactured products compared to the completely decentralized channel On the other hand, if the retailer coordinates with the remanufacturer, she will sell a lower number of new products and a higher number of remanufactured products compared to the completely decentralized structure

3 2 3 Numerical analysis

In this section, we provide a numerical analysis to further develop insight First, we explain how we set the parameters of the analysis Then we compare the CLSC optimal profits across the structures and investigate the impact of the quality of returns and the consumers' perception on these profits Finally, we compare the supplier's profit across the structures or in other words, we analyze the impact of the CLSC coordination structure decisions on the supplier's profit

3 2 3 1 Parameter setting

For setting parameters, we need to take some conditions into account One condition is that our cost factors, such as the unit cost of manufacturing, h, the unit cost of new parts, C_n, the unit cost of remanufacturing, C_{rem}, and the unit cost of reusable parts, C_r, should be set such that the prices take values smaller than 1 while resulting in feasible (positive) quantities of new and remanufactured products Another condition is related to the range of values that δ and γ can take, that is, values between (0, 1) In addition, we consider conditions 1 and 2 from expressions (3 8) and (3 9) in section 3 2 2 to determine the feasible ranges of δ and γ for the analysis In our analysis, the original set of values that we used for C_n, C_{rem}, C_r, h, and S includes {0 01, 0 02, 0 03, , 0 1}, but, since the results were consistent across these different values, we show our results based on the specific set of parameters as follows C_n = 0 03, C_{rem} = 0 01, C_r = 0 02, h = 0 03, S = 0 01, $\delta \in [0\ 66, 0\ 96]$, $\gamma \in [0\ 3, 0\ 9]$ As mentioned, we have done an extensive numerical analysis and the results that follow in the next sections are consistent across different sets of parameter values

3 2 3 2 The impact of quality of returns on optimal CLSC profits across structures

Our numerical analysis shows that the optimal CLSC profits in all structures decrease with a reduction in the quality of returns, that is, with an increase in γ This is a reasonable result since the cost of remanufacturing increases with the reduction in the quality of returns and it leads to a higher total cost in the CLSC Thus, the total profit of the CLSC will be lower

Our results show that the reduction in the CLSC profit is less than 1% in each structure when γ changes in the range from 0 3 to 0 9 This is because when the quality of returns decreases, the remanufactured product becomes less competitive to the new product (because of the higher cost of remanufacturing and resulting higher retail price) and more new products are sold at the retailer, generating more profit in the CLSC to compensate for the reduction in the profit from the remanufactured products As a result, the total CLSC profit does not change significantly This also shows that a reduction in the quality of returns reduces the remanufacturer's profit while it increases the manufacturer's profit

3 2 3 3 The impact of consumers' perceptions on optimal CLSC profits across structures

Our results show that the CLSC profits have an ascending trend with the increase in the consumers' perceptions of the remanufactured product versus new, δ More specifically, with an increase in δ in the range from 0 66 to 0 96, the profit of the CD structure increases by almost 8%, RREMC by 10 7% and MRC by 1 3% As we see, the change in the MRC structure is not as significant when compared to CD and RREMC structures This shows that the CLSC profits in the CD and RREMC structures are the most sensitive to the consumers' perception of the remanufactured product versus new As a result, if the retailer coordinates with the manufacturer, the total CLSC profit stays more stable against any possible change in the consumer's perception of the products

3 2 3 4 The comparison of the CLSC profits across structures

Looking at the CLSC profits with respect to γ, we see that the profit for the MRC structure, Π^*_{MRC}, is higher than the ones for the other structures This result is also consistent when we look at the change in the CLSC profits with respect to δ As our results show, the order for the profit of the other structures, from higher to lower, is CD and RREMC for all levels of the quality of returns when $0\ 66 \le \delta < 0\ 93$ So the following ranking holds under the aforementioned conditions

$$\Pi^*_{RRI\,MC} < \Pi^*_{CD} < \Pi^*_{MRC} \tag{3 14}$$

This suggests that the CLSC will be more profitable if the retailer coordinates with the manufacturer The coordination with the remanufacturer will decrease the CLSC's profit compared to the case of having a completely decentralized channel From a supply chain management perspective, there is a need for appropriate contracts to be in place in order to achieve the highest CLSC profit through coordinating the retailer and the manufacturer, however we do not focus on the contracts in this paper Figure 3 shows how the profits across the structures change with respect to δ These profit values almost converge when $\delta \ge 0\ 93$ and the order given in expression (3 14) changes Although the differences among the profits across the structures are not large, we find that when the new and remanufactured products are perceived as very close substitutes (i e $\delta \ge 0\ 93$), a decentralized channel could perform better than the others in terms of the total profit of the CLSC However, distinguishable new and remanufactured products under our study are perceived by consumers in a way that it makes it reasonable not to have very high δ Thus, we focus on the range $0\ 66 \le \delta < 0\ 93$ for this analysis

In the next section, we look at the supplier's side and investigate the impact of the CLSC members' decision for the coordination structure on the supplier's profit This helps us find how the second tier supplier will be affected by the structural decisions made by the CLSC downstream

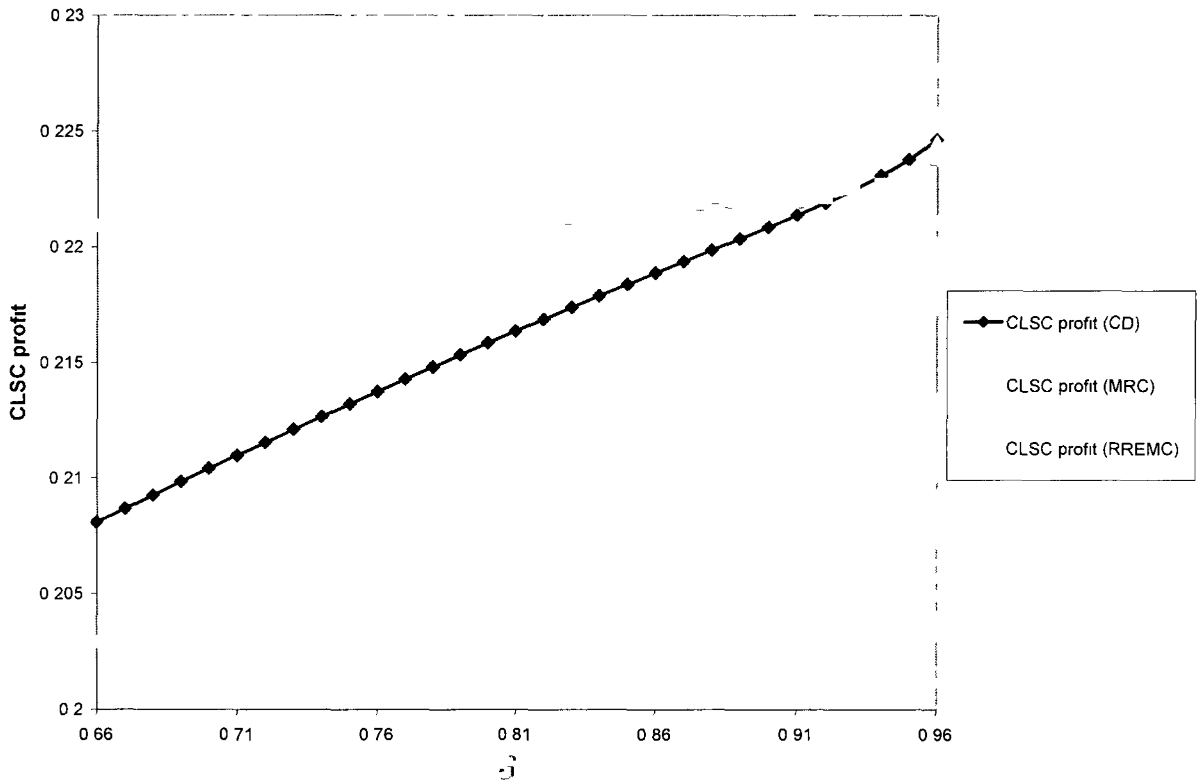


Figure 3 3 Optimal profits across structures with respect to δ

3 2 3 5 The impact of CLSC structural decisions on the supplier's profit

Our analysis shows that the supplier would rather have the retailer and the manufacturer to be coordinated, that is, the MRC structure to be in place Because it will result in a higher profit for him This is consistent with the optimal structural decision of the CLSC members to choose the MRC structure for a higher CLSC profit As a result, the optimal decision of the CLSC members will have a good impact on the supplier's profit In addition, the supplier will only prefer the RREMC structure over the CD structure if the quality of returns is low Otherwise, having the CD structure will result in a higher profit for the supplier than RREMC

3.3. Managerial Insight

This study shows that it will be more profitable for the CLSC if the retailer and the manufacturer coordinate with each other in their pricing decisions Also, from the supplier's point of view, coordination between the manufacturer and the retailer is preferred, which shows that there will be no conflict between the interests of the CLSC members and the supplier in terms of the best coordination structure In addition, our results show that the coordination between the retailer and the remanufacturer leads to a lower profit for the CLSC members than the one in a completely decentralized case We find that although the total profit of the retailer and the remanufacturer is higher when they are coordinated, the competition that exists between new and remanufactured products hurts the manufacturer's profit so much that the total CLSC profit becomes less than what it would be in the completely decentralized structure However, if the quality of returns is low, the supplier will enjoy a higher profit from the coordination between the retailer and the remanufacturer compared to the completely decentralized case

In addition, we find that a lower quality of returns decreases the total CLSC profit, but this reduction is less than 1% However, the remanufacturer faces a higher decrease in his profit while the manufacturer enjoys an increase in the profit Furthermore, we find that a higher consumers' perception for the remanufactured product versus new, which makes the new and remanufactured products closer substitutes, will result in a higher CLSC profit, especially in the completely decentralized structure and the one with coordinated retailer and remanufacturer

3.4. Conclusion and Directions for Future Research

This paper shows how the coordination options in a CLSC compare with each other To do this, we present the optimal profits for the CLSC and the supplier in addition to the optimal prices and quantities for the new and remanufactured products at the retailer The CLSC in this paper consists of a manufacturer, a remanufacturer, and a retailer We also consider a supplier to provide both the manufacturer and remanufacturer with new parts We aim to provide managers in a CLSC with insights that help them determine what coordination structure is better for the CLSC and how the optimal prices are set across the structures In addition, we consider the structural decision from a supplier's perspective, that is, to find which option the supplier would choose were it up to him to decide on the coordination structure Motivated by real-life practice, we model the CLSC for vertically differentiated new and remanufactured products We capture this differentiation by the consumers' perception or relative willingness-to-pay for the remanufactured product versus new We also consider the quality of returns and its impact on the optimal profits

We acknowledge that this research has certain limitations that could be relaxed for future research For instance, we did not consider any direct cost of collection for the used product returns and we did not include any decisions related to the acquisition cost of the returns as it was not the focus of this research However, this cost can be included in the cost of reusable parts in our models In addition, we determine the core acquisition price and the direct cost of collection in our second and third papers Another limitation in our models is that we assumed no capacity constraint for the CLSC members and the supplier Using capacitated inventory models jointly with pricing can be considered for future research In addition, we assumed negligible cost of coordination among members of the CLSC This may not be the case when the necessary infrastructure for communication and coordination is not in place, which makes the coordination more difficult and costly The cost of coordination could easily be added as a fixed cost to our models for those cases

CHAPTER 4

PAPER 2:

OPTIMAL CORE ACQUISITION AND PRODUCT PRICES FOR HYBRID MANUFACTURING/ REMANUFACTURING SYSTEMS

4.1. Model Description and Assumptions

In this chapter, we investigate the pricing decisions that a firm needs to make for new and remanufactured products and also for the end of life/use core acquisition Examples of such a firm are *Fenco* and *Cardone* in the aftermarket automotive parts industry These companies are involved in producing both new and remanufactured products We use simple economic models to show the impact of the substitutability between new and remanufactured products on all pricing decisions We also consider the cases in which the firm sells her products directly to the end-consumers, that is, we do not consider any retailer or other intermediary supply chain members in our models This helps us focus on the decisions that the company needs to make and the influencing factors in place, as we do not want any other factors like the retailer's ordering policy and so on have an impact on the real demand that should finally be satisfied at the end-consumer level Our models could also be applied to the cases in which the firm sells her products through a retailer where the firm and the retailer are fully coordinated and make pricing decisions as a joint unit

We assume that the supply of cores is a deterministic linear function of the acquisition price, P_a, paid to the end-consumers to return their used products The deterministic linear function takes the form $S(P_a) = \alpha + \beta P_a$, indicating that with an increase in the acquisition price, more cores will be expected to be collected, where α and β are positive coefficients We do not consider cases in which the consumers have to pay a fee to return their end-of-use or end-of-life products (which would require a negative value for P_a), assumed by some of the researchers in the literature (Bakal and Akcalı, 2006, Guide et al , 2003)

In addition, we look at the product as a single part that can be remanufactured and we do not consider multiple parts in our models After the cores are purchased, they have to go through a cleaning and inspection process We assume that this process costs c_I per unit for the company The percentage of the parts that conform to the quality specifications is a random variable Parts that are not remanufacturable or not remanufactured are salvaged (i e sold to a material recycler) The unit salvage price is s per unit The salvage price is not dependent on the quality of parts to be recycled, but is proportional to the recyclable material content in the parts which is the same for a single part We also consider the same remanufacturing cost per unit for all remanufacturable parts, denoted by c_r in our models Similar modeling assumptions are used in the literature for related practical examples (for example, see Bakal and Akcalı, 2006) The cost of manufacturing a unit of new product is denoted by c_n The demand quantities for new and remanufactured products are denoted by q_n and q_r, and prices by p_n and p_r respectively

The company sets the prices for new and remanufactured products to maximize his own profit as a monopoly, that is, we do not consider the competition with other companies to be able to focus on the main aspects of this research As it is observed in industries such as automotive parts, new and remanufactured products are distinguishable and each consumer's willingness-to-pay (WTP) or valuation for the remanufactured product can be defined as a fraction (δ) of their WTP for the new product In our models, we denote the market size by M The market size is an estimation of all potential consumers that could be reached by the firm We denote the maximum WTP of any consumer for any product (which obviously will be for the new product) by $\bar{\varphi}$ In addition, the consumers' WTP is distributed uniformly in the interval $[0, \bar{\varphi}]$ and, in any period, each consumer uses at most one unit Similar models are used in Ferguson and Toktay (2006) Equations (4 1) and (4 2) hold as the inverse demand functions Appendix A explains the derivation of these functions in more detail

$$P_n = \frac{\bar{\varphi}}{M}(M - q_n - \delta q_r) \tag{4 1}$$

$$P_r = \frac{\bar{\varphi}}{M}(\delta M - \delta q_n - \delta q_r) \tag{4 2}$$

The condition of the cores to be acquired has a probability distribution *g(r)*, with a cumulative distribution function *G(r)* Unlike what was used by Bakal and Akcalı (2006), we use a yield rate that is absolutely random and independent of the acquisition price This is due to the fact that there is no guarantee that increasing the core acquisition price will bring in cores with higher quality However, it is intuitive to assume that the number of returns will indeed increase with a higher acqisition price, because, the consumers will gain a higher incentive by returning their end of life/use products Table 4 1 presents the notation used in this paper

c_n	Unit cost of manufacturing new products
c_r	Unit cost of remanufacturing
c_i	Unit cost of cleaning and inspection

P_a	Unit acquisition price paid to the end-consumers
$S(P_a)$	Supply of returns
s	Unit salvage price
δ	The ratio of consumers' WTP for remanufactured products to their WTP for new products, $\delta \in (0,1)$
q_n	Demand quantity for the new product
q_r	Demand quantity for the remanufactured product
P_n	Price for the new product
P_r	Price for the remanufactured product
$\Pi(P_a, P_n, P_r)$	Profit function of the manufacturer/remanufacturer
$\overline{\varphi}$	Maximum consumers' valuation of the new product
R	Random variable denoting the yield rate
r	A realization of the random variable R
g(r), G(r)	p d f and c d f of random variable R

Table 4 1 Notation

The profit of the firm is a function of the prices that he should determine optimally The following model shows the firm's total profit considering new and remanufactured products and the core acquisition process

$$\text{Max } \Pi(P_a, P_n, P_r) = q_n(P_n - c_n) + q_r(P_r - s - c_r) + S(P_a)(s - P_a - c_l) \quad (4\ 3)$$

$$\text{Subject to } q_r \leq rS(P_a) \quad (4\ 4)$$

As mentioned earlier, our aim is to develop insight regarding the optimal prices for the new and remanufactured products and also for the core acquisition when the competition between the products is taken into account We would also like to know more about the impact of some of the model parameters, such as the consumers' perceptions of the remanufactured products versus new, the quality of returns (i e the yield rate), and the salvage value on the optimal prices and the profit of the firm For this purpose, we have to solve the maximization problem above to find the optimal prices and from there, the optimal quantities and profit of the firm In the next section, we describe our analysis and the results

4.2. Model Analysis and Results

We use a simultaneous optimization in which the optimal prices for the new and remanufactured products as well as the acquisition price for the core supply are found simultaneously As Bakal and Akcalı (2006) mention, there could be other ways to find the optimal prices For example, the firm could use a two-stage decision process in which she sets the core acquisition price first, and after collecting the cores and realization of the yield rate, she would set the price for the new and remanufactured products Since we are maximizing a concave function over a convex set of constraints, either the solution that satisfies the first order condition is optimal or the constraint is binding As a result, under each condition, we can find the optimal solution In addition, due to the fact that the closed form solutions are generally not available, we use a numerical analysis to investigate the impact of the model parameters on the optimal values As mentioned earlier, we assume that the core supply function can be represented by a linear function such as $S(P_a) = \alpha + \beta P_a$ Substituting this linear function into expression (4 3) and solving the first order conditions, we have

$$P_n^* = \frac{c_n + \overline{\varphi}}{2}$$

$$P_r^* = \frac{\delta\overline{\varphi} + c_r + s}{2}$$

$$P_a^* = \frac{\beta s - \beta c_l - \alpha}{2\beta} = \frac{s - c_l}{2} - \frac{\alpha}{2\beta}$$

$$q_n^* = M[\frac{1}{2} - \frac{c_n - c_r - s}{2\varphi(1-\delta)}]$$

$$q_r^* = M[\frac{\delta c_n - c_r - s}{2\varphi\delta(1-\delta)}]$$

This solution is feasible (and optimal) if it satisfies constraint (4 4), that is, $q_r \leq rS(P_a)$ If it does not satisfy the constraint, it means that the constraint is binding, that is $q_r = rS(P_a)$ If the constraint is binding, we find P_a in terms of q_r and thus in terms of P_n and P_r Then, we can solve for the optimal P_n and P_r, and consequently, the core acquisition price (P_a)

The numerical analysis of the model is performed using an algorithm The algorithm considers the constraint that we have in this model, that is $q_r \leq rS(P_a)$ We can re-arrange the constraint to $r \geq \frac{q_r}{S(P_a)}$ We need to find under what conditions the solution to the first order conditions satisfies the constraint For that, we solve the first order conditions and find the values of q_r^* and $S(P_a)^*$ corresponding to that solution Then, we find the value of $\frac{q_r^*}{S(P_a)^*}$ Let's assume that K denotes this value (which is associated with the solution for the first order conditions) As a result, whenever $r \geq K$, the solution to the first order conditions satisfies the constraint and it is the optimal solution to the model Note that both q_r^* and $S(P_a)^*$ above are found in terms of the model parameters Thus, K is defined by the model parameters This gives us the idea that we can first calculate K based on the model parameters, and then see under what condition, the solution to the first order conditions would satisfy the constraint and become the optimal solution to the model Considering this with the fact that r is a realization for the random variable R as the yield rate (which can fluctuate between 0 and 1), the following algorithm is used for the numerical analysis

Step 1) Find the solution to the first order conditions

Step 2) Calculate q_r^* and $S(P_a)^*$, and from there, $K = \frac{q_r^*}{S(P_a)^*}$

Step 3) If $K \leq 1 \Rightarrow$ go to step 4 Otherwise, go to Step 7

Step 4) If $r \geq K \Rightarrow$ the solution to the first order conditions is optimal and the expected profit for this part can be obtained by $\int_K^1 \Pi_1 g(r)dr$ in which Π_1 is the firm's profit for each realization of the yield rate in the range [K, 1) which is calculated using equation (4 3) and based on the optimal values

Step 5) If $r < K \Rightarrow$ the constraint should be binding As a result, the solution based on this will define Π_2 as the firm's profit for each realization of the yield rate in the range (0, K) which is used to calculate the expected profit for this case as $\int_0^K \Pi_2 g(r)dr$

Step 6) Calculate the total expected profit as $\int_K^1 \Pi_1 g(r)dr + \int_0^K \Pi_2 g(r)dr$

Step 7) If $K > 1 \Rightarrow r \ngeq K$ or in other words $r \leq 1 < K$ In this case, the constraint has to be binding for the optimal solution which results in Π_3 as the firm's profit for each realization of the yield rate in the range (0, 1) So, calculate the total expected profit as

$\int_0^1 \Pi_3 g(r)dr$

In addition, we calculate the expected values for all three prices (i e the ones for the new and remanufactured products as well as the core acquisition) and the optimal quantities in a similar way Thus, all the values used in our analysis in this paper refer to the expected optimal prices, quantities and profit of the firm and we investigate the impacts of some of the model parameters on these values

4 2 1 Parameter Setting for the Numerical Analysis

In our analysis, the original set of values that we use for c_n included {50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100} and the one for c_r included {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}, the one for c_l included {3, 5, 7, 9, 11, 13, 15, 17, 19}, and the one for $\overline{\varphi}$ included {100, 200, 300, 400, 500} But, since the results were consistent across different sets of these values, we present our results based on the specific parameter set as follows

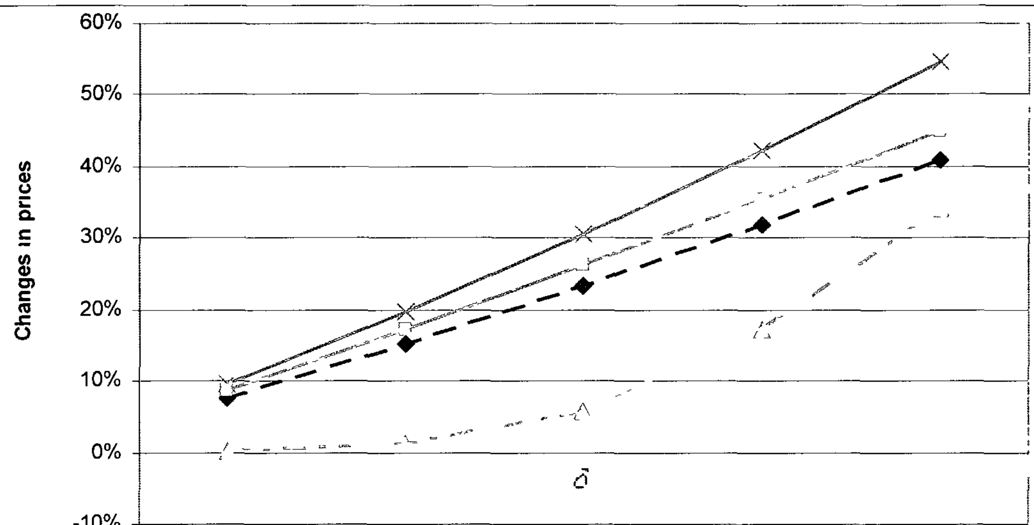
$c_n = 70$
$c_r = 15$
$c_l = 5$
$s = \{7, 9, 11, 13, 15, 17\}$
$\delta = \{0\ 6, 0\ 65, 0\ 7, 0\ 75, 0\ 8, 0\ 85\}$
$\overline{\varphi} = \{100, 500\}$
$M = 500$
$g(r) \sim N(\mu, \sigma^2)$
$\mu = \{0\ 3, 0\ 35, 0\ 4, 0\ 45, 0\ 5, 0\ 55\}$
$\sigma = \{0\ 04, 0\ 07, 0\ 1, 0\ 13, 0\ 16\}$

