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Higher-Order, Multiplicatively Weighted Voronoi Diagrams: A New Approach To Trade Area Analysis

by

Robert W. South

THESIS

Submitted to the Department of Geography
in partial fulfilment of the requirements
for the Master of Arts
Wilfrid Laurier University
1996

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ABSTRACT

Trade area models have traditionally been classified into one of two groups: spatial monopoly models or market penetration approaches. Spatial monopoly models define trade areas in a deterministic manner, while market penetration models construct market areas that are probabilistic. Past research has grouped Voronoi diagram models exclusively with the spatial monopoly approach. However, the research contained within this thesis demonstrates that Voronoi diagrams can be used to generate trade areas that are consistent with market penetration models.

This thesis introduces two new Voronoi diagrams: the order-k, multiplicatively weighted Voronoi diagram (OKMWVD) and the ordered order-k, multiplicatively weighted Voronoi diagram (OOKMWVD). When interpreted as trade area models, these diagrams generate market areas for a set of facilities which are overlapping and probabilistic. These new Voronoi diagrams allow for the simultaneous inclusion of a weight (measuring facility attraction) and consumer choice sets.

The new models are demonstrated on data collected for the supermarkets located within the cities of Kitchener and Waterloo. This application shows how Voronoi diagrams can be used to generate sales estimates for retail facilities. The OOKMWVD allows for the examination of the effect of different consumer preference levels on the sales estimates of an individual facility.

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1.0 INTRODUCTION

Retailing is an extremely complex and competitive industry. In order to be successful, it is essential for store owners, especially those of chain stores, to gain a competitive advantage over the other retailers that offer similar products. One method that retailers use to gain this position of superiority is trade area (or market area) analysis. Jones and Simmons (1993) point out the importance of trade area analysis to retailers:

> Trade area analysis permits [retailers] to make better use of their existing facilities; it becomes increasingly important to many retailers as competition increases, operating costs rise, and the growth of population and income stabilizes (Jones and Simmons, 1993, 329).

A retail market area can be defined as "The [set of] locations served by a particular facility" (Jones and Simmons, 1993, 452). Essentially, a trade area defines the region from which a store (or retail centre) draws its customers. If a retailer is able to identify his/her trade area, he/she is also able to determine who his/her customers are (Jones and Simmons, 1993, 329). By gaining a good understanding of the market (i.e. customers) that they serve, retailers are better equipped to formulate a marketing strategy (e.g. product mix, advertising themes, etc.) that will be effective in maximizing the market share of their store(s). As a result, the most successful retailers are those that are able to acquire detailed and accurate information about their market areas and the customers located within those regions.

Past research has lead to the development of a wide range of analytical methods that can be used to model retail market areas. These techniques are used by retailers to help them build a profile of their market so that they are able to make important decisions. A summary of how trade area models are perceived in the retail geography literature is now presented.