In practice, c_n can be estimated based on the total number of working hours required to manufacture one unit of the new product multiplied by an average wage per hour plus the material cost per unit of the new product In a similar way, the unit costs of remanufacturing (c_r) and inspection (c_i) can be estimated by the total number of working hours required per unit multiplied by the average wage per hour The number used for market size (*M*) does not have an impact on the results of this research That is, having a larger number for *M* simply magnifies the scale of the business, but the type of impact that the parameters under this study have on the optimal values would not be affected

To capture a wider range of products and consumers, we have assumed high and low values for the maximum consumer WTP for the new product, that is, $\overline{\varphi} = 500$ and $\overline{\varphi} = 100$ Considering the fact that the other parameters stay the same for each of these two values, the lower $\overline{\varphi}$ would mean that the cost factors constitute a higher percentage of the final price of the products, that is, the profit margin will be lower On the other hand, if the consumers are willing to pay significantly more for the products (i e $\overline{\varphi}$ is much higher), the profit margin will be also significantly higher In our numerical analysis, we consider these two cases as low and high profit margins cases and compare the differences between them Note that Bakal and Akcalı (2006) assume high and low salvage values in order to change the profit margins of the remanufactured products (or cores if they are not remanufactured) In addition, we consider a range for the consumers' relative WTP for the remanufactured product versus new (δ) which results in feasible solutions for all ranges of the whole parameter set that we use in this study Finally, we assume that the yield rate has a normal distribution and that the values for the mean and standard deviation are selected in a way that the probability of having a yield rate less than 0 or higher than 1 is negligible In the following sections, we discuss the impact of some of the model parameters such as the consumers' relative WTP for the remanufactured product (that captures the level of competition between new and remanufactured products), the yield rate, etc on the optimal prices and profits Note that in the figures of this section, P_n, P_r, and P_a represent the prices for the new and remanufactured product, and the core acquisition respectively In addition, q_n and q_r denote the quantities of the new and remanufactured products sold In addition, S(P_a) shows the number of cores that are acquired The terms "high" and "low" in parentheses stand for the high and low margin cases respectively

4 2 2 Impact of consumers' relative WTP for the remanufactured product (δ)

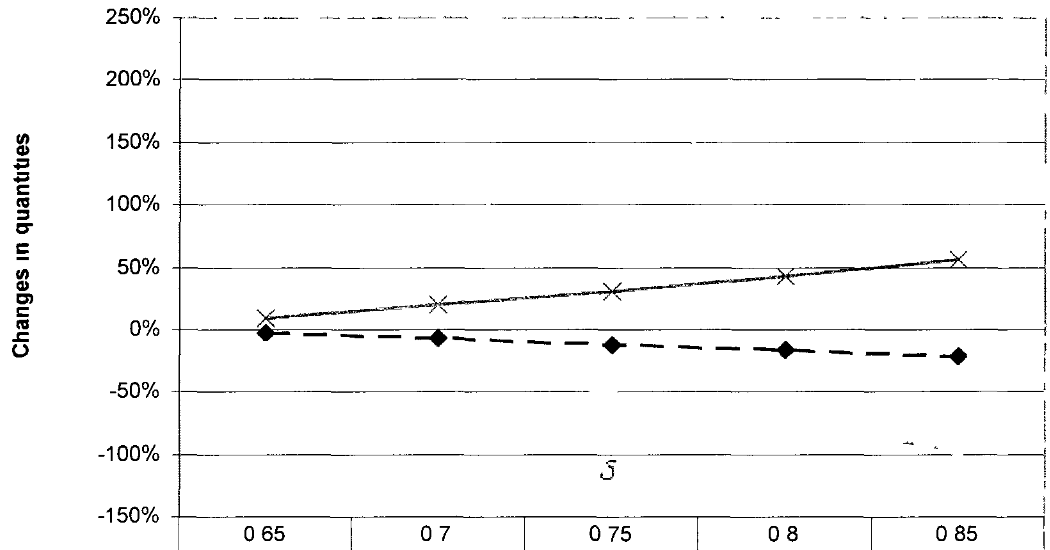
For this part of the analysis, we consider the case of $\delta = 0\ 6$ as the benchmark and vary δ parametrically in order to measure its impact on the optimal prices, quantities and the profit of the firm Our analysis shows that the price for the new product does not change if the new and remanufactured products are perceived as closer substitutes But, as we can see in Figure 4 1 and the table attached to it, when the new and remanufactured products are perceived as closer substitutes (i e having a higher δ), a higher price should be charged for the remanufactured product and a higher acquisition price should be paid for the core supply If we vary δ from 0 6 to 0 85, we observe that the percentage change in the price for the remanufactured product is consistent between the cases of high and low profit margins (i e it is around 41% – 45% in both cases) However, when the new and remanufactured products are very close substitutes, a lower increase in the acquisition price is needed for the case of high profit margin (i e around 34%) compared to the low profit margin case (i e around 54%)



δ	0 65	0 7	0 75	0 8	0 85
Change in P_r (high)	7 56%	15 23%	23 28%	31 91%	40 95%
Change in P_r (low)	8 57%	17 31%	26 25%	35 44%	44 93%
Change in Pa (high)	0 15%	1 14%	5 70%	17 31%	34 22%
Change in P_a (low)	9 64%	19 79%	30 57%	42 11%	54 60%

Figure 4 1 Impact of δ on the expected optimal prices

As for quantities of new and remanufactured products to be sold, according to Figure 4 2 and its data table, when these products are closer substitutes (i e when we have a higher δ), fewer number of new products will be sold compared to the case of having a lower δ In addition, the percentage of reduction in the sale of the new product is much higher for the case of low profit margins (i e around 97%) compared to high (i e around 21%) That is, when the profit margins are low, a higher reduction in the sales of the new product ıs expected when the new and remanufactured products are closer substıtutes As we show ın table 4 2, thıs can be explaıned by the relatıve profit margın of the new product versus the one for the remanufactured product In the case of havıng low margıns, when δ ıncreases, the profit margın for the remanufactured product ıncreases from 2 2 tımes the profit margın of the new product to 3 7 tımes The change for the case of hıgh margıns ıs from 0 7 to 1 02 As we see, the relatıve profit margın of the remanufactured product ıncreases a lot more ın the case of havıng low margıns Thıs ıs the reason that we see a sharper decrease ın the number of new products sold ın thıs case Addıtıonally, when δ ıs hıgher, ıt ıs optımal to ıncrease the sales of the remanufactured product Thıs need for addıtıonal sales of the remanufactured product calls for a hıgher number of cores to be collected Thus, payıng a hıgher core acquısıtıon prıce (mentıoned earlıer ın thıs sectıon) seems reasonable



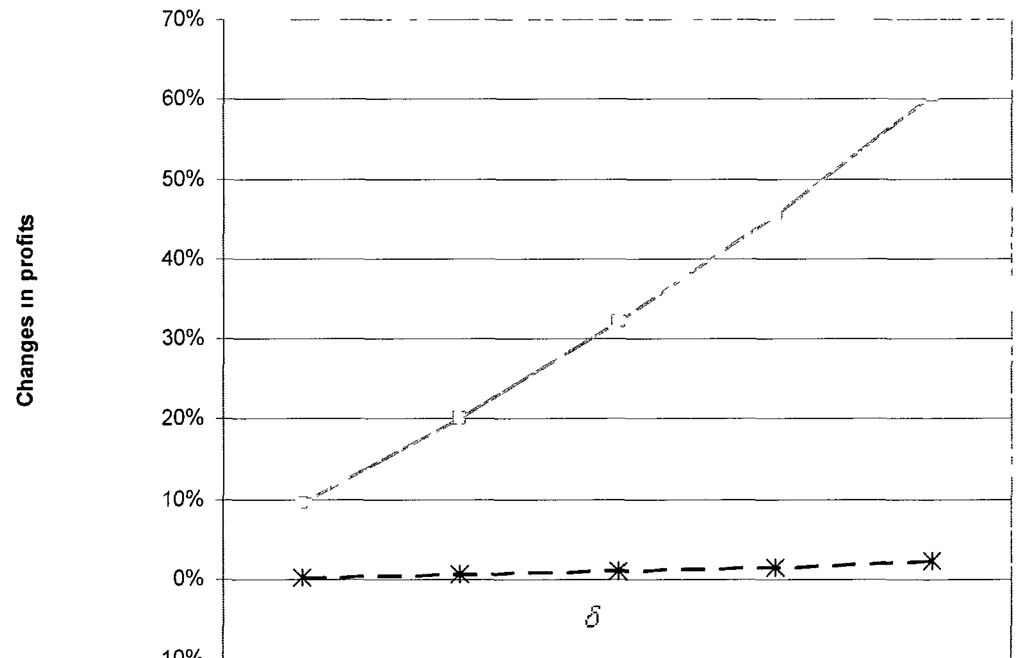
	0 65	0 7	0 75	0 8	0 85
Change in q_n (high)	-3 28%	-7 34%	-11 81%	-16 10%	-21 03%
Change in q_n (low)	-15 29%	-32 41%	-51 62%	-73 22%	-97 58%
Change in q_r (high)	41 43%	87 78%	133 32%	171 08%	211 54%
Change in q_r (low)	9 78%	20 11%	31 13%	42 98%	55 86%

Figure 4 2 Impact of δ on the expected optimal quantities

δ	Low margin				High margin		
	Margin for new (A)	Margin for reman (B)	B/A		Margin for new (C)	Margin for reman (D)	D/C
0 6	15 00	33 32	2 22		214 99	151 00	0 70
0 65	15 00	37 46	2 50		214 99	163 55	0 76
0 7	15 00	41 68	2 78		214 99	176 29	0 82
0 75	15 00	46 00	3 07		214 99	189 65	0 88
0 8	15 00	50 44	3 36		214 99	203 98	0 95
0 85	15 00	55 03	3 67		214 99	218 98	1 02

Table 4 2 Relative profit margins for new and remanufactured products in low and high margin cases with respect to δ

Regarding the expected profit of the firm, as Figure 4 3 and its table show, we can see that when the new and remanufactured products are closer substitutes, the firm can expect a higher profit This is because of the higher number of remanufactured products that can now be sold for a higher price and, as a result, a higher profit margin In addition, in the case of having low profit margins, the firm's expected profit is a lot more sensitive to the consumers' relative perception of the new and remanufactured products This can also be explained by the fact that the remanufactured product will have a much higher increase in the profit margin in the low margin case when δ goes up from 0 6 to 0 85, and this will result in a more dramatic increase in the profit



	0 65	0 7	0 75	0 8	0 85
Change in Expected profit (high)	0 20%	0 50%	0 92%	1 48%	2 19%
Change in Expected profit (low)	9 47%	20 10%	32 02%	45 42%	60 54%

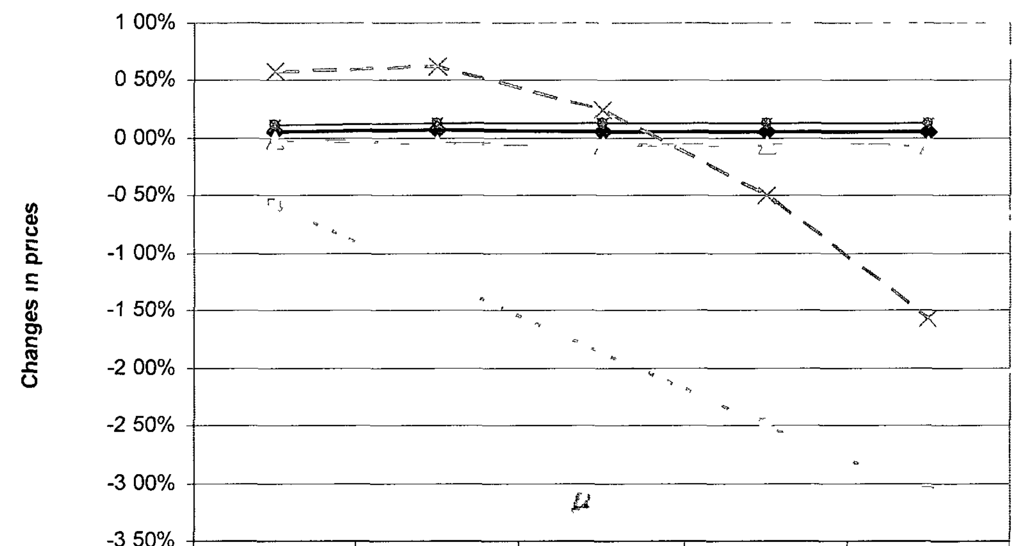
Figure 4 3 Impact of δ on the expected optimal profits of the firm

4 2 3 Impact of the yield rate and its uncertainty

As mentioned earlier, we assume that the yield rate has a Normal distribution with a mean of μ and a standard deviation of σ A higher μ implies that on average the cores have a higher quality A higher σ implies that the cores are not that consistent in terms of the quality and the uncertainty in the yield rate is higher Our analysis shows that the price of the new product is rather robust with respect to the average yield rate changes That is, in both high and low margin cases, it will increase around 0 11-0 13% and will stay almost constant at that level Figure 4 4 shows the percentages of change in all three prices (i e the ones for new and remanufactured products plus the core acquisition price) with respect to μ It is evident that when the average yield rate μ increases from 0 3 to 0 55, there is a slight decrease in the price of the remanufactured product in the low margin case (i e 3 1% reduction) But this price does not change significantly in the high margin case and it almost stays constant after a 0 06% increase We also observe that the core acquisition price decreases slightly in the low margin case (i e 1 56%) However, it stays almost constant after a slight reduction of 0 06% in the high margin case

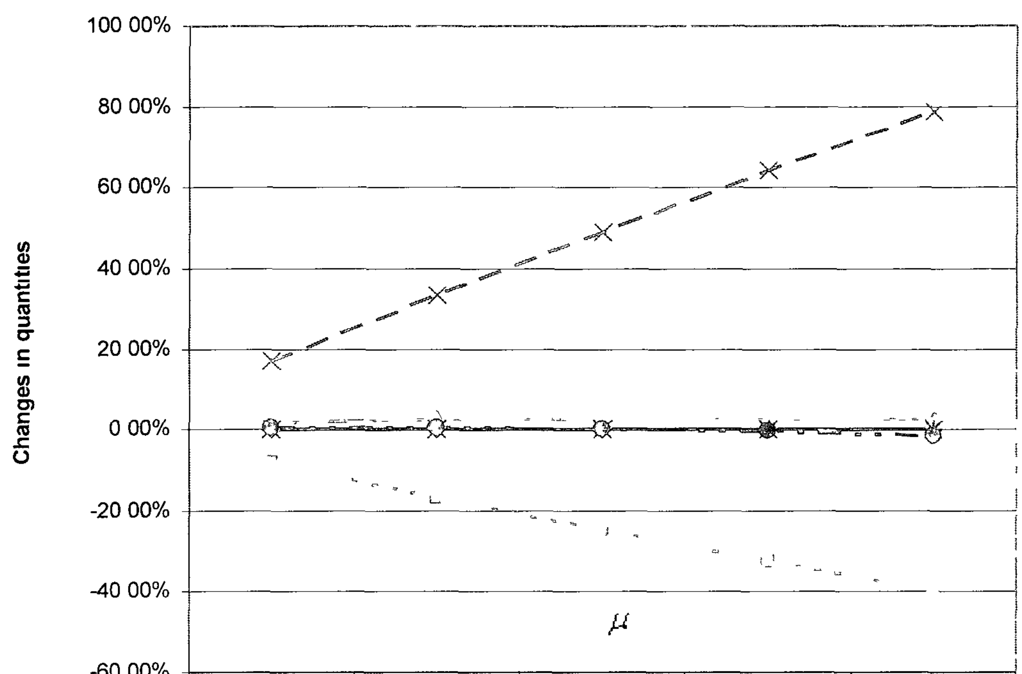
As we can see in Figure 4 5 for the percentages of change in the quantities, when the average yield rate increases from 0 3 to 0 55, it will be optimal to sell a higher number of the remanufactured product in both cases of high and low margin However, a much higher increase in the sale of the remanufactured product is expected for the case of low margin (i e 78 74%) compared to the one in the high margin case (i e 2 6%) And because the yield rate has increased, even a slight decrease of 1 51% (for the low margin case) and 0 06% (for the high margin case) in the core acquisition price will bring in enough remanufacturable cores

Furthermore, as Figure 4 6 shows, when the average yield rate increases from 0 3 to 0 55, in the high margin case, there will be a slight increase of 0 14% in the firm's profit and after that it almost stays constant This shows that when the profit margin is high, the change in the average yield rate will not affect the firm's profit that significantly However, in the low margin case, the firm can expect an increase of about 7% in his profit when the average yield rate increases from 0 3 to 0 55



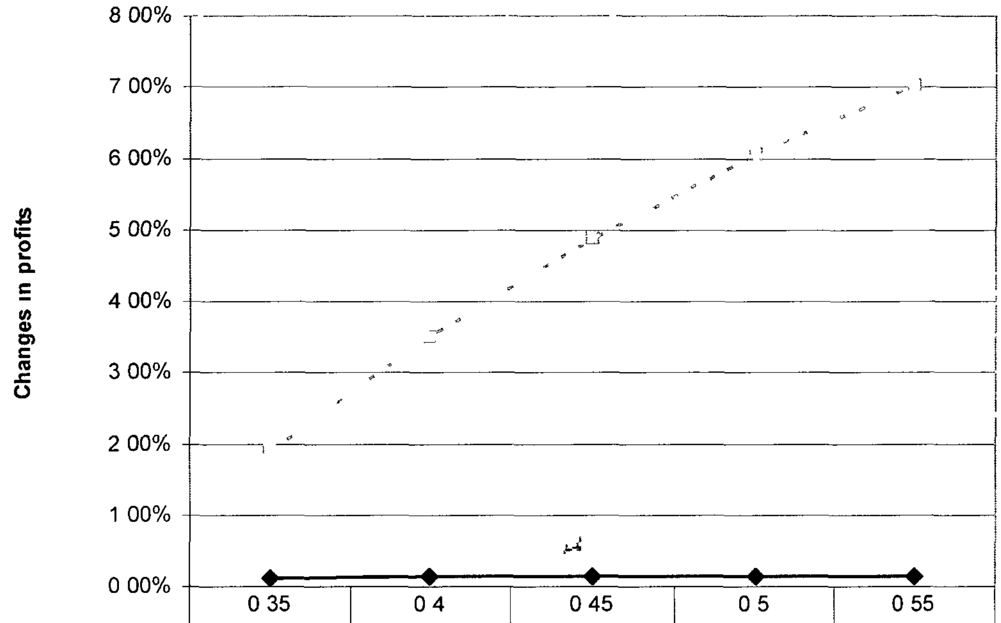
	0 35	0 4	0 45	0 5	0 55
Change in P_r (high)	0 06%	0 07%	0 06%	0 06%	0 06%
Change in P_r (low)	-0 58%	-1 23%	-1 88%	-2 50%	-3 10%
Change in P_a (high)	-0 01%	-0 04%	-0 05%	-0 06%	-0 06%
Change in P_a (low)	0 58%	0 63%	0 25%	-0 50%	-1 56%
Change in P_n	0 11%	0 13%	0 13%	0 14%	0 13%
Change in P_n (low)	0 11%	0 13%	0 13%	0 14%	0 13%

Figure 4 4 Impact of μ on the expected optimal prices



	0 35	0 4	0 45	0 5	0 55
Change in q_n (high)	0 01%	0 00%	-0 01%	-0 01%	-0 01%
Change in q_n (low)	-8 33%	-16 61%	-24 63%	-32 31%	-39 60%
Change in q_r (high)	1 79%	2 40%	2 56%	2 60%	2 60%
Change in q_r (low)	16 82%	33 25%	49 13%	64 33%	78 74%
Change in S(P_a) (high)	-0 01%	-0 04%	-0 05%	-0 05%	-0 05%
Change in S(P_a) (low)	0 55%	0 60%	0 24%	-0 48%	-1 51%

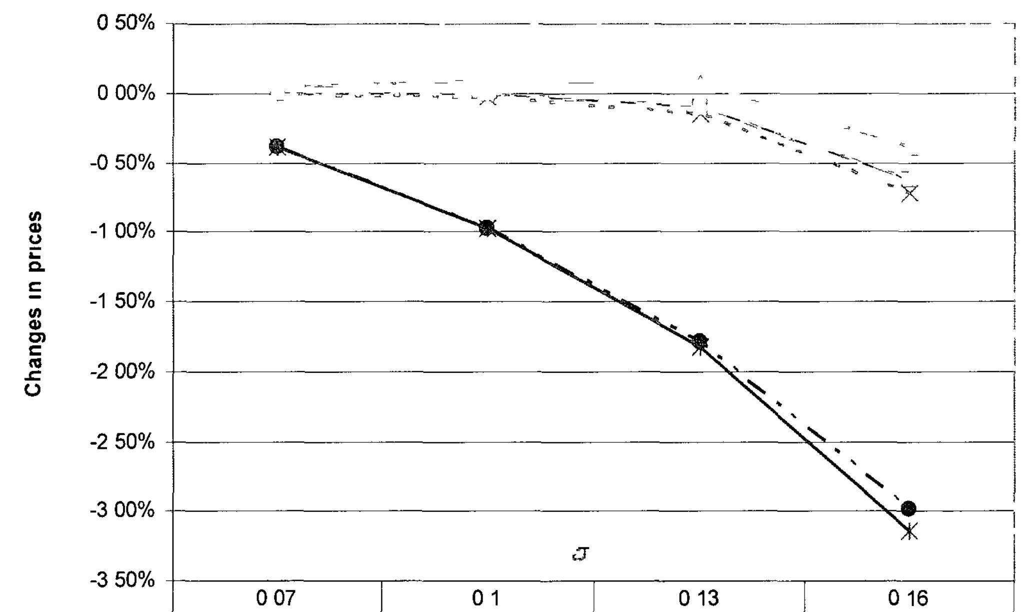
Figure 4 5 Impact of μ on the expected optimal quantities



	0 35	0 4	0 45	0 5	0 55
Change in Expected profit (high)	0 11%	0 13%	0 14%	0 14%	0 14%
Change in Expected profit (low)	1 89%	3 50%	4 88%	6 06%	7 04%

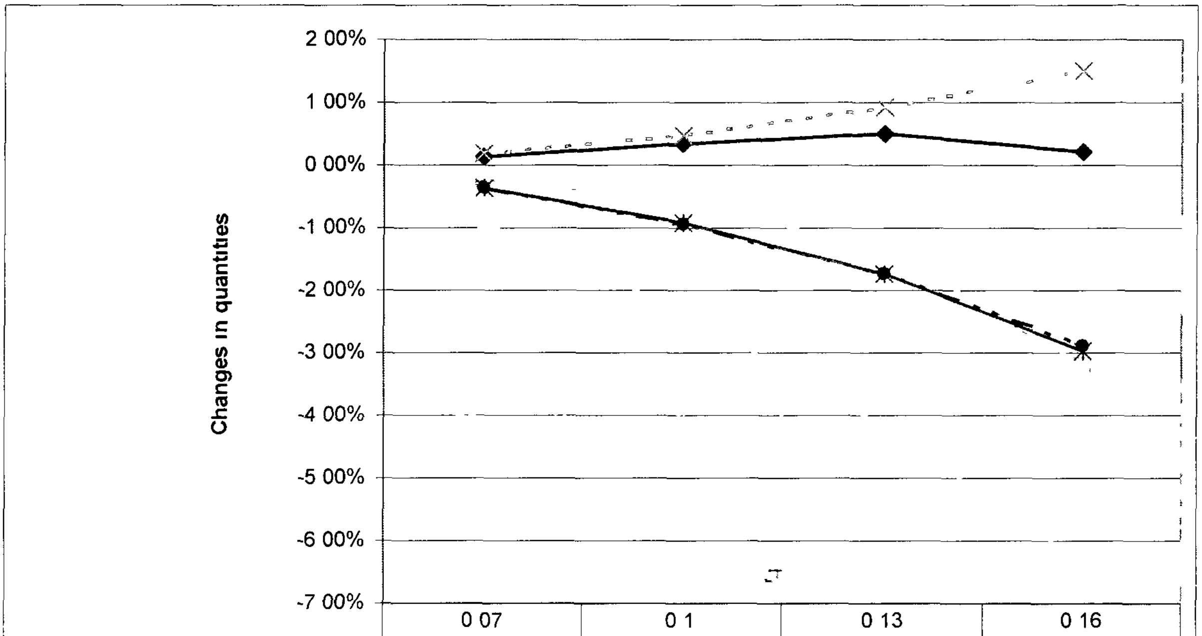
Figure 4 6 Impact of μ on the expected optimal profits of the firm

We now analyze the impact of the standard deviation of the yield rate (σ) on the expected optimal prices, quantities and profit of the firm We assume that μ is constant at 0 4 and we vary σ from 0 04 to 0 16 As we see in Figure 4 7, all prices decrease with respect to σ This includes the core acquisition prices in both high and low margin cases Reducing the acquisition price will decrease the number of core supply, but reducing the prices for new and remanufactured products should increase the demand for these products As we see in Figure 4 8, this is not necessarily the case for the new and remanufactured products For example, in the high margin case, when the price of the remanufactured product goes down, its sales quantity (or demand) decreases too Looking at the profit margins for the new and remanufactured products in table 4 3, we observe that when σ increases, the ratio of the profit margin of the remanufactured product to the one for the new product gets slightly larger, which means the remanufactured product becomes a little bit more attractive However, because in the high margin case, the new product has a higher profit margin than the remanufactured product, the firm will still be better off to sell more new products In addition, note that the core supply is decreasing which is in line with the decrease in the number of remanufactured products required to be produced



	0 07	0 1	0 13	0 16
Change in P_n (high)	0 00%	0 00%	-0 10%	-0 63%
Change in P_n (low)	0 00%	0 00%	-0 10%	-0 63%
Change in P_r (high)	0 04%	0 10%	0 07%	-0 38%
Change in P_r (low)	-0 01%	-0 02%	-0 15%	-0 72%
Change in P_a (high)	-0 38%	-0 98%	-1 82%	-3 14%
Change in P_a (low)	-0 38%	-0 97%	-1 79%	-2 99%

Figure 4 7 Impact of σ on the expected optimal prices



	0 07	0 1	0 13	0 16
Change in q_n (high)	0 14%	0 34%	0 50%	0 22%
Change in q_n (low)	-0 22%	-0 59%	-1 34%	-3 20%
Change in q_r (high)	-0 93%	-2 33%	-4 19%	-6 34%
Change in q_r (low)	0 18%	0 48%	0 91%	1 49%
Change in S(P_a) (high)	-0 36%	-0 93%	-1 73%	-2 99%
Change in S(P_a) (low)	-0 37%	-0 94%	-1 73%	-2 89%

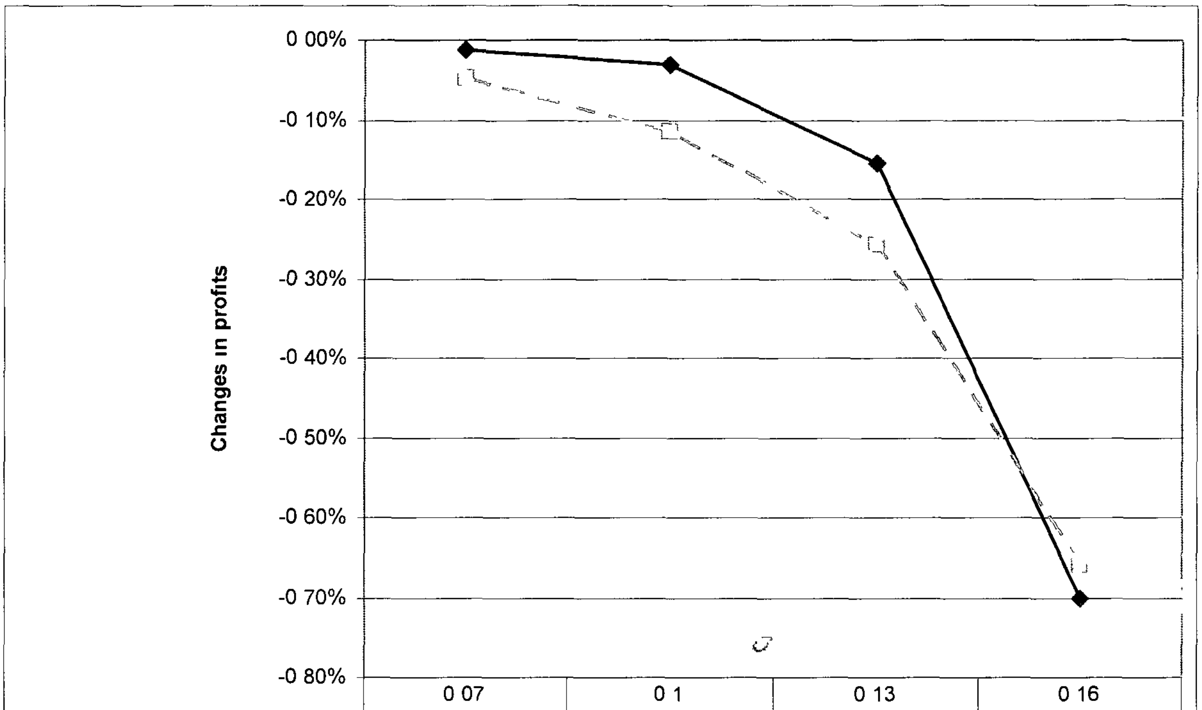
Figure 4 8 Impact of σ on the expected optimal quantities

σ	Low margin				High margin		
	Margin for new (A)	Margin for reman (B)	B/A		Margin for new (C)	Margin for reman (D)	D/C
0 04	15 000	42 032	2 802		215 000	176 173	0 819
0 07	15 000	42 028	2 802		215 000	176 251	0 820
0 1	14 997	42 018	2 802		214 991	176 361	0 820
0 13	14 911	41 947	2 813		214 701	176 313	0 821
0 16	14 465	41 621	2 877		213 205	175 444	0 823

Table 4 3 Relative profit margins for new and remanufactured products in low and high margin cases with respect to σ

Furthermore, in the low margin case, when the price of the new product goes down, its sales decrease too From Table 4 3, we observe that in the low margin case, the remanufactured is already the one with a higher profit margin and its relative profit margin versus the one for the new product gets even a bit larger with the increase in σ, which makes the remanufactured product a little more attractive to sell As a result, we can explain that the decrease in the sale of the new product is caused by the extra increase in the sales of the remanufactured product which is more desirable to sell Also, note that in this case, the sales number for the remanufactured product is going up while the number of core supply is decreasing This can be explained by the fact that the supply constraint is not binding for these cases and a decrease in the core supply will not necessarily translate to a decrease in the number of remanufactured products sold

Additionally, as we see in Figure 4 9, larger values of σ will result in a decrease in the firm's profit in both high and low margin cases, and the changes are similar in both of these two cases The reduction in the profit is intuitive due to the additional uncertainty introduced to the model by increasing the standard deviation of the yield rate

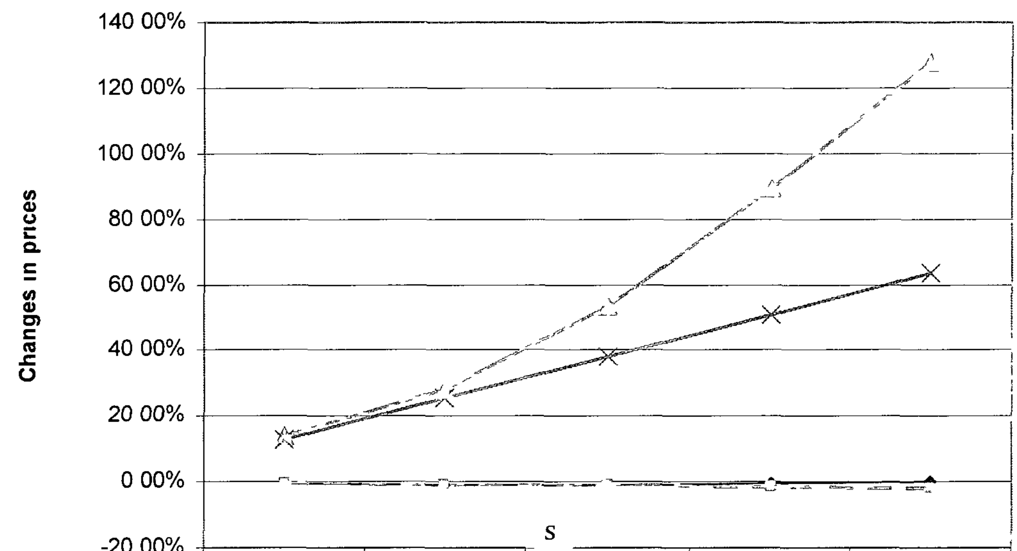


	0 07	0 1	0 13	0 16
Change in Expected profit (high)	-0 01%	-0 03%	-0 16%	-0 70%
Change in Expected profit (low)	-0 05%	-0 12%	-0 26%	-0 66%

Figure 4 9 Impact of σ on the expected optimal profits of the firm

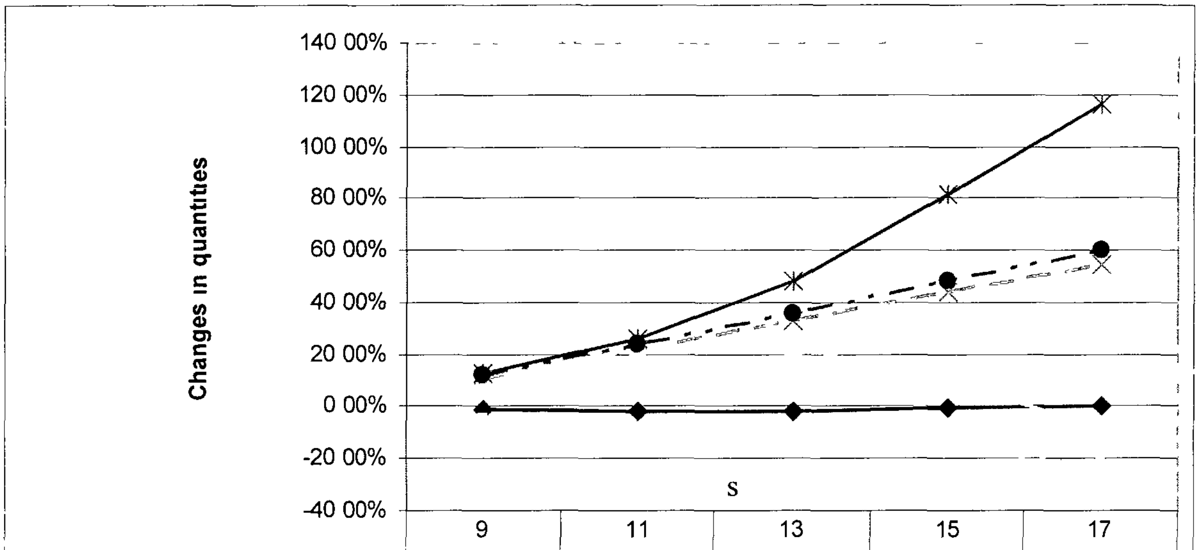
4 2 4 Impact of the salvage value (s)

To investigate the impact of the salvage value on the expected optimal values and profit of the firm, again, we assume two cases of high and low margin products by having $\overline{\varphi} = 500$ and $\overline{\varphi} = 100$ respectively We assume that $\delta = 0\ 6$, $\mu = 0\ 4$ and $\sigma = 0\ 1$ Our analysis shows that when we change the salvage value from \$7 to \$17 per unit of product, there will not be any significant changes in the price of the new product As we see in Figure 4 10, the price of the remanufactured product will slightly decrease (i e less than 2%) in the low margin case, and in the high margin case, it will decrease slightly when the salvage value ıncreases to a certaın poınt, and ıt wıll ıncrease slıghtly ıf the salvage value ıncreases any further However, the core acquısıtıon prıce ıncreases sıgnıficantly ın both hıgh (ı e 127 7%) and low (ı e 63 5%) margın cases Thıs ıs due to the fact that when the salvage value for the cores ıncreases, the potentıal loss from each extra core ın the ınventory decreases and ıt encourages the firm to acquıre more number of cores by ıncreasıng the core acquısıtıon prıce As Fıgure 4 11 shows, the firm wıll acquıre 116 4% hıgher number of cores ın the hıgh margın case and 59 9% hıgher ın the low margın case In addıtıon, as we see ın table 4 4, ın the low margın case, sınce the remanufactured product has a hıgher profit margın, the firm wıll be better off to ıncrease the sales of the remanufactured product and decrease the sales of the new product However, we find that ın the hıgh margın case, ıt wıll be optımal for the firm to ıncrease the sales of the remanufactured product ıf the salvage value ıncreases up to a certaın poınt, and ıf ıt ıncreases any further, ıt wıll be more profitable for the firm not to remanufacture the extra acquıred cores and ȷust to salvage them As a result, the quantıty of the remanufactured products decreases for those hıgher salvage values Furthermore, the firm can expect a hıgher profit when the salvage value ıncreases As we see ın Fıgure 4 12, the ıncrease ın the firm's expected profit ıs hıgher ın the low margın case (ı e 42 67%) compared to the hıgh margın (ı e 1 21%)



	9	11	13	15	17
Change in P_r (high)	-0 38%	-0 74%	-0 76%	-0 31%	0 26%
Change in P_r (low)	-0 38%	-0 77%	-1 15%	-1 54%	-1 92%
Change in P_a (high)	13 84%	28 22%	53 17%	89 31%	127 76%
Change in P_a (low)	12 71%	25 42%	38 13%	50 84%	63 55%

Figure 4 10 Impact of the salvage value on the expected optimal prices

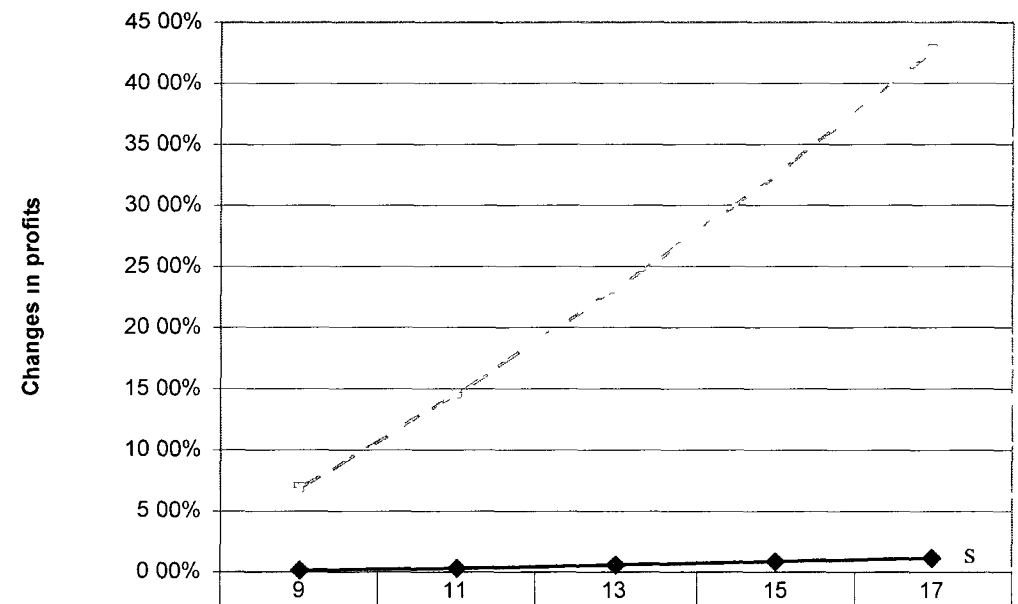


	9	11	13	15	17
Change in q_n (high)	-0 77%	-1 52%	-1 56%	-0 65%	0 54%
Change in q_n (low)	-4 44%	-8 89%	-13 33%	-17 78%	-22 22%
Change in q_r (high)	11 49%	22 55%	23 16%	9 60%	-8 01%
Change in q_r (low)	10 95%	21 89%	32 84%	43 78%	54 73%
Change in S(P_a) (high)	12 62%	25 73%	48 48%	81 42%	116 47%
Change in S(P_a) (low)	11 98%	23 97%	35 95%	47 93%	59 92%

Figure 4 11 Impact of the salvage value on the expected optimal quantities

Salvage Value	Low margin			High margin		
	Margin for new (A)	Margin for reman (B)	B/A	Margin for new (C)	Margin for reman (D)	D/C
7	15 00	34 27	2 2848	214 99	150 57	0 7004
9	15 00	34 08	2 2721	214 99	149 95	0 6975
11	15 00	33 89	2 2595	214 99	149 35	0 6947
13	15 00	33 70	2 2469	214 99	149 31	0 6945
15	15 00	33 51	2 2342	214 99	150 05	0 6979
17	15 00	33 32	2 2216	214 99	151 00	0 7024

Table 4 4 Relative profit margins for new and remanufactured products in low and high margin cases with respect to the salvage value (s)



	9	11	13	15	17
Change in Expected profit (high)	0 16%	0 33%	0 54%	0 83%	1 21%
Change in Expected profit (low)	6 89%	14 60%	23 13%	32 49%	42 67%

Figure 4 12 Impact of the salvage value on the expected optimal profits

4.3. Managerial Insight

Our analysis shows that when the new and remanufactured products are perceived as closer substitutes, higher prices should be set for the remanufactured product and the core acquisition and the firm can expect a higher total profit In other words, if the firm can increase the consumers' relative WTP for the remanufactured product, she can improve her total profit, especially in the case of having a low margin product For example, additional marketing efforts could be considered to promote the remanufactured product and improve the consumers' perception of it versus the new product Making some changes in the original product designs so that they do not lose their value too quickly (particularly by the time they get remanufactured) could also be considered In either case, the additional total profit must surpass the extra marketing and/or product design costs to keep it profitable

In addition, we find that when the firm sells low margin products, if the average yield rate increases, there will be no need to change the price of the new product But the firm should charge a lower price for the remanufactured product to increase its sales This increase in sales will automatically decrease the sales for the new product, but it will still increase the firm's profit More specifically, the firm can expect an increase of about 7% in her profit when the average yield rate increases from 0 30 to 0 55 If the firm sells high profit margin products, there will almost be no need to change the prices and sales quantities with an increase in the average yield rate In other words, in the case of high profit margin, the change in the firm's profit will be negligible with respect to the average yield rate We also find that when the uncertainty in the yield rate increases (i e through increasing its standard deviation), the firm's profit will decrease in both high and low profit margin cases Furthermore, we show that the firm's profit will increase when the salvage value increases We also find that in the low profit margin case, the firm's profit is more sensitive to the changes in the salvage value Moreover, when the firm sells high margin products, if the salvage value is higher than a certain amount, it will be more profitable for the firm to reduce the number of remanufactured products and instead take advantage of salvaging the acquired cores In the next section, we conclude this chapter and provide some future research directions

4.4. Conclusion and Directions for Future Research

This chapter considers a firm that collects end of use/life cores and produces both new and remanufactured products The firm determines the optimal prices for the new and remanufactured product as well the core acquisition price We capture the quality of returns by assuming a stochastic collection yield rate and find the optimal expected prices and quantities in addition to the firm's expected profit We show how the competition between new and remanufactured products affects the optimal expected prices and quantities as well as the firm's expected profit In addition, we investigate the impacts of the yield rate and the salvage value on the optimal values and provide managerial insight on how the firm should set the optimal prices under different circumstances (with respect to the parameters under study)

To extend the current research (in this chapter), a non-linear core supply function can be assumed In addition, the cases in which the yield rate can be affected by the acquisition price can be considered in the future Furthermore, different probability distribution functions can be used for the yield rate and their impacts on the optimal solutions can be investigated Finally, more complex models can be developed to capture joint pricing and inventory management decisions The latter is considered in chapter 5 where the firm jointly determines the prices and lot sizes for the differentiated new and remanufactured products

CHAPTER 5

PAPER 3:

THE IMPACT OF DEMAND UNCERTAINTY AND VERTICAL DIFFERENTIATION ON THE OPTIMAL REVERSE CHANNEL CHOICE

5.1. Model Description and Assumptions

In this chapter, we model a manufacturer who produces both new and remanufactured products that are distinguishable from each other and sells them to the end-consumers directly She is faced with two options for collecting the end of use/life products (i e cores) from consumers First, she can directly collect the cores from the consumers, inspect them and use the ones that are remanufacturable in remanufacturing This option is called the "Centralized Channel (C)" Second, she can let a third party take care of the collection and inspection This option is called the "Decentralized Channel (D)" Note that in the first option, the firm can have the cores at a lower cost per unit, but she will have to incur the cost of inspection and any associated costs related to the core acquisition including the possible loss from non-remanufacturable cores

We model each of these options and find the optimal prices and lot sizes for the new and remanufactured products as well as the optimal core acquisition price and profits in the supply chain Note that in the centralized channel, the firm's profit is equal to the total supply chain profit (i e the profit from selling new and remanufactured products) which includes both forward and reverse channels In the decentralized channel, the total profit is the sum of the firm's profit and the collector's profit We assume that the demands for new and remanufactured products are stochastic We define each demand as the summation of a deterministic part that is determined by prices and a random part which is independent of the prices A similar approach is used by Petruzzi and Dada (1999) to define the stochastic demand We also assume that the randomness in the demand for the new product is independent of the randomness in the demand for the remanufactured product

Following chapter 4, we assume that the supply of cores is a deterministic linear function of the acquisition price, P_a, paid to the end-consumers to return their used products The deterministic linear function takes the form $S(P_a) = \alpha + \beta P_a$, indicating that with an increase in the acquisition price, more cores will be expected to be collected, where α and β are positive coefficients In addition, we look at the product as a single part that can be remanufactured and we do not consider multiple parts in our models After the cores are purchased, they have to go through a cleaning and inspection process We assume that this process costs c_I per unit for the company The percentage of the parts that conform to the quality specifications is known as the yield rate and is denoted by r We assume an average yield rate level and later on we analyze how the changes in the yield rate would impact the optimal values Parts that are not remanufacturable or not remanufactured are salvaged (for example, sold to a material recycler) The unit salvage price is v per unit The salvage price is not dependent on the quality of parts to be recycled, but is proportional to the recyclable material content in the parts which is the same for a single part

The firm sets the prices for new and remanufactured products to maximize her own profit as a monopoly, that is, we do not consider the competition with other companies to be able to focus on the main aspects of this research as explained earlier As it is observed in industries such as automotive parts, computer systems and office equipment, the remanufactured products are dıstınguıshable from the new products and they are prıced lower than the new ones (Ferrer, 1997, Ayres et al , 1997, Ferrer and Swamınathan, 2010) In such cases, each consumer's wıllıngness-to-pay (WTP) or valuatıon for the remanufactured product can be defined as a fractıon (δ) of theır WTP for the new product (Ferguson and Toktay, 2006) In our models, we denote the market sıze by M and ıt ıs an estımatıon of all potentıal consumers that could be reached by the firm We also show the maxımum WTP of any consumer for any product (whıch obvıously wıll be for the new product) by $\overline{\varphi}$ In addıtıon, the consumers' WTP ıs dıstrıbuted unıformly ın the ınterval $[0,\overline{\varphi}]$ and each consumer uses at most one unıt Sımılar models are used ın Ferguson and Toktay (2006) Table 5 1 summarızes the notatıon used ın thıs chapter

q_n	Number of new products to be stocked
q_r	Number of remanufactured products to be stocked
q_a	Number of cores to be acquıred (ı e supply of returns)
P_n	Prıce for the new product
P_r	Prıce for the remanufactured product
P_a	Unıt acquısıtıon prıce paıd to the end-consumers
$D_i(P_n, P_r, \varepsilon_i) = D_i$	Demand for product type i (i = n (new), r (remanufactured))
$y_i(P_n, P_r) = y_i$	The portıon of the demand for product type i that changes wıth prıces - for the new and remanufactured products (i = n, r)
ε_i	A random varıable that captures randomness for the demand of the product type i and changes ın the range $[A_i, B_i]$, with i = n, r
$\Pi(P_n, P_r, P_a, q_n, q_r) = \Pi$	Profit functıon of the manufacturer/remanufacturer
c_n	Unıt cost of manufacturıng the new product
c_r	Unıt cost of remanufacturıng and stockıng ıt for the perıod
c_I	Unıt cost of cleanıng and ınspectıon
h_i	Unıt dısposal cost / salvage prıce for product type i (i = n, r)
s_i	Unıt shortage cost for product type i (i = n, r)
v	Unıt salvage value for the cores that are not remanufactured

δ	The ratio of consumers' WTP for remanufactured products to their WTP for new products, $\delta \in (0,1)$
$\overline{\varphi}$	Maximum consumers' valuation of the new product
r	Yield rate
$f_i(\varepsilon_i), F_i(\varepsilon_i)$	p d f and c d f of the random variable ε_i $(i = n, r)$

Table 5 1 Notation

The profit that each product brings in for the firm (without consideration of the core acquisition and any profit or loss associated with it) is defined as follows

$$\text{Profit from the new product} = \Pi_n = \begin{cases} P_n D_n - c_n q_n - h_n(q_n - D_n), & D_n \le q_n \\ P_n D_n - c_n q_n - s_n(D_n - q_n), & D_n > q_n \end{cases} \quad (5\ 1)$$

$$\text{Profit from the remanufactured product} = \Pi_r = \begin{cases} P_r D_r - c_r q_r - h_r(q_r - D_r), & D_r \le q_r \\ P_r D_r - c_r q_r - s_r(D_r - q_r), & D_r > q_r \end{cases} \quad (5\ 2)$$

We define $D_n(P_n, P_r, \varepsilon_n) = y_n(P_n, P_r) + \varepsilon_n$ and $D_r(P_n, P_r, \varepsilon_r) = y_r(P_n, P_r) + \varepsilon_r$ or in a shorter form $D_n = y_n + \varepsilon_n$ and $D_r = y_r + \varepsilon_r$ As we explain in Appendix A, y_n and y_r, which are the deterministic parts of the demand for the new and remanufactured products, can be determined in terms of the prices for the new and remanufactured products That is, $y_n = M\left[1 - \frac{P_n - P_r}{\overline{\varphi}(1-\delta)}\right]$ and $y_r = M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right]$ Random variable ε_i captures the randomness for the demand of the product type i and changes in the range $[A_i, B_i]$, with i = n, r denoting the new and remanufactured products respectively We also define $z_n = q_n - y_n$ and $z_r = q_r - y_r$, which is consistent with Ernst (1970), Thowsen (1975) and Petruzzi and Dada (1999) Substituting these expressions in (1) and (2), we have

$$\Pi_n = \begin{cases} P_n(y_n+\varepsilon_n) - c_n(y_n+z_n) - h_n(z_n-\varepsilon_n), & \varepsilon_n \le z_n \\ P_n(y_n+\varepsilon_n) - c_n(y_n+z_n) - s_n(\varepsilon_n-z_n), & \varepsilon_n > z_n \end{cases} \quad (5\ 3)$$

$$\Pi_r = \begin{cases} P_r(y_r+\varepsilon_r) - c_r(y_r+z_r) - h_r(z_r-\varepsilon_r), & \varepsilon_r \le z_r \\ P_r(y_r+\varepsilon_r) - c_r(y_r+z_r) - s_r(\varepsilon_r-z_r), & \varepsilon_r > z_r \end{cases} \quad (5\ 4)$$

This transformation of variables provides an alternative interpretation of the stocking decision That is, if the choice of z_i is larger than the realized value of ε_i, then leftovers occur for product type i If the choice of z_i is smaller than the realized value of ε_i, then shortages occur for product type i The corresponding optimal stocking levels and pricing policy are to stock $q_n^* = y_n(P_n^*, P_r^*) + z_n^*$ units of the new product (to sell at the unit price P_n^*) and $q_r^* = y_r(P_n^*, P_r^*) + z_r^*$ units of the remanufactured product (to sell at the unit price P_i^*), where P_n^*, P_r^*, z_n^* and z_r^* maximize the expected profit of the firm In the next section, we analyze the option of the Centralized Channel

5.2. Centralized Channel Models

In this section, we assume that the firm collects the cores herself, and thus, we can define her total profit as follows

$$\Pi(P_n, P_r, P_a, z_n, z_r) = \Pi_n + \Pi_r + (q_a - q_r)v - q_a(P_a + c_l) \quad (5\ 5)$$

Subject to $q_r \le r q_a$ (5 6)

As we explained earlier in this chapter, we define the core supply as a linear function of the acquisition price That is, $q_a = \alpha + \beta P_a$ Using a linear function for the core supply is reasonable for cases in which there are enough cores available to be acquired We aim to find the optimal values of P_n^*, P_r^*, P_a^*, z_n^* and z_r^* such that they maximize the expected profit of the firm To do this, first, we find the solution to the first order conditions If this solution satisfies the constraint, it will be the optimal solution If it does not satisfy the constraint, the optimal solution will be found by assuming that the constraint is binding and solving for the optimal values based on that Based on expressions (5 3), (5 4) and (5 5) above, the firm's expected profit is

$$E(\Pi) = \int_{A_n}^{z_n} [P_n(y_n + u_n) - h_n(z_n - u_n)] f_n(u_n) du_n + \int_{z_n}^{B_n} [P_n(y_n + z_n) - s_n(u_n - z_n)] f_n(u_n) du_n +$$

$$+ \int_{A_r}^{z_r} [P_r(y_r + u_r) - h_r(z_r - u_r)] f_r(u_r) du_r + \int_{z_r}^{B_r} [P_r(y_r + z_r) - s_r(u_r - z_r)] f_r(u_r) du_r -$$

$$- c_n(y_n + z_n) - c_r(y_r + z_r) + (q_a - y_r - z_r)v - q_a(P_a + c_I) \qquad (5\ 7)$$

Defining $\Lambda(z_i) = \int_{A_i}^{z_i} (z_i - u_i) f_i(u_i) du_i$ and $\Theta(z_i) = \int_{z_i}^{B_i} (u_i - z_i) f_i(u_i) du_i$ for $i = n, r$, similar to Petruzzi and Dada (1999), we can re-write expression (5 7) as follows

$$E(\Pi) = \psi_n - L_n + \psi_r - L_r + \Delta \qquad (5\ 8)$$

where

$$\psi_i = (P_i - c_i)(y_i + \mu_i), \qquad (5\ 9)$$

$$L_i = (c_i + h_i)\Lambda(z_i) + (P_i + s_i - c_i)\Theta(z_i) \quad \text{for } i = n, r \qquad (5\ 10)$$

and

$$\Delta = (q_a - y_r - z_r)v - q_a(P_a + c_l)$$

$$= (\alpha + \beta P_a - y_r - z_r)v - (\alpha + \beta P_a)(P_a + c_l) \quad (5\ 11)$$

Expression (5 9) represents the riskless profit functions (Mills, 1959) related to the new and remanufactured products (i e without considering the core acquisition), which are the profits from the new and remanufactured products for a given set of prices in the equivalent problem in which ε_n and ε_r are replaced by their mean values of μ_n and μ_r respectively Expression (5 10) shows the loss functions (Silver and Peterson, 1985), which assess an overage cost $(c_i + h_i)$ for each of the $\Lambda(z_i)$ expected leftovers when z_i is too high, and an underage cost $(P_i + s_i - c_i)$ for each of the $\Theta(z_i)$ expected shortages when z_i is too low Expression (5 11) captures the profit or loss that the firm faces as a result of acquiring q_a units of cores (which is the salvage value for the cores that are not remanufactured minus the collection and inspection costs)

The objective is to maximize the expected profit of the firm by finding the optimal prices and lot sizes for the new and remanufactured products We have

$$E(\Pi) = (P_n - c_n)(M\left[1 - \frac{P_n - P_r}{\overline{\varphi}(1-\delta)}\right] + \mu_n) - (c_n + h_n)\Lambda_n(z_n) - (P_n + s_n - c_n)\Theta_n(z_n)$$
$$+ (P_r - c_r)(M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \mu_r) - (c_r + h_r)\Lambda_r(z_r) - (P_r + s_r - c_r)\Theta_r(z_r)$$
$$+ (\alpha + \beta P_a - M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] - z_r)v - (\alpha + \beta P_a)(P_a + c_l)$$

In Appendix D, we show that the Hessian matrix for $E(\Pi)$ is negative semidefinite and thus, the expected profit function for the firm is strictly concave with respect to the model variables and the solution to the first order conditions maximizes the expected profit In addition, the expected profit is concave in P_n, P_r and P_a for any given set of z_n and z_r (see Appendix D for details) Thus, it is possible to reduce this optimization problem to one over z_n and z_r Assuming that the solution to the first order conditions satisfies the model constraint (i e constraint (5 6)), first, we find the optimal values of P_n^*, P_r^* and P_a^* for each set z_n and z_r Then, we substitute these optimal values in $E(\Pi)$ and find the optimal values of z_n^* and z_r^* that maximize $E(\Pi)$ As a result, we have

$$\frac{\partial E(\Pi)}{\partial P_n} = 0 \Rightarrow -2MP_n + 2MP_r + Mc_n - Mc_r - Mv + [M + \mu_n - \Theta_n(z_n)]\overline{\varphi}(1-\delta) = 0 \quad (5\ 12)$$

$$\frac{\partial E(\Pi)}{\partial P_r} = 0 \Rightarrow 2\delta MP_n - 2MP_r - \delta Mc_n + Mc_r + Mv + [\mu_r - \Theta_r(z_r)]\overline{\varphi}\delta(1-\delta) = 0 \quad (5\ 13)$$

Solving equations (12) and (13) for P_n and P_r, we have

$$P_n^* = \frac{c_n + \overline{\varphi}}{2} + \frac{(\mu_n + \delta\mu_r)\overline{\varphi}}{2M} - \frac{[\Theta_n(z_n) + \delta\Theta_r(z_r)]\overline{\varphi}}{2M} \quad (5\ 14)$$

$$P_r^* = \frac{c_r + v + \delta\overline{\varphi}}{2} + \frac{(\mu_n + \mu_r)\delta\overline{\varphi}}{2M} - \frac{[\Theta_n(z_n) + \Theta_r(z_r)]\delta\overline{\varphi}}{2M} \quad (5\ 15)$$

In addition, to find the optimal acquisition price, we have

$$\frac{\partial E(\Pi)}{\partial P_a} = \beta v - \alpha - \beta c_l - 2\beta P_a = 0 \Rightarrow$$

$$P_a^* = \frac{\beta v - \alpha - \beta c_l}{2\beta} = \frac{v - c_l}{2} - \frac{\alpha}{2\beta} \tag{5 16}$$

Note that when constraint (5 6) is not binding, the acquisition price is determined independently from z_n and z_r In addition, if the firm were riskless, then $\Theta_n(z_n) = 0$ and $\Theta_r(z_r) = 0$ resulting in the optimal riskless prices $P_n^0 = \frac{c_n + \overline{\varphi}}{2} + \frac{(\mu_n + \delta\mu_r)\overline{\varphi}}{2M}$ and $P_r^0 = \frac{c_r + v + \delta\overline{\varphi}}{2} + \frac{(\mu_n + \mu_r)\delta\overline{\varphi}}{2M}$ Lemma 1 and Lemma 2 summarize these results

Lemma 1 *For a fixed set of z_n and z_r, the optimal prices for new and remanufactured products are determined uniquely as functions of z_n and z_r*

$$P_n^* = P_n(z_n, z_r) = P_n^0 - \frac{[\Theta_n(z_n) + \delta\Theta_r(z_r)]\overline{\varphi}}{2M} \quad \text{and}$$

$$P_r^* = P_r(z_n, z_r) = P_r^0 - \frac{[\Theta_n(z_n) + \Theta_r(z_r)]\delta\overline{\varphi}}{2M}$$

Lemma 2 *The optimal acquisition price does not depend on z_n and z_r*

$$P_a^* = \frac{v - c_l}{2} - \frac{\alpha}{2\beta}$$

Since both $\Theta_n(z_n)$ and $\Theta_r(z_r)$ are nonnegative, $P_n^* \le P_n^0$ and $P_r^* \le P_r^0$ That is, when there is uncertainty in the demands, the firm will set lower prices than the riskless ones Now, we substitute P_n^* and P_r^* in $E(\Pi)$, and then maximize $E(\Pi)$ with respect to

z_n and z_r Theorem 1 shows that depending on the parameters of the problem, $E(\Pi)$ might have different points that satisfy the first order conditions

Theorem 1 The single period optimal stocking and pricing policy is to stock $q_n^* = y_n(P_n^*, P_r^*) + z_n^*$ units of new products to be sold at the unit price P_n^* and $q_r^* = y_r(P_n^*, P_r^*) + z_r^*$ units of remanufactured products to be sold at the unit price P_r^*, and to pay the unit price P_a^* for each core acquired, where P_n^*, P_r^* and P_a^* are specified by Lemma 1, and z_n^* and z_r^* are determined as follows

(*a*) Assuming that $F_n()$ is a distribution function that either satisfies $\frac{\partial f_n(z_n)}{\partial z_n} > -3 f_n(z_n) r(z_n)$ or $\frac{\partial f_n(z_n)}{\partial z_n} < -3 f_n(z_n) r(z_n)$ (i e only one could happen for different values of z_n in the region $[A_n, B_n]$), and $F_r()$ is a distribution function that either satisfies $\frac{\partial f_r(z_r)}{\partial z_r} > -f_r(z_r) r(z_r)$ or $\frac{\partial f_r(z_r)}{\partial z_r} < -f_r(z_r) r(z_r)$ where $r() = \frac{f()}{1-F()}$ is defined as the hazard rate, when sufficient conditions

$$\frac{\overline{\varphi}}{2M}(4\mu_n - 2A_n - \delta A_r) + \frac{\delta}{2(1-\delta)} c_n + s_n \geq \frac{1}{2(1-\delta)}(c_r + v)$$

and

$$\frac{\delta \overline{\varphi}}{2M}\left[M + A_n + A_r\right] + s_r \geq \frac{1}{2}(c_r + v)$$

are met, then z_n^* is the largest z_n in the region $[A_n, B_n]$ that satisfies $\frac{\partial E(\Pi)}{\partial z_n} = 0$, and z_r^* is the largest z_r in the region $[A_r, B_r]$ that satisfies $\frac{\partial E(\Pi)}{\partial z_r} = 0$, and they are unique

(*b*) If $F()$ is an arbitrary distribution function that does not fit into the description in (*a*), then an exhaustive search over all values of z_n and z_r in the regions $[A_n,\ B_n]$ and $[A_r,\ B_r]$ respectively will determine z_n^* and z_r^*

Proof See Appendix E

The above lemmas and the theorem hold for the case in which the solution to the first order conditions satisfies constraint (5 6) When the solution to the first order conditions does not satisfy the constraint, the optimal solution can be found where the constraint is binding So, we have

$$q_r = rq_a \Rightarrow M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r = r(\alpha + \beta P_a) \Rightarrow$$

$$P_a^* = \frac{M}{r\beta}\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \frac{z_r - r\alpha}{r\beta} \qquad (5\ 17)$$

Substituting P_a in $E(\Pi)$, we have

$$\Rightarrow E(\Pi) = (P_n - c_n)(M\left[1 - \frac{P_n - P_r}{\overline{\varphi}(1-\delta)}\right] + \mu_n) - (c_n + h_n)\Lambda_n(z_n) - (P_n + s_n - c_n)\Theta_n(z_n)$$
$$+ (P_r - c_r)(M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \mu_r) - (c_r + h_r)\Lambda_r(z_r) - (P_r + s_r - c_r)\Theta_r(z_r) \quad (5\ 18)$$
$$+ (M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r)(\frac{1-r}{r})v - \frac{1}{r}\left[M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r\right]\left[\frac{M}{r\beta}\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \frac{z_r - r\alpha}{r\beta} + c_l\right]$$

In Appendix F, we show that the Hessian matrix for $E(\Pi)$ is negative semidefinite and thus, the expected profit function for the firm is strictly concave and the solution to the first order conditions maximizes the expected profit As a result, we can use a similar procedure as we used for the case of non-binding constraint to find the optimal solution First, we need to find the optimal values of P_n^* and P_r^* for each set z_n and z_r Then, we substitute these optimal values in $E(\Pi)$ and find the optimal values of z_n^* and z_r^* that maximize $E(\Pi)$ To find the optimal values of P_n^* and P_r^*, we have to solve the first order conditions with respect to P_n and P_r After some simplifications we have

$$P_n^* = \frac{(\overline{\varphi} + c_n)r^2}{2} + \frac{1}{2\overline{\varphi}\delta(1-\delta)^2\beta}[\beta v - \beta c_l + \alpha][1 - \overline{\varphi}\delta(1-\delta)]r + \frac{\overline{\varphi}r^2}{2M}[\mu_n + \delta\mu_r]$$
$$- \frac{\overline{\varphi}r^2}{2M}[\Theta_n(z_n) + \delta\Theta_r(z_r)] \quad (5\ 19)$$

If we define

$P_n^0 = \frac{(\overline{\varphi} + c_n)r^2}{2} + \frac{1}{2\overline{\varphi}\delta(1-\delta)^2\beta}[\beta v - \beta c_l + \alpha][1 - \overline{\varphi}\delta(1-\delta)]r + \frac{\overline{\varphi}r^2}{2M}[\mu_n + \delta\mu_r]$ as the optimal riskless prices, we can re-write P_n^* as follows

$$P_n^* = P_n^0 - \frac{\bar{\varphi} r^2}{2M}\left[\Theta_n(z_n) + \delta\Theta_r(z_r)\right] \tag{5 20}$$

$$\Rightarrow P_r = \frac{1}{2(1-\delta)M\left[\bar{\varphi}\delta(1-\delta)r^2\beta + M\right]}\left\{\begin{matrix} 2\delta(1-\delta)M^2 P_n^0 + \bar{\varphi}^2\delta^2(1-\delta)^2 r^2\beta\left[M + \mu_n + \mu_r\right] \\ + \bar{\varphi}\delta(1-\delta)^2 r^2 \beta M(c_r + v) - (\beta v - \beta c_l + \alpha)\left[\bar{\varphi}(1-\delta) - 1\right]\delta r M \end{matrix}\right\}$$
$$+ \frac{\bar{\varphi}\delta}{2}\left[(1-r^2)\Theta_n(z_n) + (1-\delta r^2)\Theta_r(z_r)\right] + \frac{\bar{\varphi}\delta(1-\delta)}{\left[\bar{\varphi}\delta(1-\delta)r^2\beta + M\right]} z_r$$

If we define

$$P_r^0 = \frac{1}{2(1-\delta)M\left[\bar{\varphi}\delta(1-\delta)r^2\beta + M\right]}\left\{\begin{matrix} 2\delta(1-\delta)M^2 P_n^0 + \bar{\varphi}^2\delta^2(1-\delta)^2 r^2\beta\left[M + \mu_n + \mu_r\right] \\ + \bar{\varphi}\delta(1-\delta)^2 r^2 \beta M(c_r + v) - (\beta v - \beta c_l + \alpha)\left[\bar{\varphi}(1-\delta) - 1\right]\delta r M \end{matrix}\right\}$$

as the optimal riskless prices, we can re-write P_r^* as follows

$$P_r^* = P_r^0 + \frac{\bar{\varphi}\delta}{2}\left[(1-r^2)\Theta_n(z_n) + (1-\delta r^2)\Theta_r(z_r)\right] + \frac{\bar{\varphi}\delta(1-\delta)}{\left[\bar{\varphi}\delta(1-\delta)r^2\beta + M\right]} z_r \tag{5 21}$$

We observe that if $z_r \geq 0$, then $P_r^* \geq P_r^0$ This shows that in the case of having a binding constraint, we could have $P_r^* \geq P_r^0$ which was not the case for the case of non-binding constraint That is, the firm sets a higher optimal price for the remanufactured product compared to a riskless firm This is reasonable due to the fact that when the constraint is binding, it shows that the number of available cores is limited (i e all remanufacturable cores are to be remanufactured) or to acquire more cores, a higher acquisition price needs to be paid to the consumers (which may not be ideal for the firm) As a result, the firm sets a higher price for the remanufactured product to reduce its demand (which could reduce possible shortage costs or additional core acquisition costs if the shortage is to be avoided) Depending on the model parameters and functions

$\Theta_n(z_n)$ and $\Theta_r(z_r)$, for some negative values of z_r we could have $P_r^* \le P_r^0$ However, the optimal price charged for the new product is lower than the one charged by a riskless firm (i e $P_n^* \le P_n^0$)

To find the final optimal values in terms of z_n and z_r, we need to substitute the price values in (20) and (21) in equation (18) for the expected profit of the firm and solve $\frac{\partial E(\Pi)}{\partial z_n} = 0$ and $\frac{\partial E(\Pi)}{\partial z_r} = 0$ Since we cannot have a simple closed-form solution at this point, we will further analyze this part using a numerical analysis that is explained in section 5 4 Next, we explain the models for the Decentralized Channel in which the core collection is done by a third party known as the collector

5.3. Decentralized Channel Models

In the decentralized channel, there is a third party who collects cores from the consumers by paying an acquisition price P_a Then, he inspects the cores and sells the remanufacturable ones to the firm (who is in charge of manufacturing new products and remanufacturing) for a unit price w The firm sets the optimal prices and the production lot sizes for the new and remanufactured products Since we assume that each remanufactured product consists of one unit of remanufacturable core, the order size that the firm places to the collector is equal to the production lot size of the remanufacturing (i e q_r) We can define the firms expected profit (i e $E(\Pi_M)$) and the collector's expected profit (i e $E(\Pi_C)$) as follows

$$E(\Pi_M) = (P_n - c_n)(M\left[1 - \frac{P_n - P_r}{\overline{\varphi}(1-\delta)}\right] + \mu_n) - (c_n + h_n)\Lambda_n(z_n) - (P_n + s_n - c_n)\Theta_n(z_n)$$
$$+ (P_r - c_r - w)(M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \mu_r) - (c_r + w + h_r)\Lambda_r(z_r) - (P_r + s_r - c_r - w)\Theta_r(z_r) \quad (5\ 22)$$

$$E(\Pi_C) = wq_r + (\alpha + \beta P_a - q_r)v - (\alpha + \beta P_a)(P_a + c_l) \quad (5\ 23)$$

Since $q_r = y_r + z_r = M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r$, we have

$$E(\Pi_C) = w(M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r) + (\alpha + \beta P_a - M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] - z_r)v - (\alpha + \beta P_a)(P_a + c_l)$$

In Appendix G, we show that the Hessian matrix for $E(\Pi_M)$ is negative semidefinite and thus, the expected profit function for the firm is strictly concave and the solution to the first order conditions maximizes the expected profit The first order conditions are

$$\frac{\partial E(\Pi_M)}{\partial P_n} = 0 \Rightarrow -2MP_n + 2MP_r + Mc_n - Mc_r - Mw + [M + \mu_n - \Theta_n(z_n)]\overline{\varphi}(1-\delta) = 0 \quad (5\ 24)$$

$$\frac{\partial E(\Pi_M)}{\partial P_r} = 0 \Rightarrow 2\delta MP_n - 2MP_r - \delta Mc_n + Mc_r + Mw + [\mu_r - \Theta_r(z_r)]\overline{\varphi}\delta(1-\delta) = 0 \quad (5\ 25)$$

Solving equations (5 24) and (5 25) for P_n and P_r, we have

$$P_n^* = \frac{c_n + \overline{\varphi}}{2} + \frac{(\mu_n + \delta\mu_r)\overline{\varphi}}{2M} - \frac{[\Theta_n(z_n) + \delta\Theta_r(z_r)]\overline{\varphi}}{2M} \quad (5\ 26)$$

$$P_r^* = \frac{c_r + w + \delta\overline{\varphi}}{2} + \frac{(\mu_n + \mu_r)\delta\overline{\varphi}}{2M} - \frac{[\Theta_n(z_n) + \Theta_r(z_r)]\delta\overline{\varphi}}{2M} \tag{5 27}$$

$$\Rightarrow q_r = \frac{M}{\overline{\varphi}\delta(1-\delta)}\left[\frac{\delta c_n - c_r - w}{2}\right] - \frac{1}{2\delta}[\mu_r - \Theta_r(z_r)] + z_r \tag{5 28}$$

We also know that the collector has to provide the number of remanufacturable cores requested by the firm That is, for any order of the size q_r, the collector needs to collect $q_a = \frac{q_r}{r}$ Thus, we have $q_a = \alpha + \beta P_a \Rightarrow \frac{q_r}{r} = \alpha + \beta P_a \Rightarrow P_a = \frac{q_r}{r\beta} - \frac{\alpha}{\beta} \Rightarrow$

$$P_a = \frac{q_r}{r\beta} - \frac{\alpha}{\beta} = \frac{1}{r\beta}\left\{\frac{M}{\overline{\varphi}\delta(1-\delta)}\left[\frac{\delta c_n - c_r - w}{2}\right] - \frac{1}{2\delta}[\mu_r - \Theta_r(z_r)] + z_r\right\} - \frac{\alpha}{\beta} \tag{5 29}$$

Substituting q_r and P_a from (5 28) and (5 29) in equation (5 23), and solving $\frac{\partial E(\Pi_C)}{\partial w} = 0$, we find the optimal price for each core that the collector charges the firm as follows

$$\begin{aligned}
\Rightarrow w^* = (\delta c_n - c_r) + \frac{\overline{\varphi}\delta(1-\delta)}{M}\left[z_r - \frac{1}{2\delta}[\mu_r - \Theta_r(z_r)]\right] \\
+ \frac{1}{1 + \frac{M}{2r^2\beta\overline{\varphi}\delta(1-\delta)}} \times \\
\times\left\{-\frac{1}{2r}\left[(1-r)v - c_l + \frac{\alpha}{\beta}\right] - \left[\frac{\delta c_n - c_r}{2}\right] - \frac{1}{2r^2\beta\delta}[\mu_r - \Theta_r(z_r)] + \frac{z_r}{r^2\beta}\right\}
\end{aligned} \tag{5 30}$$

We can also re-write equation (5 30) as follows

$$\Rightarrow w^* = w^0 + \left[\frac{\overline{\varphi}\delta(1-\delta)}{M} + \frac{1}{r^2\beta + \frac{M}{2\overline{\varphi}\delta(1-\delta)}} \right] \times \left[z_r + \frac{1}{2\delta}\Theta_r(z_r) \right] \text{ where}$$

$$w^0 = (\delta c_n - c_r) - \frac{\overline{\varphi}(1-\delta)}{2M}\mu_r - \frac{1}{1 + \frac{M}{2r^2\beta\overline{\varphi}\delta(1-\delta)}} \times$$

$$\times \left\{ \frac{1}{2r}\left[(1-r)v - c_l + \frac{\alpha}{\beta} \right] + \left[\frac{\delta c_n - c_r}{2} \right] + \frac{1}{2r^2\beta\delta}\mu_r \right\}$$

and w^0 is the riskless price that the collector would charge the firm for each remanufacturable core We observe that if $\frac{1}{2\delta}\Theta_r(z_r) \geq -z_r$, then $w^* \geq w^0$ Otherwise, $w^* < w^0$ Considering w^*, the firm will order

$q_r^* = \frac{M}{\overline{\varphi}\delta(1-\delta)}\left[\frac{\delta c_n - c_r - w^*}{2} \right] - \frac{1}{2\delta}\left[\mu_r - \Theta_r(z_r) \right] + z_r$ units of cores Next, we substitute w^* from equation (30) in equation (27) to find P_r^* in terms of the model parameters and z_n and z_r Finally, we calculate the optimal z_n and z_r (i e z_n^* and z_r^*) that maximize the firm's expected profit and find optimal values of the model variables (i e optimal prices and lot sizes) based on z_n^* and z_r^* Since we cannot have a simple closed-form solution at this point, we will further analyze the decentralized channel models using a numerical analysis In addition, we analyze the impact of some of the model parameters on the optimal prices and quantities, and the expected profit of the firm

5.4. Numerical Analysis

In this section, we further analyze the models and the optimal values for both centralized and decentralized channels First, we provide the set of parameters that we use for the numerical analysis Next, we analyze the impact of consumers' perception of the remanufactured product versus new and the collection yield rate on the optimal solutions In addition, we investigate how the centralized and decentralized channels compare with each other under different conditions Furthermore, we show numerically how the demand uncertainty changes the optimal values of the models

5 4 1 Parameter Setting and Optimization Procedure

The original sets of parameter values that we considered for our extensive numerical analysis include c_n= {50, 55, 60, 65, 70, 75, 80, 85, 90}, c_r= {5, 10, 15, 20, 25, 30, 35, 40}, c_l = {3, 5, 7, 9, 11, 13, 15}, h_n={-55, -45, -35, -25, -15}, h_r= {-10, -8, -6, -4, -2}, s_n= {8, 10, 12, 14, 16}, s_r= {3, 5, 7, 9, 11}, v= {20, 25, 30, 35, 40, 45, 50}, α = {5, 10, 15, 20}, β = {15, 20, 25, 30, 35}, $\overline{\varphi}$ = {500, 800, 1100, 1400}, M = {500, 1000, 1500, 2000, 2500, 3000} However, since the results are consistent across these sets of values, we present our results based on the specific set of values for the model parameters as follows

c_n= 70, c_r= 15, c_l = 5, $h_n = -35$, $h_r = -10$, $s_n = 12$, $s_r = 3$, $v = 30$, A_n =-100 and -5, A_r =-100 and -5, $B_n = 5$ and 100, $B_r = 5$ and 100, $\alpha = 5$, $\beta = 20$, $\overline{\varphi} = 500$, $M = 500$, $\delta = 0\ 2, 0\ 25, 0\ 3, \quad , 0\ 97$, $r = 0\ 1, \quad , 0\ 9$,
$\varepsilon_n \sim Uniform[A_n, B_n]$ and $\varepsilon_r \sim Uniform[A_r, B_r]$

These values make it possible to find feasible solutions for a large range of parameters such as δ and r, and they are reasonable from a practical For example, the unit cost of remanufacturing (c_r) is assumed to be small enough compared to the unit cost of manufacturing new products (c_n) to make the remanufacturing a viable option This is also consistent with the data sets used in the literature (for example, see Bakal and Akcali, 2006) To make the process of finding the optimal solutions more straightforward, we use Excel Solver to find the optimal z_n and z_r, and the optimal prices that maximize the firm's expected profit In addition, we use some of the equations presented earlier in sections 5 2 and 5 3 to define how the variables and parameters are related to each other in our models Below we analyze the impact of different values of δ and the yield rate on the optimal solutions and we compare the centralized and decentralized channels under different circumstances

5 4 2 Impact of consumers' relative WTP for the remanufactured product (δ) and the yield rate

As mentioned earlier in this chapter, when consumers perceive the new and remanufactured products as closer substitutes, δ takes a higher value Analyzing the impact of δ on the profits in the centralized and decentralized channels, we find that depending on the yield rate level, δ can have different impacts on the optimal values When the yield rate is high, in both centralized and decentralized channels, the firm's profit decreases slightly (i e for less than 1%) when δ increases to a certain value (i e around 0 65 in our experiment), and when δ increases any further (up to 0 97 in our experiment), the firm's profit increases for about 7 5% in the centralized channel and 1 5% in the decentralized one This is due to the fact that when δ is higher, the remanufactured product becomes a closer substitute for the new product and as a result, the firm's profit from the new product diminishes while her profit from the remanufactured products increases If we increase δ from a very low value (such as $\delta = 0\ 2$) to a certain value (such as $\delta = 0\ 65$ in our experiment), the decrease in the profit of the new product will be higher than the increase in the profit of the remanufacturing, and thus, the firm's total profit will decrease But when we increase the value of δ further, the profit of the remanufacturing takes a sharper increase that is higher than the reduction in the profit from the new products, which will result in an increase in the firm's total profit Note that the increase in the firm's total profit in the centralized channel is observed to be higher than the one in the decentralized channel when δ takes a value higher than a certain value which is explained above

When the yield rate is low, in the centralized channel, increasing δ changes the firm's profit in a similar way as in the case of high yield rate above The only difference is that when the firm's profit increases with respect to δ, it only increases for less than 1% compared to 7 5% in the case of having a high yield rate This is reasonable because in the case of a low yield rate, the unit cost for each remanufacturable core is higher and as a result the remanufactured product is not as profitable as it was in the case of a high yield rate Thus, the firm will not see an increase in her profit as she would in the case of a high yield rate In addition, when the yield rate is low, in the decentralized channel, the firm's profit only decreases when δ takes a higher value This is due to the fact that in the centralized channel, the firm could gain some revenue form salvaging the extra cores that were not remanufactured or remanufacturable, and this revenue could cover some the additional costs of acquiring remanufacturable cores But in the decentralized channel, this revenue in obtained by the collector and as a result, the increase in the firm's profit from remanufacturing does not surpass the decrease in the firm's profit from the new products, and the firm's total profit decreases with respect to δ

To compare the centralized (C) and decentralized (D) channels, we consider the centralized channel as the benchmark and investigate the impacts of switching from the centralized channel to the decentralized one on the optimal prices, lot sizes, and the profits Table 5 2 summarizes the result of our analysis Since we did not find any significant changes in the optimal price and the leftover and shortage costs of the new product, we excluded them from this table Note that we do not consider the firm's decision on which channel to choose What we analyze here is that under what conditions it will be less harmful (or more beneficial) to the firm if she chooses a decentralized channel over a centralized one As a result, not considering a fixed cost for the core collection does not change our results and just shifts the numbers to be more in favor of the centralized channel As we see in part (a), when the remanufactured product is perceived as a closer substitute for the new product (i e δ is higher) and the yield rate is low, it will be more advantageous to the firm to operate in a centralized channel Under these conditions (i e having a high δ and a low yield rate), a decentralized channel could reduce the firm's profit for about 7 56%, which is higher than the reduction that could happen under other circumstances But if the yield rate is low and the consumers do not perceive the remanufactured product as a close substitute to the new product, it will be less detrimental to the firm's profit to switch to the decentralized channel In addition, we observe that the change in the yield rate from low to high does not have much impact on the firm's profit (for switching from C to D), but the consumer's perception has a higher effect

Furthermore, in part (b) of Table 5 2, we show that how the total supply chain's profit (which in the centralized channel it is equal to the firm's profit) would change if a decentralized channel were chosen over the centralized one We observe that the supply chain will experience the highest reduction in total profit when the remanufactured product is perceived poorly by the consumers (i e δ is low) and the yield rate is high We also find that when the yield rate is low and δ is high, the total supply chain's profit will have the least reduction (if the decentralized channel is chosen over the centralized) As we explained earlier in part (a) of the table, having a low yield rate and a high δ are the conditions under which the firm will face a highest reduction in her profit by switching to the decentralized channel This means that if switching to the decentralized channel happens under these conditions, the firm will need to be compensated for the extra loss This could be done through appropriate contracts that are not within the focus of this research In part (c) of the table, we observe that the firm's leftover costs for the remanufactured products increase 3 to 9 folded depending on the level of the yield rate and δ When the yield rate is low and the products are perceived as closer substitutes, switching to the decentralized channel will cause the firm the highest increase in the leftover costs of the remanufactured products (i e almost 9 times) In contrary, when the yield rate is low and the remanufactured product is poorly perceived by the consumers (i e δ is low), the increase in the leftover costs of the remanufactured products will be the smallest

(a) Change in the Firm's profit when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	-7 56%	-7 22%
Low = 0 3	-6 15%	-6 20%

(b) Change in the Total profit of the chain when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	-0 59%	-2 18%
Low = 0 3	-1 51%	-6 03%

(c) Change in the Firm's Leftover cost for reman when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	883 95%	554 50%
Low = 0 3	371 07%	461 87%

(d) Change in the Firm's Shortage cost for reman when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	-0 70%	73 63%
Low = 0 3	-78 39%	-81 31%

(e) Change in the Firm's optimal lot size for the new product when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	2 61%	10 20%
Low = 0 3	-0 24%	-0 24%

(f) Change in the Firm's optimal lot size for the reman product when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	-27 83%	-16 14%
Low = 0 3	440 30%	464 06%

(g) Change in the Firm's optimal Price for the reman product when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	0 19%	0 50%
Low = 0 3	0 20%	0 21%

(h) Change in the Number of acquired cores when switching from C to D channel

Delta	Yield rate: Low = 0 1	Yield rate: High = 0 9
High = 0 9	-27 83%	-56 46%
Low = 0 3	-44 20%	-93 53%

Table 5 2 Impacts of switching from a Centralized (C) channel to Decentralized (D)

In part (d) of the table, we find that when the yield rate is high and the remanufactured product is not perceived as a close substitute to the new product (i e δ is low), switching to the decentralized channel will reduce the firm's shortage cost for the remanufactured products by around 81% But, if the yield rate and δ are both high, the firm's shortage cost will go up by around 73% In addition, as we see in parts (e) and (f) of the table, the firm's optimal lot size will increase if δ is high and it will stay almost the same (i e with a slight decrease of 0 24%) if δ is low The opposite is true for the firm's optimal lot size for the remanufactured product When δ is high, the firm's optimal lot size for the remanufactured product decreases and its highest reduction happens when the yield rate is low (i e for about 28%) Also, when δ is low, this optimal lot size becomes more than 4 times larger

As we mentioned earlier, the optimal price for the new product does not change significantly We can also see in part (g) of Table 5 2 that the optimal price of the remanufactured product increases slightly with the highest increase happening when both the yield rate and δ are high (i e for 0 5%) Finally, in part (h) of the table, we find that the optimal number of acquired cores decreases when the decentralized channel is chosen over the centralized The highest reduction occurs when the yield rate is high, but the remanufactured product is perceived poorly by the consumers (i e δ is low)

5 4 3 Impact of new and remanufactured product demand uncertainty

For this part, we assumed four different combinations for the demand uncertainties of the new and remanufactured products In the first one, the randomness in demands of the new and remanufactured products can change in smaller ranges, that is, the demand for each type of product is less uncertain since the variance for the randomness is smaller More specifically, we assume that $A_n = -5$, $B_n = 5$, $A_r = -5$ and $B_r = 5$ In the second case, $A_n = -5$, $B_n = 5$, $A_r = -100$ and $B_r = 100$, which means the demand for the remanufactured product is more uncertain while the demand for the new product is more certain In the third case, $A_n = -100$, $B_n = 100$, $A_r = -5$ and $B_r = 5$, which implies the opposite of the situation in the second case, that is, the demand for the new product is more uncertain while the remanufactured product has a more certain demand Finally, in the fourth case, $A_n = -100$, $B_n = 100$, $A_r = -100$ and $B_r = 100$, which indicates that both products have very uncertain demands We consider the first case as the benchmark and investigate how the optimal solution changes when we change the demand uncertainties to the ones in any of the other three cases Note that the magnitude of the change in most cases depends on the model parameters, but the direction of the changes (i e increase or reduction) is reasonable to be considered in our analysis We also consider two cases of model parameters under which we could have binding and non-binding constraints (for the centralized channel) In the following, we present the results for both of these cases

In both centralized and decentralized channels, when the uncertainty in any type of product increases, the total profit decreases We observe that the amount of reduction is similar between centralized and decentralized channels In addition, as we expected, the reduction in the total profit is the highest in the fourth case where the demand uncertainties for both new and remanufactured products are higher However, we find that when the uncertainties are higher, the reduction in the firm's profit is slightly less in the centralized channel compared to the decentralized one (i e 2-3% lower reduction in the firm's profit in the centralized channel in our experiments) Since in the centralized channel the firm makes all the decisions on the optimal prices and lot sizes, she could reduce the amount of loss that she has to incur due to higher uncertainties But in the decentralized channel, the firm does not determine the optimal acquisition price and as result, the collector's decision will not necessarily be the best for the firm when reacting to higher uncertainties

Regarding the optimal prices, as we noted in the analytical parts of sections 5 2 and 5 3, the firm will set lower prices for the new product in both centralized and decentralized channels, and a lower price for the remanufactured product in the centralized channel when the uncertainty in the demand for the new and/or remanufactured product is higher In the decentralized channel, when the uncertainty in the demand for the new product is higher, the firm sets a lower price for the remanufactured product, but she sets a higher price for the remanufactured product only when the uncertainty in the demand of the remanufactured product is higher She sets a higher price in this case because the higher uncertainty in the demand of the remanufactured product increases the unit core price that the collector charges the firm, and as a result, the firm has to charge a higher price This is consistent with the analytical results in section 5 3

In addition, in the centralized channel, when the constraint is not binding, the optimal core acquisition price and, as a result, the optimal number of acquired cores do not depend on any demand uncertainties We also observe in this case that the optimal lot sizes for the new and remanufactured products are set higher only when the uncertainty in their respective demand increases Furthermore, we observe that in the case of having a decentralized channel, the uncertainty in the demand of the new product does not have any impact on the optimal core acquisition price A larger lot size is set for the new product when the uncertainty in its demand is higher But the uncertainty in the demand of the remanufactured product does not have any impact on this lot size In the centralized channel, when the constraint is binding, a smaller optimal core acquisition price is set with a higher uncertainty in the demand of the new product Also in this case, when the uncertainty in the demand of the new product increases, the optimal lot size for the remanufactured product is reduced, but a larger lot size is set for the new product when the uncertainty in the new and/or remanufactured product is higher It can be shown analytically that all these results hold independent of the model parameters

We also find that the leftover and shortage costs for the new and remanufactured products increase significantly only when the uncertainty in their respective demands increase However, when the uncertainty in the demand of the other product increases, it changes the leftover and shortage costs slightly This is because of the fact that the higher uncertainty in the demand of the other product changes the optimal prices for both new and remanufactured products and thus it changes the optimal z_n and z_r which consequently affect the leftover and shortage costs In the next section, we provide some managerial insight based on the results of our analysis

5.4. Managerial Insight

Our findings show that in both centralized and decentralized channels, when the uncertainty in the demand of any of the two products increases, the total profit decreases (as expected), but, we do not see any significant difference between centralized and decentralized channels in the amount of reduction in their total profit We also observe that when the uncertainties are higher, the reduction in the firm's profit is about 2-3% less in the centralized channel compared to the decentralized one In addition, the firm should set lower optimal prices for the new and remanufactured products in both centralized and decentralized channels when the uncertainties in the demands of the new and remanufactured products increase, except for the remanufactured product in the decentralized channel for which the firm sets a higher price when facing a higher uncertainty in the demand of the remanufactured product

We also observe that the leftover and shortage costs for the new and remanufactured products increase significantly only when the uncertainty in their respective demands increase However, when the uncertainty in the demand of the other product increases, it changes the leftover and shortage costs slightly Furthermore, depending on the consumers' perception of the remanufactured product versus new and the yield rate for the core collection, switching from the centralized channel to a decentralized one, could have different impacts on the optimal prices, quantities and profits For example, if the yield rate is low and the consumers do not perceive the remanufactured product as a close substitute to the new product, it will be less detrimental or more beneficial (in the case that the decentralized channel is more profitable with the consideration of the fixed costs of the core collection) to the firm's profit to switch to the decentralized channel But if the yield rate is low and the consumers perceive the products as close substitutes, it will be the least desirable condition to the firm to switch from a centralized channel to a decentralized one Next, we conclude this chapter and present possible future research directions

5.5. Conclusion and Directions for Future Research

In this chapter, we consider a firm that produces distinguishable new and remanufactured products with uncertainty in the demands She has the option of collecting the cores herself in a centralized channel or using a third party collector to provide her with any number of remanufacturable cores as she may require in a decentralized channel In each channel, we jointly find the optimal prices and lot sizes for the new and remanufactured products as well as the optimal core acquisition price that needs to be paid to the consumers to return their end of life/use products We investigate the impact of uncertainties in the demands of the new and remanufactured products on the optimal prices, lot sizes and profits in each channel

The current study has assumed a single period model for a product that already exists in the market and the market for such a product is mature enough to allow for the new and remanufactured products to co-exist in the market while there are enough end of life/use products available for collection Multi-period and infinite-horizon joint pricing and inventory management could be considered as an extension in the future research This will extensively add to the complexity of the models, but it could capture the impact of the decisions in one period on the optimal policies in the future periods In addition, the impact of the OEM's initial decisions for the price and lot size of the new product (when the product is just introduced to the market) on the future optimal prices and lot sizes of the new and remanufactured products could also be taken into account in a multi-period modeling structure

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

Pricing and structural decisions for differentiated new and remanufactured products (that are sold in the same market) have not been well addressed in the literature Examples of such products are computer systems, automotive parts and office equipment Most of the existing literature deals with new and remanufactured products that are not distinguishable by the consumers, that is, they are assumed to be perfect substitutes, and as a result, the same price can be set for both products However, in this thesis, we model different levels of competition (substitution) between new and remanufactured products by considering the relative willingness to pay of the consumers for the remanufactured product versus new This thesis consists of three research papers

In the first paper, we take into account a retailer that sells vertically differentiated new and remanufactured products More specifically, we consider the cases in which the new and remanufactured products are produced by separate firms (i e the manufacturer and the remanufacturer respectively) The problem here is whether it is better that the retailer collaborates more closely (be coordinated) with the manufacturer or the remanufacturer Note that we assume that when two members of the supply chain are coordinated with each other, they determine the retail price for the respective product as a joint unit For example, if the retailer and the manufacturer are coordinated, they jointly define the optimal retail price for the new product We find which coordination structure performs better in terms of the total CLSC profit In addition, we analyze different conditions under which any of the structures would lead some of the CLSC members out of business Finally, we do a more detailed analysis under conditions where all members of the supply chain exist in the market More specifically, we analyze the impacts of the consumers' perception of the remanufactured product versus new (i e the level of competition or substitution between products) and the quality of returns on the optimal pricing and structural decisions

To extend the research in the first paper, in the second paper, we consider business cases in which one firm produces both new and remanufactured products (which are distinguishable) and sells them to the same market As a result, the firm is to determine the optimal prices for both products In addition, we include the decision making for the acquisition price that the firm needs to pay to collect the end of life or end of use products (known as the core acquisition price) We find these three prices simultaneously and investigate the impacts of some of the model parameters on the expected optimal prices and quantities for both products, and the expected profit of the firm The parameters under study include the competition between new and remanufactured products (which is captured by the consumers' willingness to pay for the remanufactured product versus new), quality of returns (which is modeled by the stochastic core collection yield rate), and the salvage value of the cores that are not remanufactured or remanufacturable Furthermore, we compare the cases in which the firms deal with high profit margin products versus low profit margin ones We show how the optimal decisions could be different for high versus low margin products under different conditions

In both the first and second papers, we assumed deterministic demand functions for the new and remanufactured products But, in the third paper, we extend our models by assuming stochastic demand functions for the new and remanufactured products while they are still influenced by the product prices Another extension in this paper is that we develop models to jointly find the optimal prices and lot sizes for differentiated new and remanufactured products Similar to the second paper, we consider a firm who produces distinguishable new and remanufactured products and sells them to the same market In addition, we investigate two types of reverse channels for the core collection In the first channel, which is called centralized, the firm collects the cores directly from the consumers Thus, she needs to determine the optimal core acquisition price in addition to the optimal prices and lot sizes for the new and remanufactured products In the second channel, which is called decentralized, a separate third party collects the cores and sells them to the firm as required As a result, the optimal core acquisition price is determined by the third party collector, and the firm sets the optimal prices and lot sizes for the new and remanufactured products We find the impacts of some of the model parameters, such as the competition between the products, the quality of returns (i e the core collection yield rate) and the level of uncertainty in the demand of each product, on the optimal prices and lot sizes, and the expected profits We do the analysis for each channel choice and compare them with respect to changes in the optimal values under different conditions

To extend the research in this thesis, non-linear core supply functions can be assumed although they add to the complexity of the models significantly In modeling the core collection yield rate, we considered the cases in which that the yield rate was independent of the acquisition price This can be changed in a future research to include cases in which the acquisition price affects the yield rate In addition, different probability distribution functions can be used to model the stochastic yield rate and their impact of the optimal solutions can be analyzed Furthermore, multi-period and infinite horizon joint pricing and inventory management models can be considered in the future research to investigate the impact of the optimal decisions made in each period on the future periods One of the research streams that can also be considered as the extension of the current thesis is the one that includes the decisions related to the initial product design and its impact on the optimal recovery policies (i e including the optimal prices and lot sizes as well as the optimal recovery options available to the firm and the third party competitors) that a firm could plan for ahead of the time Depending on the type of products, different researches can be conducted to help the OEM make more sustainable decisions from the very beginning when she designs the new products

Appendix A: Derivation of Inverse Demand Functions

To come up with the inverse demand functions (i.e. expressions 1 and 2 in the first paper or 23 and 24 in the second paper), first we assume a more general case in which the market size is equal to M and the consumers' willingness-to-pay is heterogeneous and uniformly distributed in the interval $\varphi \in [0,\bar{\varphi}]$ (where $\bar{\varphi} < \infty$) with the cumulative distribution function $F()$. As a result, $F(\varphi) = \varphi/\bar{\varphi}$, for $\varphi \in [0,\bar{\varphi}]$. Based on the consumers' preferences, we can divide them into three groups: first, the consumers who prefer to buy the new product, the consumers who prefer to buy the remanufactured product, and third, the consumers who prefer not to buy any of the products. We assume that the consumer who is indifferent between buying the new and remanufactured products, has a willingness-to-pay of φ_1. For this consumer, the utility that he gains from buying a unit of the new product is equal to the utility from buying a unit of the remanufactured product. In addition, the consumer who is indifferent between buying a remanufactured product and not buying anything, has a willingness-to-pay of φ_2. Again, the utility of this consumer from buying a unit of the remanufactured product is equal to the utility that he will have from not buying anything, that is zero. It is evident that $\varphi_2 < \varphi_1 < \bar{\varphi}$. We find the prices for new (P_n) and remanufactured (P_r) products based on φ_1 and φ_2 by solving the indifference conditions. The conditions are as follows:

$$\alpha_n \varphi_1 - P_n = \alpha_r \varphi_1 - P_r \tag{A1}$$

$$\alpha_r \varphi_2 - P_r = 0 \tag{A2}$$

α_n and α_r show the quality perception of the consumers towards new and remanufactured products respectively In our models, we have $\alpha_n = 1$ and $\alpha_r = \delta$, $\delta \in (0,1)$, which shows that if the consumer's willingness-to-pay for the new product is φ_1, his willingness to pay for the remanufactured product will be $\delta\varphi_1$ In the more general terms, if the consumer's willingness-to-pay for the new product is $\alpha_n\varphi_1$, his willingness-to-pay for the remanufactured product will be $\alpha_r\varphi_1$ In condition (A1), the left side of the equation shows the utility of the consumer type 1 (who is indifferent between buying a unit of the new product and buying a unit of the remanufactured product) from buying a unit of the new product, and the right side represents the consumer's utility from buying a unit of the remanufactured product In condition (A2), the left side of the equation shows the utility that the consumer type 2 (who is indifferent between buying a remanufactured product and not buying anything) gains from buying a unit of the remanufactured product, and the right side is his utility from not buying anything, which is equal to zero

From condition (A2) we have

$$P_r = \alpha_r \varphi_2 \tag{A3}$$

We substitute P_r in condition (A1), and we will have

$$\alpha_n \varphi_1 - P_n = \alpha_r \varphi_1 - \alpha_r \varphi_2 \Rightarrow$$

$$P_n = (\alpha_n - \alpha_r)\varphi_1 + \alpha_r \varphi_2 \tag{A4}$$

Now we find the relationship between the quantities (demands) for new (q_n) and remanufactured (q_r) products and φ_1 and φ_2 Assuming the market size of M and the cumulative distribution function $F()$ for φ ($\varphi \sim U[0,\bar{\varphi}]$) as explained earlier, we have

$$q_n = M[1 - F(\varphi_1)] = \frac{M}{\bar{\varphi}}(\bar{\varphi} - \varphi_1)$$

$$q_r = M[F(\varphi_1) - F(\varphi_2)] = \frac{M}{\bar{\varphi}}(\varphi_1 - \varphi_2)$$

Solving for φ_1 and φ_2 in terms of quantities, we will have

$$\varphi_1 = \bar{\varphi}(1 - \frac{1}{M}q_n) \tag{A5}$$

$$\varphi_2 = \bar{\varphi}[1 - \frac{1}{M}(q_n + q_r)] \tag{A6}$$

Now, as we assumed in this research, if $\alpha_n = 1$, $\alpha_r = \delta$, where $\delta \in (0,1)$, by substituting these values and φ_1 and φ_2 from (A5) and (A6) in expressions (A3) and (A4) we will have

$$P_r = \delta\varphi_2 = \delta\frac{\bar{\varphi}}{M}(M - q_n - q_r) \tag{A7}$$

$$P_n = (1-\delta)\varphi_1 + \delta\varphi_2 = \frac{\bar{\varphi}}{M}(M - q_n - \delta q_r) \tag{A8}$$

Equations (A7) and (A8) give values of P_r and P_n in a general format, that is, they depend on the total market size M and the maximum possible willingness to pay by any consumer, $\bar{\varphi}$ As a result, they do not have to be less than 1 in this general format

However, in this research, where we have $\overline{\varphi} = 1$ and the market size is normalized to 1 (i e $M = 1$), we will have

$$\varphi_1 = 1 - q_n \tag{A9}$$

$$\varphi_2 = 1 - q_n - q_r \tag{A10}$$

And from equations (A7) and (A8), we will have

$$P_n = 1 - q_n - \delta q_r \tag{A11}$$

$$P_r = \delta(1 - q_n - q_r) \tag{A12}$$

Expressions (A11) and (A12) are the inverse demand functions that we use in this research and are consistent with the ones in Ferguson and Toktay (2006)

Now if somebody wants to find the general values of the prices for new (P_n^G) and remanufactured (P_r^G) products from the values in (A11) and (A12), here are the required calculations

From (A11) $q_n + \delta q_r = 1 - P_n$

From (A12) $q_n + q_r = 1 - \dfrac{P_r}{\delta}$

From (A7) we have $P_r^G = \delta \dfrac{\overline{\varphi}}{M}(M - q_n - q_r) \Rightarrow$

$$P_r^G = \delta \frac{\overline{\varphi}}{M}(M - 1 + \frac{P_r}{\delta}) \tag{A13}$$

And from (A8) we have $P_n^G = \frac{\bar{\varphi}}{M}(M - q_n - \delta q_r) \Rightarrow$

$$P_n^G = \frac{\bar{\varphi}}{M}(M - 1 + P_n) \qquad \text{(A14)}$$

Equations (A13) and (A14) give the general price values for the remanufactured and new products respectively, knowing the ones that we find in this research (i e P_r and P_n)

Appendix B. Analysis of the cases for KKT conditions

Case 1 $\_q_n > 0$, $q_r > 0$, $\mu_n = 0$ and $\mu_r = 0$

$$q_r^{MRC} > 0 \Rightarrow \delta(C_n + h) > B \qquad (B1)$$
$$q_r^{RRLMC} > 0 \Rightarrow \delta(1-\delta) + \delta(C_n + h) - (2-\delta)B > 0 \Rightarrow \delta(C_n + h) > (2-\delta)B - \delta(1-\delta) \Rightarrow$$
$$\Rightarrow \delta(C_n + h) > B + (1-\delta)(B-\delta) \qquad (B2)$$

$$\Rightarrow C_n + h > \frac{B}{\delta} + \frac{(1-\delta)(B-\delta)}{\delta}$$

If $B > \delta \Rightarrow$ in (4) $\Rightarrow C_n + h > \frac{B}{\delta} > 1$, but $C_n + h$ cannot be larger than 1

$\Rightarrow B < \delta \Rightarrow (1-\delta)(B-\delta) < 0$

$$q_r^{CD} > 0 \Rightarrow \delta(1-\delta) + \delta(C_n + h) - (2-\delta)B > 0 \Rightarrow \delta(C_n + h) > (2-\delta)B - \delta(1-\delta) \Rightarrow$$

$$\Rightarrow \delta(C_n + h) > B + (1-\delta)(B-\delta) \qquad (B3)$$

If (B1) holds, (B2) and (B3) will hold because the right side of (B1) is the largest of the three So, we choose (B1) as the condition to be in place

$$q_n^{RRLMC} > 0 \Rightarrow C_n + h < B + (1-\delta) \qquad (B4)$$
$$q_n^{MRC} > 0 \Rightarrow 2(1-\delta) - (2-\delta)(C_n + h) + B > 0 \Rightarrow (2-\delta)(C_n + h) < B + 2(1-\delta) \qquad (B5)$$
$$q_n^{CD} > 0 \Rightarrow 2(1-\delta) - (2-\delta)(C_n + h) + B > 0 \Rightarrow (2-\delta)(C_n + h) < B + 2(1-\delta) \qquad (B6)$$

$$(B4) \Rightarrow (2-\delta)(C_n + h) < (2-\delta)B + (2-\delta)(1-\delta)$$
$$(B5) \Rightarrow (2-\delta)(C_n + h) < B + 2(1-\delta)$$
$$(B6) \Rightarrow (2-\delta)(C_n + h) < B + 2(1-\delta)$$

(B5) and (B6) are equivalent If (B4) holds, (B6) will hold, because (B4) has a smaller value in its right side of the inequation, which makes it a tighter condition

$$(2-\delta)B + (2-\delta)(1-\delta) - B - 2(1-\delta) = (1-\delta)B - \delta(1-\delta) = (1-\delta)(B-\delta) < 0$$

So, we choose (B4) from these three conditions to be in place

As a result, we have two conditions that must hold in order for all values to be positive (i e feasible) These two conditions are (B1) and (B4) from above

$\delta(C_n + h) > B$
and
$C_n + h < B + (1-\delta)$

Case 2 $q_n > 0$, $q_r = 0$, $\mu_n = 0$ and $\mu_r \geq 0$

$$q_n^{CD} = \frac{2(1-\delta)-(2-\delta)(C_n+h)+B-\mu_r}{2(1-\delta)(4-\delta)} > 0 \Rightarrow 2(1-\delta)-(2-\delta)(C_n+h)+B-\mu_r > 0$$

$$\Rightarrow (2-\delta)(C_n+h) < B + 2(1-\delta) - \mu_r \qquad \text{(B7)}$$

$$q_r^{CD} = \frac{\delta(1-\delta)+\delta(C_n+h)-(2-\delta)B+(2-\delta)\mu_r}{2\delta(1-\delta)(4-\delta)} = 0$$
$$\Rightarrow \mu_r = \frac{-\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B}{2-\delta}$$

$$\mu_r \geq 0 \Rightarrow -\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B \geq 0 \Rightarrow \delta(C_n+h) \leq (2-\delta)B - \delta(1-\delta)$$
(B8)

Substituting μ_r in (B7) we have

$$(2-\delta)^2(C_n+h) < (2-\delta)B + 2(2-\delta)(1-\delta) + \delta(1-\delta) + \delta(C_n+h) - (2-\delta)B \Rightarrow$$
$$(4-5\delta+\delta^2)(C_n+h) < (4-\delta)(1-\delta) \Rightarrow$$
$$\Rightarrow C_n + h < 1$$

This is true all the time since the unit cost of manufacturing has to be less than 1 when we are dealing with normalized prices

$$q_n^{MRC} = \frac{1}{2} + \frac{B-(2-\delta)(C_n+h)-\mu_r}{4(1-\delta)} > 0 \Rightarrow 2(1-\delta)-(2-\delta)(C_n+h)+B-\mu_r > 0 \Rightarrow$$
$$\Rightarrow (2-\delta)(C_n+h) < B + 2(1-\delta) - \mu_r \qquad \text{(B9)}$$

$$q_r^{MRC} = \frac{\delta(C_n + h) - B + \mu_r}{4\delta(1-\delta)} = 0 \Rightarrow \mu_r = B - \delta(C_n + h) \geq 0 \Rightarrow$$

$$\delta(C_n + h) \leq B \qquad \text{(B10)}$$

Substituting μ_r in (B9) we have

$$\Rightarrow (2-\delta)(C_n + h) < B + 2(1-\delta) - B + \delta(C_n + h) \Rightarrow 2(1-\delta)(C_n + h) < 2(1-\delta) \Rightarrow$$

$\Rightarrow C_n + h < 1$ which is always true

$$q_n^{RRLMC} = \frac{1}{4} - \frac{C_n + h - B - \mu_r}{4(1-\delta)} > 0 \Rightarrow (1-\delta) - (C_n + h) + B - \mu_r > 0 \Rightarrow$$

$$\Rightarrow (C_n + h) < B + (1-\delta) - \mu_r \qquad \text{(B11)}$$

$$q_r^{RRLMC} = \frac{1}{4} + \frac{\delta(C_n + h) - (2-\delta)B + (2-\delta)\mu_r}{4\delta(1-\delta)} = 0 \Rightarrow$$

$$\Rightarrow \delta(1-\delta) + \delta(C_n + h) - (2-\delta)B + (2-\delta)\mu_r = 0 \Rightarrow$$

$$\Rightarrow \mu_r = \frac{-\delta(1-\delta) - \delta(C_n + h) + (2-\delta)B}{2-\delta} \geq 0 \Rightarrow$$

$$\delta(C_n + h) \leq (2-\delta)B - \delta(1-\delta) \qquad \text{(B12)}$$

Substituting μ_r in (B11) we have

$$\Rightarrow (2-\delta)(C_n + h) < (2-\delta)B + (1-\delta)(2-\delta) + \delta(1-\delta) + \delta(C_n + h) - (2-\delta)B \Rightarrow$$

$$\Rightarrow 2(1-\delta)(C_n + h) < 2(1-\delta) \Rightarrow$$

$\Rightarrow C_n + h < 1$ which is always true

In summary we have

From (B8) and (B12) $\delta(C_n + h) \leq (2-\delta)B - \delta(1-\delta)$

From (B10) $\delta(C_n + h) \leq B$

Case 3 $q_n = 0$, $q_r > 0$, $\mu_n \geq 0$ and $\mu_r = 0$

$$q_r^{CD} = \frac{\delta(1-\delta) + \delta(C_n + h) - (2-\delta)B - \delta\mu_n}{2\delta(1-\delta)(4-\delta)} > 0$$

$$\Rightarrow \delta(C_n + h) > (2-\delta)B - \delta(1-\delta) + \delta\mu_n \qquad \text{(B13)}$$

$$q_n^{CD} = \frac{2(1-\delta) - (2-\delta)(C_n + h) + B + (2-\delta)\mu_n}{2(1-\delta)(4-\delta)} = 0$$

$$\Rightarrow \mu_n = \frac{(2-\delta)(C_n + h) - B - 2(1-\delta)}{2-\delta} \geq 0 \Rightarrow (2-\delta)(C_n + h) \geq B + 2(1-\delta)$$

(B14)

Substituting μ_n in (B13) we have

$$\delta(2-\delta)(C_n + h) > (2-\delta)^2 B - \delta(1-\delta)(2-\delta) + \delta(2-\delta)(C_n + h) - \delta B - 2\delta(1-\delta) \Rightarrow$$

$$\Rightarrow (4 - 5\delta + \delta^2)B < \delta(1-\delta)(4-\delta) \Rightarrow (1-\delta)(4-\delta)B < \delta(1-\delta)(4-\delta) \Rightarrow$$

$\Rightarrow B < \delta$ which we have already proved this to be true for (B3)

$$q_r^{MRC} = \frac{\delta(C_n + h) - B - \delta\mu_n}{4\delta(1-\delta)} > 0 \Rightarrow \delta(C_n + h) > B + \delta\mu_n \qquad \text{(B15)}$$

$$q_n^{MRC} = \frac{1}{2} + \frac{-(2-\delta)(C_n + h) + B + (2-\delta)\mu_n}{4(1-\delta)} = 0 \Rightarrow$$

$$2(1-\delta) - (2-\delta)(C_n + h) + B + (2-\delta)\mu_n = 0 \Rightarrow$$

$$\mu_n = \frac{(2-\delta)(C_n + h) - B - 2(1-\delta)}{2-\delta} \geq 0 \Rightarrow (2-\delta)(C_n + h) \geq B + 2(1-\delta) \qquad \text{(B16)}$$

Substituting μ_n in (B15) we have

$$\delta(2-\delta)(C_n + h) > (2-\delta)B + \delta(2-\delta)(C_n + h) - \delta B - 2\delta(1-\delta) \Rightarrow 2(1-\delta)B < 2\delta(1-\delta) \Rightarrow$$

$\Rightarrow B < \delta$ which is already shown to be true

$$q_r^{RRCMC} = \frac{1}{4} + \frac{\delta(C_n + h) - (2-\delta)B - \delta\mu_n}{4\delta(1-\delta)} > 0 \Rightarrow \delta(1-\delta) + \delta(C_n + h) - (2-\delta)B - \delta\mu_n > 0 \Rightarrow$$

$$\Rightarrow \delta(C_n + h) > (2-\delta)B - \delta(1-\delta) + \delta\mu_n \qquad \text{(B17)}$$

$$q_n^{RRLMC} = \frac{1}{4} - \frac{C_n + h - B - \mu_n}{4(1-\delta)} = 0 \Rightarrow \mu_n = C_n + h - B - (1-\delta) \geq 0 \Rightarrow$$

$$C_n + h \geq B + (1-\delta) \qquad \text{(B18)}$$

Substituting μ_n in (B17) we have

$\Rightarrow \delta(C_n + h) > (2-\delta)B - \delta(1-\delta) + \delta(C_n + h) - \delta B - \delta(1-\delta) \Rightarrow$

$2(1-\delta)B < 2\delta(1-\delta) \Rightarrow$

$\Rightarrow B < \delta$ which is already shown to be true

In summary we have

From (B14) and (B16) $(2-\delta)(C_n + h) \geq B + 2(1-\delta)$

From (B18) $C_n + h \geq B + (1-\delta)$

Considering the conditions from cases 1, 2 and 3, we can create figure 3 2 to show how the feasible solution area can be divided into different regions based on the values for the unit costs of manufacturing and remanufacturing The following inequalities define the different regions

Inequality 1 $\delta(C_n + h) \leq (2-\delta)B - \delta(1-\delta)$

Inequality 2 $(2-\delta)(C_n + h) \geq B + 2(1-\delta)$

Inequality 3 $\delta(C_n + h) \leq B$

Inequality 4 $C_n + h \geq B + (1-\delta)$

And the regions are define as follows

Region 1 $\delta(C_n + h) > B$ and $C_n + h < B + (1-\delta)$

Region 2 $C_n + h \geq B + (1-\delta)$ and $(2-\delta)(C_n + h) < B + 2(1-\delta)$

Region 3 $(2-\delta)(C_n+h) \geq B+2(1-\delta)$

Region 4 $\delta(C_n+h) \leq B$ and $\delta(C_n+h) > (2-\delta)B-\delta(1-\delta)$

Region 5 $\delta(C_n+h) \leq (2-\delta)B-\delta(1-\delta)$

Appendix C. Comparison of prices and quantities across structures

Now, considering conditions 1 and 2 in expressions (3 8) and (3 9), we look at the optimal prices across different structures and compare them with each other

$$P_n^{RRLMC} - P_n^{MRC} = \frac{1-\delta-(C_n+h-B)}{4}$$

From (B4) in Appendix B we know $C_n + h - B < 1 - \delta$

$$\Rightarrow 1-\delta-(C_n+h-B) > 0 \Rightarrow P_n^{MRC} < P_n^{RREMC}$$

$$P_n^{CD} - P_n^{MRC} = \frac{2(C_n+h)+2(1-\delta)+B}{2(4-\delta)} - \frac{C_n+h}{2} = \frac{2(1-\delta)+B-(2-\delta)(C_n+h)}{2(4-\delta)}$$

According to (B6) $(2-\delta)(C_n+h) < B + 2(1-\delta)$

$$\Rightarrow 2(1-\delta)+B-(2-\delta)(C_n+h) > 0 \Rightarrow P_n^{MRC} < P_n^{CD}$$

$$P_n^{CD} - P_n^{RREMC} = \frac{2(C_n+h)+2(1-\delta)+B}{2(4-\delta)} - \frac{1-\delta+C_n+h+B}{4} =$$

$$= \frac{4(C_n+h)+4(1-\delta)+2B-(4-\delta)(1-\delta)-(4-\delta)(C_n+h)-(4-\delta)B}{4(4-\delta)}$$

$$= \frac{\delta(C_n+h)+\delta(1-\delta)-(2-\delta)B}{4(4-\delta)}$$

From (B2) in Appendix B we know that

$$\delta(C_n+h) > (2-\delta)B - \delta(1-\delta) \Rightarrow \delta(C_n+h)+\delta(1-\delta)-(2-\delta)B > 0$$

$$P_n^{CD} - P_n^{RREMC} > 0 \Rightarrow P_n^{RRLMC} < P_n^{CD}$$

As a result, we have found that under conditions 1 and 2, the following always holds

$$\boxed{P_n^{MRC} < P_n^{RREMC} < P_n^{CD}}$$

Now, we look at the remanufactured product prices across different sructures

$$P_r^{MRC} - P_r^{RREMC} = \frac{2\delta+\delta(C_n+h)+B}{4} - \frac{\delta+B}{2} = \frac{\delta(C_n+h)-B}{4}$$

We know that $\delta(C_n + h) > B \Rightarrow P_r^{MRC} > P_r^{RREMC}$

$$P_r^{CD} - P_r^{RREMC} = \frac{\delta}{2} + \frac{\delta(1-\delta) + \delta(C_n + h) + 2B}{2(4-\delta)} - \frac{\delta + B}{2} =$$
$$= \frac{\delta(1-\delta) + \delta(C_n + h) + 2B - (4-\delta)B}{2(4-\delta)} = \frac{\delta(1-\delta) - (2-\delta)B + \delta(C_n + h)}{2(4-\delta)}$$

From (B2) in Appendix B we know that

$$\delta(C_n + h) > (2-\delta)B - \delta(1-\delta) \Rightarrow \delta(C_n + h) + \delta(1-\delta) - (2-\delta)B > 0$$

$$P_r^{CD} - P_r^{RREMC} > 0 \Rightarrow P_r^{RREMC} < P_r^{CD}$$

$$P_r^{CD} - P_r^{MRC} = \frac{\delta}{2} + \frac{\delta(1-\delta) + \delta(C_n + h) + 2B}{2(4-\delta)} - \frac{2\delta + \delta(C_n + h) + B}{4} =$$
$$= \frac{2\delta(1-\delta) + 2\delta(C_n + h) + 4B - \delta(4-\delta)(C_n + h) - (4-\delta)B}{4(4-\delta)} =$$
$$= \frac{2\delta(1-\delta) - \delta(2-\delta)(C_n + h) + \delta B}{4(4-\delta)} = \frac{\delta[2(1-\delta) - (2-\delta)(C_n + h) + B]}{4(4-\delta)}$$

$$(B6) \quad \Rightarrow \quad (2-\delta)(C_n + h) < B + 2(1-\delta) \Rightarrow \delta[2(1-\delta) - (2-\delta)(C_n + h) + B] > 0$$

$$\Rightarrow P_r^{CD} - P_r^{MRC} > 0 \Rightarrow P_r^{MRC} < P_r^{CD}$$

To summarize, we are able to show that the following rankings hold between the prices for the remanufactured product in different structures

$$\boxed{P_r^{RREMC} < P_r^{MRC} < P_r^{CD}}$$

In a similar way, we are able to show how the quantities of new and remanufactured products across structures compare with each other, as follows

$$q_n^{RRIMC} - q_n^{CD} = \frac{1-\delta-(C_n+h)+B}{4(1-\delta)} - \frac{2(1-\delta)-(2-\delta)(C_n+h)+B}{2(1-\delta)(4-\delta)} =$$

$$= \frac{(1-\delta)(4-\delta)-(4-\delta)(C_n+h)+(4-\delta)B-4(1-\delta)+2(2-\delta)(C_n+h)-2B}{4(1-\delta)(4-\delta)} =$$

$$= \frac{-\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B}{4(1-\delta)(4-\delta)}$$

From (B2) in Appendix B we know that

$$\delta(C_n+h) > (2-\delta)B-\delta(1-\delta) \Rightarrow -\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B < 0$$

$$\Rightarrow q_n^{RRLMC} - q_n^{CD} < 0 \Rightarrow q_n^{RRLMC} < q_n^{CD}$$

$$q_n^{MRC} - q_n^{RRLMC} = \frac{1}{2} + \frac{B-(2-\delta)(C_n+h)}{4(1-\delta)} - \frac{1}{4} + \frac{(C_n+h)-B}{4(1-\delta)} =$$

$$= \frac{(1-\delta)+B-(2-\delta)(C_n+h)+(C_n+h)-B}{4(1-\delta)} =$$

$$= \frac{(1-\delta)-(1-\delta)(C_n+h)}{4(1-\delta)} = \frac{1-(C_n+h)}{4} > 0 \Rightarrow q_n^{RRLMC} < q_n^{MRC}$$

$$q_n^{MRC} - q_n^{CD} = \frac{2(1-\delta)+B-(2-\delta)(C_n+h)}{4(1-\delta)} - \frac{2(1-\delta)-(2-\delta)(C_n+h)+B}{2(1-\delta)(4-\delta)} =$$

$$= \frac{2(1-\delta)(4-\delta)+B(4-\delta)-(2-\delta)(4-\delta)(C_n+h)-4(1-\delta)+2(2-\delta)(C_n+h)-2B}{4(1-\delta)(4-\delta)} =$$

$$= \frac{2(1-\delta)(2-\delta)+B(2-\delta)-(2-\delta)(2-\delta)(C_n+h)}{4(1-\delta)(4-\delta)} =$$

$$= \frac{(2-\delta)[2(1-\delta)+B-(2-\delta)(C_n+h)]}{4(1-\delta)(4-\delta)}$$

From (B6) in Appendix B we know that $(2-\delta)(C_n+h) < B+2(1-\delta)$

$$\Rightarrow 2(1-\delta)+B-(2-\delta)(C_n+h) > 0 \Rightarrow q_n^{MRC} - q_n^{CD} > 0 \Rightarrow q_n^{CD} < q_n^{MRC}$$

We can summarize the comparisons above as follows

$$\boxed{q_n^{RREMC} < q_n^{CD} < q_n^{MRC}}$$

Finally, we look at the optimal quantities for the remanufactured product across structures Considering the optimal values in table 2, we have

$$q_r^{CD} - q_r^{RRIMC} = \frac{\delta(1-\delta)+\delta(C_n+h)-(2-\delta)B}{2\delta(1-\delta)(4-\delta)} - \frac{1}{4} - \frac{\delta(C_n+h)-(2-\delta)B}{4\delta(1-\delta)} =$$
$$= \frac{2\delta(1-\delta)+2\delta(C_n+h)-2(2-\delta)B-\delta(1-\delta)(4-\delta)-\delta(4-\delta)(C_n+h)+(2-\delta)(4-\delta)B}{4\delta(1-\delta)(4-\delta)} =$$
$$= \frac{-\delta(1-\delta)(2-\delta)-\delta(2-\delta)(C_n+h)+(2-\delta)^2 B}{4\delta(1-\delta)(4-\delta)} = \frac{(2-\delta)[-\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B]}{4\delta(1-\delta)(4-\delta)}$$

From (B2) we know that
$\delta(C_n+h) > (2-\delta)B-\delta(1-\delta) \Rightarrow -\delta(1-\delta)-\delta(C_n+h)+(2-\delta)B < 0$

$\Rightarrow q_r^{CD} - q_r^{RRIMC} < 0 \Rightarrow q_r^{CD} < q_r^{RRLMC}$

$$q_r^{CD} - q_r^{MRC} = \frac{\delta(1-\delta)+\delta(C_n+h)-(2-\delta)B}{2\delta(1-\delta)(4-\delta)} - \frac{\delta(C_n+h)-B}{4\delta(1-\delta)} =$$
$$= \frac{2\delta(1-\delta)+2\delta(C_n+h)-2(2-\delta)B-\delta(4-\delta)(C_n+h)+(4-\delta)B}{4\delta(1-\delta)(4-\delta)} =$$
$$= \frac{2\delta(1-\delta)-\delta(2-\delta)(C_n+h)+\delta B}{4\delta(1-\delta)(4-\delta)} = \frac{2(1-\delta)-(2-\delta)(C_n+h)+B}{4(1-\delta)(4-\delta)}$$

According to (B6) $(2-\delta)(C_n+h) < B+2(1-\delta)$
$\Rightarrow 2(1-\delta)+B-(2-\delta)(C_n+h) > 0 \Rightarrow q_r^{CD} - q_r^{MRC} > 0 \Rightarrow q_r^{MRC} < q_r^{CD}$

Finally, we can summarize the comparisons above as the following

$$\boxed{q_r^{MRC} < q_r^{CD} < q_r^{RREMC}}$$

Appendix D. Concavity test for the firm's expected profit in the Centralized Channel when the constraint is non-binding

To check for the concavity of the firm's expected profit function, we need to calculate the Hessian matrix First and second derivatives of the firm's expected profit with respect to P_n, P_r, P_a, z_n and z_r are as follows

$$\frac{\partial E(\Pi)}{\partial z_n} = -(c_n + h_n) + (P_n + s_n + h_n)[1 - F_n(z_n)] \tag{D1}$$

$$\frac{\partial^2 E(\Pi)}{\partial z_n^2} = -(P_n + s_n + h_n) f_n(z_n) \tag{D2}$$

$$\frac{\partial E(\Pi)}{\partial z_r} = -(c_r + h_r + v) + (P_r + s_r + h_r)[1 - F_r(z_r)] \tag{D3}$$

$$\frac{\partial^2 E(\Pi)}{\partial z_r^2} = -(P_r + s_r + h_r) f_r(z_r) \tag{D4}$$

$$\frac{\partial E(\Pi)}{\partial P_n} = M\left[1 - \frac{2P_n - P_r}{\overline{\varphi}(1-\delta)}\right] + \frac{c_n M}{\overline{\varphi}(1-\delta)} + \mu_n - \Theta_n(z_n) + \frac{(P_r - c_r)M}{\overline{\varphi}(1-\delta)} - \frac{vM}{\overline{\varphi}(1-\delta)} \tag{D5}$$

$$\frac{\partial^2 E(\Pi)}{\partial P_n^2} = \frac{-2M}{\overline{\varphi}(1-\delta)} \tag{D6}$$

$$\frac{\partial E(\Pi)}{\partial P_r} = \frac{(P_n - c_n)M}{\overline{\varphi}(1-\delta)} + M\left[\frac{\delta P_n - 2P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \frac{c_r M}{\overline{\varphi}\delta(1-\delta)} + \mu_r - \Theta(z_r) + \frac{vM}{\overline{\varphi}\delta(1-\delta)} \quad \text{(D7)}$$

$$\frac{\partial^2 E(\Pi)}{\partial P_r^2} = \frac{-2M}{\overline{\varphi}\delta(1-\delta)} \quad \text{(D8)}$$

$$\frac{\partial E(\Pi)}{\partial P_a} = \beta v - \alpha - \beta c_l - 2\beta P_a \quad \text{(D9)}$$

$$\frac{\partial^2 E(\Pi)}{\partial P_a^2} = -2\beta \quad \text{(D10)}$$

If we define $g(\,) = E(\Pi)$, we have

$$g_{11} = \frac{\partial^2 E(\Pi)}{\partial z_n^2} = -(P_n + s_n + h_n) f_n(z_n),\ g_{12} = \frac{\partial^2 E(\Pi)}{\partial z_n \partial z_r} = 0,\ g_{13} = \frac{\partial^2 E(\Pi)}{\partial z_n \partial P_n} = 1 - F_n(z_n),$$

$$g_{14} = \frac{\partial^2 E(\Pi)}{\partial z_n \partial P_r} = 0,\ g_{15} = \frac{\partial^2 E(\Pi)}{\partial z_n \partial P_a} = 0,\ g_{21} = \frac{\partial^2 E(\Pi)}{\partial z_r \partial z_n} = 0,$$

$$g_{22} = \frac{\partial^2 E(\Pi)}{\partial z_r^2} = -(P_r + s_r + h_r) f_r(z_r),\ g_{23} = \frac{\partial^2 E(\Pi)}{\partial z_r \partial P_n} = 0,\ g_{24} = \frac{\partial^2 E(\Pi)}{\partial z_r \partial P_r} = 1 - F_r(z_r),$$

$$g_{25} = \frac{\partial^2 E(\Pi)}{\partial z_r \partial P_a} = 0,\ g_{31} = \frac{\partial^2 E(\Pi)}{\partial P_n \partial z_n} = 1 - F_n(z_n),\ g_{32} = \frac{\partial^2 E(\Pi)}{\partial P_n \partial z_r} = 0,$$

$$g_{33} = \frac{\partial^2 E(\Pi_M)}{\partial P_n^2} = \frac{-2M}{\overline{\varphi}(1-\delta)},\ g_{34} = \frac{\partial^2 E(\Pi)}{\partial P_n \partial P_r} = \frac{2M}{\overline{\varphi}(1-\delta)},\ g_{35} = \frac{\partial^2 E(\Pi)}{\partial P_n \partial P_a} = 0,$$

$$g_{41} = \frac{\partial^2 E(\Pi)}{\partial P_r \partial z_n} = 0,\ g_{42} = \frac{\partial^2 E(\Pi)}{\partial P_r \partial z_r} = 1 - F_r(z_r),\ g_{43} = \frac{\partial^2 E(\Pi)}{\partial P_r \partial P_n} = \frac{2M}{\overline{\varphi}(1-\delta)},$$

$$g_{44} = \frac{\partial^2 E(\Pi)}{\partial P_r^2} = \frac{-2M}{\overline{\varphi}\delta(1-\delta)},\ g_{45} = \frac{\partial^2 E(\Pi)}{\partial P_r \partial P_a} = 0,\ g_{51} = \frac{\partial^2 E(\Pi)}{\partial P_a \partial z_n} = 0,\ g_{52} = \frac{\partial^2 E(\Pi)}{\partial P_a \partial z_r} = 0,$$

$$g_{53} = \frac{\partial^2 E(\Pi)}{\partial P_a \partial P_n} = 0,\ g_{54} = \frac{\partial^2 E(\Pi)}{\partial P_a \partial P_r} = 0,\ g_{55} = \frac{\partial^2 E(\Pi)}{\partial P_a^2} = -2\beta$$

The Hessian matrix is defined as follows $H = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} \end{bmatrix}$

$$|H_1| = g_{11} = -(P_n + s_n + h_n) f_n(z_n) < 0,$$

$$|H_2| = \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = (P_n + s_n + h_n) f_n(z_n)\ \ (P_r + s_r + h_r) f_r(z_r) > 0$$

$$|H_3| = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} = g_{11}(g_{22}g_{33} - g_{23}g_{32}) - 0 + g_{13}(g_{21}g_{32} - g_{22}g_{31}) =$$

$$= -(P_n + s_n + h_n) f_n(z_n) \left[(P_r + s_r + h_r) f_r(z_r) \frac{2M}{\overline{\varphi}(1-\delta)} - 0 \right]$$

$$+ [1 - F_n(z_n)][0 + (P_r + s_r + h_r) f_r(z_r)[1 - F_n(z_n)]] =$$

$$= (P_r + s_r + h_r) f_r(z_r) \left[[1 - F_n(z_n)]^2 - (P_n + s_n + h_n) \frac{2M}{\overline{\varphi}(1-\delta)} f_n(z_n) \right]$$

Since $[1 - F_n(z_n)]^2 < 1$ and $(P_n + s_n + h_n) \frac{2M}{\overline{\varphi}(1-\delta)} f_n(z_n) > 1$ in our analysis, this

principal minor is negative Thus, $|H_3| < 0$

$$|H_4| = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} = \begin{vmatrix} -(P_n + s_n + h_n) f_n(z_n) & 0 & 1 - F_n(z_n) & 0 \\ 0 & -(P_r + s_r + h_r) f_r(z_r) & 0 & 1 - F_r(z_r) \\ 1 - F_n(z_n) & 0 & \frac{-2M}{\overline{\varphi}(1-\delta)} & \frac{2M}{\overline{\varphi}(1-\delta)} \\ 0 & 1 - F_r(z_r) & \frac{2M}{\overline{\varphi}(1-\delta)} & \frac{-2M}{\overline{\varphi}\delta(1-\delta)} \end{vmatrix}$$

$$|H_4| = -(P_n + s_n + h_n) f_n(z_n) \begin{bmatrix} -(P_r + s_r + h_r) f_r(z_r) [\frac{4M^2}{\bar{\varphi}^2 \delta (1-\delta)^2} - \frac{4M^2}{\bar{\varphi}^2 (1-\delta)^2}] + \\ [1 - F_r(z_r)]^2 \frac{2M}{\bar{\varphi}(1-\delta)} \end{bmatrix}$$

$$-[1 - F_n(z_n)]^2 \left[(P_r + s_r + h_r) f_r(z_r) \frac{2M}{\bar{\varphi}\delta(1-\delta)} - [1 - F_r(z_r)]^2 \right] =$$

$$= (P_n + s_n + h_n) f_n(z_n) \left[(P_r + s_r + h_r) f_r(z_r) \frac{4M^2}{\bar{\varphi}^2 \delta (1-\delta)} - [1 - F_r(z_r)]^2 \frac{2M}{\bar{\varphi}(1-\delta)} \right]$$

$$-[1 - F_n(z_n)]^2 \left[(P_r + s_r + h_r) f_r(z_r) \frac{2M}{\bar{\varphi}\delta(1-\delta)} - [1 - F_r(z_r)]^2 \right] =$$

$$\Rightarrow |H_4| = (P_r + s_r + h_r) f_r(z_r) \frac{2M}{\bar{\varphi}\delta} \left[(P_n + s_n + h_n) f_n(z_n) \frac{2M}{\bar{\varphi}(1-\delta)} - [1 - F_n(z_n)]^2 \frac{1}{1-\delta} \right]$$

$$-[1 - F_r(z_r)]^2 \left[(P_n + s_n + h_n) f_n(z_n) \frac{2M}{\bar{\varphi}(1-\delta)} - [1 - F_n(z_n)]^2 \right]$$

It can be shown numerically that for the parameters used in our analysis $|H_4| > 0$ In addition, we have $|H_5| = |H_4| \times (-2\beta)$ Since $|H_4| > 0$, we will have $|H_5| < 0$ As a result, the Hessian matrix is negative semidefinite and the expected profit function is strictly concave

Appendix E. Proof for Theorem 1

$$\frac{\partial E_R(\Pi)}{\partial z_n} = -\frac{[1-F_n(z_n)]\overline{\varphi}}{2M}\left[(y_n^0+\mu_n)-\delta(y_r^0+\mu_r)+2\Theta_n(z_n)+\delta\Theta_r(z_r)\right]$$
$$-(c_n+h_n)F_n(z_n)+(P_n^0-c_n+2s_n)\frac{[1-F_n(z_n)]}{2}$$

Where $y_n^0 = M\left[1-\frac{P_n^0-P_r^0}{\overline{\varphi}(1-\delta)}\right]$ and $y_r^0 = M\left[\frac{\delta P_n^0-P_r^0}{\overline{\varphi}\delta(1-\delta)}\right]$ If we assume $R(z_n)=\frac{\partial E_R(\Pi)}{\partial z_n}$, we will need to find the zeros of $R(z_n)$

$$\frac{\partial R(z_n)}{\partial z_n} = \frac{\partial}{\partial z_n}\left[\frac{\partial E_R(\Pi)}{\partial z_n}\right] =$$

$$= f_n(z_n)\left[(\frac{\overline{\varphi}}{2M})\left[(y_n^0+\mu_n)-\delta(y_r^0+\mu_r)+2\Theta_n(z_n)+\delta\Theta_r(z_r)\right]-\frac{1}{2}(P_n^0+c_n+2h_n+2s_n)\right]$$
$$+[1-F_n(z_n)]^2\frac{\overline{\varphi}}{M}$$

$$\Rightarrow \frac{\partial^2 R(z_n)}{\partial z_n^2} = \frac{\partial f_n(z_n)}{\partial z_n}\left[(\frac{\overline{\varphi}}{2M})\left[(y_n^0+\mu_n)-\delta(y_r^0+\mu_r)+2\Theta_n(z_n)+\delta\Theta_r(z_r)\right]-\frac{1}{2}(P_n^0+c_n+2h_n+2s_n)\right]$$
$$-3f_n(z_n)[1-F_n(z_n)]\frac{\overline{\varphi}}{M}$$

$$\Rightarrow \frac{\partial^2 R(z_n)}{\partial z_n^2} = \frac{\partial f_n(z_n)}{\partial z_n}\frac{1}{f_n(z_n)}\left[\frac{\partial R(z_n)}{\partial z_n}-[1-F_n(z_n)]^2\frac{\overline{\varphi}}{M}\right]-3f_n(z_n)[1-F_n(z_n)]\frac{\overline{\varphi}}{M}$$

For $\frac{\partial R(z_n)}{\partial z_n}=0 \Rightarrow \frac{\partial^2 R(z_n)}{\partial z_n^2} = -\frac{\partial f_n(z_n)}{\partial z_n}\frac{[1-F_n(z_n)]^2}{f_n(z_n)}\frac{\overline{\varphi}}{M}-3f_n(z_n)[1-F_n(z_n)]\frac{\overline{\varphi}}{M}$

$$\frac{\partial^2 R(z_n)}{\partial z_n^2} = -[1-F_n(z_n)]\frac{\overline{\varphi}}{M}\left[\frac{\partial f_n(z_n)}{\partial z_n}\frac{[1-F_n(z_n)]}{f_n(z_n)}+3f_n(z_n)\right]$$

Defining $r() = \frac{f()}{1-F()}$ which is known as the hazard rate (Barlow and Proschan, 1975, Petruzzi and Dada, 1999), we will have

$$\frac{\partial^2 R(z_n)}{\partial z_n^2} = -[1 - F_n(z_n)]\frac{\overline{\varphi}}{M}\left[\frac{\partial f_n(z_n)}{\partial z_n}\frac{1}{r_n(z_n)} + 3f_n(z_n)\right]$$

We find that at the point where $\frac{\partial R(z_n)}{\partial z_n} = 0$, the value of $\frac{\partial^2 R(z_n)}{\partial z_n^2}$ is independent of z_n

In addition, we have $R(B_n) = -(c_n + h_n) < 0$

I) If $\frac{\partial f_n(z_n)}{\partial z_n} > -3f_n(z_n)r(z_n)$, then $\frac{\partial^2 R(z_n)}{\partial z_n^2} < 0$ which indicates that $R(z_n)$ is either monotone or unimodal, and it has at most two roots In addition, $R(B_n) = -(c_n + h_n) < 0$ Thus, if $R(z_n)$ has only one root, it shows a change of sign from positive to negative, which corresponds to a local maximum of $E_R(\Pi)$ If $R(z_n)$ has two roots, the larger of the two corresponds to a local maximum and the smaller of the two corresponds to a local minimum Either way, $E_R(\Pi)$ has only one local maximum which is determined either as the unique value of z_n that satisfies $R(z_n) = \frac{\partial E_R(\Pi)}{\partial z_n} = 0$ or as the larger of two values of z_n that satisfy $R(z_n) = \frac{\partial E_R(\Pi)}{\partial z_n} = 0$

II) If $\frac{\partial f_n(z_n)}{\partial z_n} < -3f_n(z_n)r(z_n)$, then $\frac{\partial^2 R(z_n)}{\partial z_n^2} > 0$ Since $R(B_n) < 0$, it means that $R(z_n)$ has to change sign from positive to negative when z_n increases up to B_n Note that this needs a sufficient condition such as $R(A_n) > 0$ Thus, the only possibility of this happening is if $R(z_n)$ has only one root which corresponds to a local maximum for $E_R(\Pi)$

In either I or II, we can claim that the largest value of z_n that satisfies $\frac{\partial E_R(\Pi)}{\partial z_n} = 0$ (which in the case of having one root for $R(z_n)$ is the only point) should be chosen as z_n^*

Now, we calculate the derivatives of $E_R(\Pi)$ with respect to z_r

$$\Rightarrow \frac{\partial E_R(\Pi)}{\partial z_r} = [1 - F_r(z_r)]\frac{\delta\overline{\varphi}}{2M}\left[(y_n^0 + \mu_n) + (y_r^0 + \mu_r) - \Theta_n(z_n) - \Theta_r(z_r)\right]$$
$$-(c_r + h_r + v)F_r(z_r) + \frac{1}{2}(P_r^0 - c_r + 2s_r - v)[1 - F_r(z_r)]$$

If we assume $L(z_r) = \frac{\partial E_R(\Pi)}{\partial z_r}$, we have

$$\frac{\partial^2 E_R(\Pi)}{\partial z_r^2} = \frac{\partial L(z_r)}{\partial z_r} = -f_r(z_r)\frac{\delta\overline{\varphi}}{2M}\left[(y_n^0 + \mu_n) + (y_r^0 + \mu_r) - \Theta_n(z_n) - \Theta_r(z_r)\right]$$
$$+ [1 - F_r(z_r)]^2 \frac{\delta\overline{\varphi}}{2M} - (c_r + h_r + v)f_r(z_r) - \frac{1}{2}f_r(z_r)(P_r^0 - c_r + 2s_r - v)$$

$$\Rightarrow \frac{\partial L(z_r)}{\partial z_r} = -f_r(z_r)\left[\frac{\delta\overline{\varphi}}{2M}\left[(y_n^0 + \mu_n) + (y_r^0 + \mu_r) - \Theta_n(z_n) - \Theta_r(z_r)\right] + \frac{1}{2}(P_r^0 + c_r + v + 2s_r + 2h_r)\right]$$
$$+ [1 - F_r(z_r)]^2 \frac{\delta\overline{\varphi}}{2M}$$

$$\Rightarrow \frac{\partial L^2(z_r)}{\partial z_r^2} = -\frac{\partial f_r(z_r)}{\partial z_r}\left[\frac{\delta\overline{\varphi}}{2M}\left[(y_n^0+\mu_n)+(y_r^0+\mu_r)-\Theta_n(z_n)-\Theta_r(z_r)\right]+\frac{1}{2}(P_r^0+c_r+v+2s_r+2h_r)\right]$$

$$+\frac{\delta\overline{\varphi}}{2M}f_r(z_r)[1-F_r(z_r)]-2f_r(z_r)[1-F_r(z_r)]\frac{\delta\overline{\varphi}}{2M}=$$

$$=-\frac{\partial f_r(z_r)}{\partial z_r}\left[\frac{\delta\overline{\varphi}}{2M}\left[(y_n^0+\mu_n)+(y_r^0+\mu_r)-\Theta_n(z_n)-\Theta_r(z_r)\right]+\frac{1}{2}(P_r^0+c_r+v+2s_r+2h_r)\right]$$

$$-\frac{\delta\overline{\varphi}}{2M}f_r(z_r)[1-F_r(z_r)]$$

$$\Rightarrow \frac{\partial L^2(z_r)}{\partial z_r^2} = -\frac{\partial f_r(z_r)}{\partial z_r}\left[\frac{\frac{\partial L(z_r)}{\partial z_r}-[1-F_r(z_r)]^2\frac{\delta\overline{\varphi}}{2M}}{-f_r(z_r)}\right]-\frac{\delta\overline{\varphi}}{2M}f_r(z_r)[1-F_r(z_r)]$$

For $\frac{\partial L(z_r)}{\partial z_r}=0$ we have

$$\frac{\partial L^2(z_r)}{\partial z_r^2} = -\frac{\partial f_r(z_r)}{\partial z_r}\left[\frac{[1-F_r(z_r)]^2}{f_r(z_r)}\right]\frac{\delta\overline{\varphi}}{2M}-f_r(z_r)[1-F_r(z_r)]\frac{\delta\overline{\varphi}}{2M}\Rightarrow$$

$$\frac{\partial L^2(z_r)}{\partial z_r^2} = [1-F_r(z_r)]\frac{\delta\overline{\varphi}}{2M}\left[-\frac{\partial f_r(z_r)}{\partial z_r}\frac{1}{r(z_r)}-f_r(z_r)\right]$$

If $\frac{\partial f_r(z_r)}{\partial z_r}\frac{1}{r(z_r)}+f_r(z_r)>0 \Rightarrow \frac{\partial L^2(z_r)}{\partial z_r^2}<0 \Rightarrow$ If

$$\frac{\partial f_r(z_r)}{\partial z_r} > -f_r(z_r)\ r(z_r) \Rightarrow \frac{\partial L^2(z_r)}{\partial z_r^2}<0$$

We also have $R(B_r)=-(c_r+h_r+v)<0$

As a result, a similar analysis (to what we had for the new product) is applicable here Now, we define the sufficient conditions $R(A_n)\geq 0$ and $L(A_r)\geq 0$ that we used in the analysis earlier

$$R(A_n) = -\frac{\overline{\varphi}}{2M}\left[(y_n^0 + \mu_n) - \delta(y_r^0 + \mu_r) + 2\mu_n - 2A_n + \delta\Theta_r(z_r)\right] + \frac{1}{2}(P_n^0 - c_n + 2s_n)$$

We substitute $y_n^0 = M\left[1 - \frac{P_n^0 - P_r^0}{\overline{\varphi}(1-\delta)}\right]$ and $y_r^0 = M\left[\frac{\delta P_n^0 - P_r^0}{\overline{\varphi}\delta(1-\delta)}\right]$ while we consider $z_r = A_r$

$$R(A_n) = -\frac{\overline{\varphi}}{2M}\left[(M\left[1 - \frac{P_n^0 - P_r^0}{\overline{\varphi}(1-\delta)} - \frac{\delta P_n^0 - P_r^0}{\overline{\varphi}(1-\delta)}\right] + \mu_n - \delta\mu_r) + 2\mu_n - 2A_n + \delta\mu_r - \delta A_r\right]$$
$$+\frac{1}{2}(P_n^0 - c_n + 2s_n) = -\frac{\overline{\varphi}}{2M}\left[M\left[1 - \frac{(1+\delta)P_n^0 - 2P_r^0}{\overline{\varphi}(1-\delta)}\right] + 3\mu_n - 2A_n - \delta A_r\right]$$
$$+\frac{1}{2}(P_n^0 - c_n + 2s_n)$$

We know that $P_n^0 = \frac{c_n + \overline{\varphi}}{2} + \frac{(\mu_n + \delta\mu_r)\overline{\varphi}}{2M}$ and $P_r^0 = \frac{c_r + v + \delta\overline{\varphi}}{2} + \frac{(\mu_n + \mu_r)\delta\overline{\varphi}}{2M}$ We substitute the values for P_n^0 and P_r^0 in the equation above and find the relationship among the model parameters that need to be in place so that $R(A_n) \geq 0$ So, we have

$$R(A_n) = -\frac{\overline{\varphi}}{2M}\left[M\left[1 - \frac{(1+\delta)P_n^0 - 2P_r^0}{\overline{\varphi}(1-\delta)}\right] + 3\mu_n - 2A_n - \delta A_r\right] + \frac{1}{2}(P_n^0 - c_n + 2s_n) \geq 0$$

$$\Rightarrow R(A_n) = \frac{\overline{\varphi}}{2M}(4\mu_n - 2A_n - \delta A_r) + \frac{1}{2(1-\delta)}\left[\delta c_n - c_r - v + 2(1-\delta)s_n\right] \geq 0$$

$$\Rightarrow \frac{\overline{\varphi}}{2M}(4\mu_n - 2A_n - \delta A_r) + \frac{\delta}{2(1-\delta)}c_n + s_n \geq \frac{1}{2(1-\delta)}(c_r + v)$$

$$L(A_r)=\frac{\delta\overline{\varphi}}{2M}\left[(M\left[1-\frac{P_n^0-P_r^0}{\overline{\varphi}(1-\delta)}\right]+\mu_n)+(M\left[\frac{\delta P_n^0-P_r^0}{\overline{\varphi}\delta(1-\delta)}\right]+\mu_r)-\mu_n+A_n-\mu_r+A_r\right]$$

$$+\frac{1}{2}(P_r^0-c_r-v+2s_r)=$$

$$=\frac{\delta\overline{\varphi}}{2M}\left[M\left[1-\frac{P_r^0}{\overline{\varphi}\delta}\right]+A_n+A_r\right]+\frac{1}{2}(P_r^0-c_r-v+2s_r)=\frac{\delta\overline{\varphi}}{2}+\frac{\delta\overline{\varphi}}{2M}\left[A_n+A_r\right]+\frac{1}{2}(-c_r-v+2s_r)$$

$$\Rightarrow L(A_r)=\frac{\delta\overline{\varphi}}{2}+\frac{\delta\overline{\varphi}}{2M}\left[A_n+A_r\right]+\frac{1}{2}(-c_r-v+2s_r)\geq 0$$

$$\Rightarrow\frac{\delta\overline{\varphi}}{2M}\left[M+A_n+A_r\right]+s_r\geq\frac{1}{2}(c_r+v)$$

Sufficient conditions for $R(z_n)$ and $L(z_r)$ to have at least one root, are $R(A_n)\geq 0$ and $L(A_r)\geq 0$ respectively Thus, we need to have

$$\frac{\overline{\varphi}}{2M}(4\mu_n-2A_n-\delta A_r)+\frac{\delta}{2(1-\delta)}c_n+s_n\geq\frac{1}{2(1-\delta)}(c_r+v)$$

$$\frac{\delta\overline{\varphi}}{2M}\left[M+A_n+A_r\right]+s_r\geq\frac{1}{2}(c_r+v)$$

Appendix F. Concavity test for the firm's expected profit in the Centralized Channel when constraint is binding

To check for the concavity of the firm's expected profit function, we need to calculate the Hessian matrix First and second derivatives of the firm's expected profit with respect to P_n, P_r, z_n and z_r are as follows

$$\frac{\partial E(\Pi_M)}{\partial z_n} = -(c_n + h_n) + (P_n + s_n + h_n)[1 - F_n(z_n)] \tag{F1}$$

$$\frac{\partial^2 E(\Pi_M)}{\partial z_n^2} = -(P_n + s_n + h_n) f_n(z_n) \tag{F2}$$

$$\begin{aligned}\frac{\partial E_R(\Pi)}{\partial z_r} &= -(c_r + h_r)F_r(z_r) + (P_r + s_r - c_r)[1 - F_r(z_r)] \\ &- \frac{2M}{r^2\beta}\left[\frac{\delta P_n - P_r}{\varphi\delta(1-\delta)}\right] - \frac{2}{r^2\beta} z_r + (\frac{1}{r})(v - c_l + \frac{\alpha}{\beta}) - v\end{aligned} \tag{F3}$$

$$\frac{\partial^2 E_R(\Pi)}{\partial z_r^2} = -(P_r + s_r + h_r) f_r(z_r) - \frac{2}{r^2\beta} \tag{F4}$$

$$\begin{aligned}\frac{\partial E_R(\Pi)}{\partial P_n} &= \frac{M}{\overline{\varphi}(1-\delta)}\left\{\overline{\varphi}(1-\delta) - 2P_n + 2P_r + c_n - c_r + \frac{1}{r}(v - c_l + \frac{\alpha}{\beta}) - v - \frac{2}{r^2\beta}\left[M\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + z_r\right]\right\} \\ &+ \mu_n - \Theta_n(z_n)\end{aligned} \tag{F5}$$

$$\frac{\partial^2 E_R(\Pi)}{\partial P_n^2} = \frac{-2M}{\overline{\varphi}(1-\delta)}\left\{1+\frac{M}{r^2\beta}\left[\frac{1}{\overline{\varphi}(1-\delta)}\right]\right\} \tag{F6}$$

$$\frac{\partial E_R(\Pi)}{\partial P_r} = \frac{M}{\overline{\varphi}\delta(1-\delta)}\left[2\delta P_n - 2P_r - \delta c_n + c_r - \frac{1}{r}(v - c_I + \frac{\alpha}{\beta}) + v + \frac{2M}{r^2\beta}\left[\frac{\delta P_n - P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \frac{2z_r}{r^2\beta}\right] + \mu_r - \Theta_r(z_r) \tag{F7}$$

$$\frac{\partial^2 E_R(\Pi)}{\partial P_r^2} = \frac{-M}{\overline{\varphi}\delta(1-\delta)}\left\{2+\frac{2M}{r^2\beta}\left[\frac{1}{\overline{\varphi}\delta(1-\delta)}\right]\right\} \tag{F8}$$

If we define $g() = E(\Pi_M)$, we have

$g_{11} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_n^2} = -(P_n + s_n + h_n) f_n(z_n)$, $g_{12} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_n \partial z_r} = 0$,

$g_{13} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_n \partial P_n} = 1 - F_n(z_n)$, $g_{14} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_n \partial P_r} = 0$, $g_{21} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_r \partial z_n} = 0$,

$g_{22} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_r^2} = -(P_r + s_r + h_r) f_r(z_r) - \dfrac{2}{r^2\beta}$, $g_{23} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_r \partial P_n} = -\dfrac{2M}{r^2\beta\overline{\varphi}(1-\delta)}$,

$g_{24} = \dfrac{\partial^2 E(\Pi_M)}{\partial z_r \partial P_r} = \dfrac{2M}{r^2\beta\overline{\varphi}\delta(1-\delta)} + 1 - F_r(z_r)$, $g_{31} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_n \partial z_n} = 1 - F_n(z_n)$,

$g_{32} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_n \partial z_r} = -\dfrac{2M}{r^2\beta\overline{\varphi}(1-\delta)}$, $g_{33} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_n^2} = \dfrac{-2M}{\overline{\varphi}(1-\delta)}\left\{1 + \dfrac{M}{r^2\beta\overline{\varphi}(1-\delta)}\right\}$,

$g_{34} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_n \partial P_r} = \dfrac{2M}{\overline{\varphi}(1-\delta)}\left[1 + \dfrac{M}{r^2\beta\overline{\varphi}\delta(1-\delta)}\right]$, $g_{41} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_r \partial z_n} = 0$,

$g_{42} = \dfrac{\partial^2 E(\Pi_M)}{\partial P_r \partial z_r} = 1 - F_r(z_r) + \dfrac{2M}{r^2\beta\overline{\varphi}\delta(1-\delta)}$,

$$g_{43} = \frac{\partial^2 E(\Pi_M)}{\partial P_r \partial P_n} = \frac{2M}{\overline{\varphi}(1-\delta)}\left[1 + \frac{M}{r^2 \beta\overline{\varphi}\delta(1-\delta)}\right],$$

$$g_{44} = \frac{\partial^2 E(\Pi_M)}{\partial P_r^2} = \frac{-M}{\overline{\varphi}\delta(1-\delta)}\left\{2 + \frac{2M}{r^2\beta}\left[\frac{1}{\overline{\varphi}\delta(1-\delta)}\right]\right\}$$

The Hessian matrix is defined as follows $H = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}$

$$|H_1| = g_{11} = -(P_n + s_n + h_n) f_n(z_n) < 0,$$

$$|H_2| = \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = (P_n + s_n + h_n) f_n(z_n)\ [(P_r + s_r + h_r) f_r(z_r) + \frac{2}{r^2\beta}] > 0$$

$$|H_3| = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} = g_{11}(g_{22}g_{33} - g_{23}g_{32}) - 0 + g_{13}(g_{21}g_{32} - g_{22}g_{31}) =$$

$$= -(P_n + s_n + h_n) f_n(z_n)\left[[(P_r + s_r + h_r) f_r(z_r) + \frac{2}{r^2\beta}]\frac{2M}{\overline{\varphi}(1-\delta)}\left\{1 + \frac{M}{r^2\beta\overline{\varphi}(1-\delta)}\right\} - [\frac{2M}{r^2\beta\overline{\varphi}(1-\delta)}]^2\right]$$
$$+ [1 - F_n(z_n)]\left[0 + [(P_r + s_r + h_r) f_r(z_r) + \frac{2}{r^2\beta}][1 - F_n(z_n)]\right]$$

$$= -(P_n + s_n + h_n) f_n(z_n)\left[(P_r + s_r + h_r) f_r(z_r)\frac{2M}{\overline{\varphi}(1-\delta)}\left\{1 + \frac{M}{r^2\beta\overline{\varphi}(1-\delta)}\right\} + \frac{4M}{r^2\beta\overline{\varphi}(1-\delta)}\right]$$
$$+ [1 - F_n(z_n)]^2\left[(P_r + s_r + h_r) f_r(z_r) + \frac{2}{r^2\beta}\right]$$

$$= -(P_r + s_r + h_r) f_r(z_r)\left\{(P_n + s_n + h_n) f_n(z_n)\frac{2M}{\overline{\varphi}(1-\delta)}\left\{1 + \frac{M}{r^2\beta\overline{\varphi}(1-\delta)}\right\} - [1 - F_n(z_n)]^2\right\}$$
$$- (P_n + s_n + h_n) f_n(z_n)\left[\frac{4M}{r^2\beta\overline{\varphi}(1-\delta)}\right] + [1 - F_n(z_n)]^2\left[\frac{2}{r^2\beta}\right] < 0$$

$$\Rightarrow |H_3| < 0$$

$$|H_4| = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} =$$

$$= \begin{vmatrix} -(P_n+s_n+h_n)f_n(z_n) & 0 & 1-F_n(z_n) & 0 \\ 0 & -(P_r+s_r+h_r)f_r(z_r)-\dfrac{2}{r^2\beta} & -\dfrac{2M}{r^2\beta\bar{\varphi}(1-\delta)} & \dfrac{2M}{r^2\beta\bar{\varphi}\delta(1-\delta)}+1-F_r(z_r) \\ 1-F_n(z_n) & -\dfrac{2M}{r^2\beta\bar{\varphi}(1-\delta)} & \dfrac{-2M}{\bar{\varphi}(1-\delta)}\left\{1+\dfrac{M}{r^2\beta\bar{\varphi}(1-\delta)}\right\} & \dfrac{2M}{\bar{\varphi}(1-\delta)}\left[1+\dfrac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right] \\ 0 & \dfrac{2M}{r^2\beta\bar{\varphi}\delta(1-\delta)}+1-F_r(z_r) & \dfrac{2M}{\bar{\varphi}(1-\delta)}\left[1+\dfrac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right] & \dfrac{-2M}{\bar{\varphi}\delta(1-\delta)}\left\{1+\dfrac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right\} \end{vmatrix}$$

$$|H_4| = -(P_n+s_n+h_n)f_n(z_n)\left\{ \begin{array}{l} -\left[(P_r+s_r+h_r)f_r(z_r)+\dfrac{2}{r^2\beta}\right]\left[\dfrac{2M}{\bar{\varphi}(1-\delta)}\right]^2\left[1+\dfrac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right](\dfrac{1}{\delta}-1) \\ -\dfrac{1}{r^2\beta}\left[\dfrac{2M}{\bar{\varphi}(1-\delta)}\right]^2\left[1+\dfrac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right][1-F_r(z_r)] \\ -\left[\dfrac{2M}{r^2\beta\bar{\varphi}\delta(1-\delta)}+1-F_r(z_r)\right]\times\left\{ \begin{array}{l} \dfrac{2M}{\bar{\varphi}(1-\delta)}\left[\dfrac{1}{\delta}\left[\dfrac{M}{r^2\beta\bar{\varphi}(1-\delta)}\right]^2+1+\dfrac{M}{r^2\beta\bar{\varphi}\delta}\right] \\ +\dfrac{2M}{\bar{\varphi}(1-\delta)}\left\{1+\dfrac{M}{r^2\beta\bar{\varphi}(1-\delta)}\right\}F_r(z_r) \end{array} \right\} \end{array} \right\}$$

$$-[1-F_n(z_n)]^2\left[\frac{2M}{\bar{\varphi}\delta(1-\delta)}\left[1+\frac{M}{r^2\beta\bar{\varphi}\delta(1-\delta)}\right](P_r+s_r+h_r)f_r(z_r)+\frac{4M}{r^2\beta\bar{\varphi}\delta(1-\delta)}F_r(z_r)-[1-F_r(z_r)]^2\right]$$

It can be shown numerically that for the parameters used in our study $|H_4| > 0$ and as a result the Hessian matrix is negative semidefinite Thus, the firm's expected profit in the Centralized Channel is strictly concave when the constraint is binding

Appendix G. Concavity test for the firm's expected profit in the Decentralized Channel

To check for the concavity of the firm's expected profit function, we need to calculate the Hessian matrix First and second derivatives of the firm's expected profit with respect to P_n, P_r, z_n and z_r are as follows

$$\frac{\partial E(\Pi_M)}{\partial z_n} = -(c_n + h_n) + (P_n + s_n + h_n)[1 - F_n(z_n)] \tag{G1}$$

$$\frac{\partial^2 E(\Pi_M)}{\partial z_n^2} = -(P_n + s_n + h_n) f_n(z_n) \tag{G2}$$

$$\frac{\partial E(\Pi_M)}{\partial z_r} = -(c_r + w + h_r) + (P_r + s_r + h_r)[1 - F_r(z_r)] \tag{G3}$$

$$\frac{\partial^2 E(\Pi_M)}{\partial z_r^2} = -(P_r + s_r + h_r) f_r(z_r) \tag{G4}$$

$$\frac{\partial E(\Pi_M)}{\partial P_n} = M\left[1 - \frac{2P_n - P_r}{\overline{\varphi}(1-\delta)}\right] + \frac{c_n M}{\overline{\varphi}(1-\delta)} + \mu_n - \Theta_n(z_n) + \frac{(P_r - c_r - w)M}{\overline{\varphi}(1-\delta)} \tag{G5}$$

$$\frac{\partial^2 E(\Pi_M)}{\partial P_n^2} = \frac{-2M}{\overline{\varphi}(1-\delta)} \tag{G6}$$

$$\frac{\partial E(\Pi_M)}{\partial P_r} = \frac{(P_n - c_n)M}{\overline{\varphi}(1-\delta)} + M\left[\frac{\delta P_n - 2P_r}{\overline{\varphi}\delta(1-\delta)}\right] + \frac{(c_r + w)M}{\overline{\varphi}\delta(1-\delta)} + \mu_r - \Theta(z_r) \tag{G7}$$

$$\frac{\partial^2 E(\Pi_M)}{\partial P_r^2} = \frac{-2M}{\overline{\varphi}\delta(1-\delta)} \tag{G8}$$

If we define $g(\,) = E(\Pi_M)$, we have

$g_{11} = \frac{\partial^2 E(\Pi_M)}{\partial z_n^2} = -(P_n + s_n + h_n)f_n(z_n)$, $g_{12} = \frac{\partial^2 E(\Pi_M)}{\partial z_n \partial z_r} = 0$,

$g_{13} = \frac{\partial^2 E(\Pi_M)}{\partial z_n \partial P_n} = 1 - F_n(z_n)$, $g_{14} = \frac{\partial^2 E(\Pi_M)}{\partial z_n \partial P_r} = 0$, $g_{21} = \frac{\partial^2 E(\Pi_M)}{\partial z_r \partial z_n} = 0$,

$g_{22} = \frac{\partial^2 E(\Pi_M)}{\partial z_r^2} = -(P_r + s_r + h_r)f_r(z_r)$, $g_{23} = \frac{\partial^2 E(\Pi_M)}{\partial z_r \partial P_n} = 0$,

$g_{24} = \frac{\partial^2 E(\Pi_M)}{\partial z_r \partial P_r} = 1 - F_r(z_r)$, $g_{31} = \frac{\partial^2 E(\Pi_M)}{\partial P_n \partial z_n} = 1 - F_n(z_n)$, $g_{32} = \frac{\partial^2 E(\Pi_M)}{\partial P_n \partial z_r} = 0$,

$g_{33} = \frac{\partial^2 E(\Pi_M)}{\partial P_n^2} = \frac{-2M}{\overline{\varphi}(1-\delta)}$, $g_{34} = \frac{\partial^2 E(\Pi_M)}{\partial P_n \partial P_r} = \frac{2M}{\overline{\varphi}(1-\delta)}$, $g_{41} = \frac{\partial^2 E(\Pi_M)}{\partial P_r \partial z_n} = 0$,

$g_{42} = \frac{\partial^2 E(\Pi_M)}{\partial P_r \partial z_r} = 1 - F_r(z_r)$, $g_{43} = \frac{\partial^2 E(\Pi_M)}{\partial P_r \partial P_n} = \frac{2M}{\overline{\varphi}(1-\delta)}$, $g_{44} = \frac{\partial^2 E(\Pi_M)}{\partial P_r^2} = \frac{-2M}{\overline{\varphi}\delta(1-\delta)}$

The Hessian matrix is defined as follows $H = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}$

$|H_1| = g_{11} = -(P_n + s_n + h_n)f_n(z_n) < 0$,

$|H_2| = \begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = (P_n + s_n + h_n)f_n(z_n)\ \ (P_r + s_r + h_r)f_r(z_r) > 0$

$$|H_3| = \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} = g_{11}(g_{22}g_{33} - g_{23}g_{32}) - 0 + g_{13}(g_{21}g_{32} - g_{22}g_{31}) =$$

$$= -(P_n + s_n + h_n)f_n(z_n)\left[(P_r + s_r + h_r)f_r(z_r)\frac{2M}{\overline{\varphi}(1-\delta)} - 0\right]$$

$$+ [1 - F_n(z_n)]\big[0 + (P_r + s_r + h_r)f_r(z_r)[1 - F_n(z_n)]\big] =$$

$$= (P_l + s_r + h_r)f_r(z_r)\left[[1 - F_n(z_n)]^2 - (P_n + s_n + h_n)\frac{2M}{\overline{\varphi}(1-\delta)}f_n(z_n)\right]$$

Since $[1 - F_n(z_n)]^2 < 1$ and $(P_n + s_n + h_n)\frac{2M}{\overline{\varphi}(1-\delta)}f_n(z_n) > 1$ in our analysis, this principal minor is negative Thus, $|H_3| < 0$

$$|H_4| = \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} = \begin{vmatrix} -(P_n + s_n + h_n)f_n(z_n) & 0 & 1 - F_n(z_n) & 0 \\ 0 & -(P_r + s_r + h_r)f_r(z_r) & 0 & 1 - F_r(z_r) \\ 1 - F_n(z_n) & 0 & \frac{-2M}{\overline{\varphi}(1-\delta)} & \frac{2M}{\overline{\varphi}(1-\delta)} \\ 0 & 1 - F_r(z_r) & \frac{2M}{\overline{\varphi}(1-\delta)} & \frac{-2M}{\overline{\varphi}\delta(1-\delta)} \end{vmatrix}$$

$$|H_4| = -(P_n + s_n + h_n)f_n(z_n)\begin{bmatrix} -(P_r + s_r + h_r)f_r(z_r)[\frac{4M^2}{\overline{\varphi}^2\delta(1-\delta)^2} - \frac{4M^2}{\overline{\varphi}^2(1-\delta)^2}] + \\ [1 - F_r(z_r)]^2\frac{2M}{\overline{\varphi}(1-\delta)} \end{bmatrix}$$

$$-[1 - F_n(z_n)]^2\left[(P_r + s_r + h_r)f_r(z_r)\frac{2M}{\overline{\varphi}\delta(1-\delta)} - [1 - F_r(z_r)]^2\right] =$$

$$= (P_n + s_n + h_n)f_n(z_n)\left[(P_r + s_r + h_r)f_r(z_r)\frac{4M^2}{\overline{\varphi}^2\delta(1-\delta)} - [1 - F_r(z_r)]^2\frac{2M}{\overline{\varphi}(1-\delta)}\right]$$

$$-[1 - F_n(z_n)]^2\left[(P_r + s_r + h_r)f_r(z_r)\frac{2M}{\overline{\varphi}\delta(1-\delta)} - [1 - F_r(z_r)]^2\right] =$$

$$\Rightarrow |H_4| = (P_r + s_r + h_r)f_r(z_r)\frac{2M}{\overline{\varphi}\delta(1-\delta)}\left[(P_n + s_n + h_n)f_n(z_n)\frac{2M}{\overline{\varphi}} - [1 - F_n(z_n)]^2\right]$$

$$-[1 - F_r(z_r)]^2\left[(P_n + s_n + h_n)f_n(z_n)\frac{2M}{\overline{\varphi}(1-\delta)} - [1 - F_n(z_n)]^2\right]$$

It can be shown numerically that for the parameters used in our analysis $|H_4| > 0$ As a result, the Hessian matrix is negative semidefinite and the expected profit function is strictly concave

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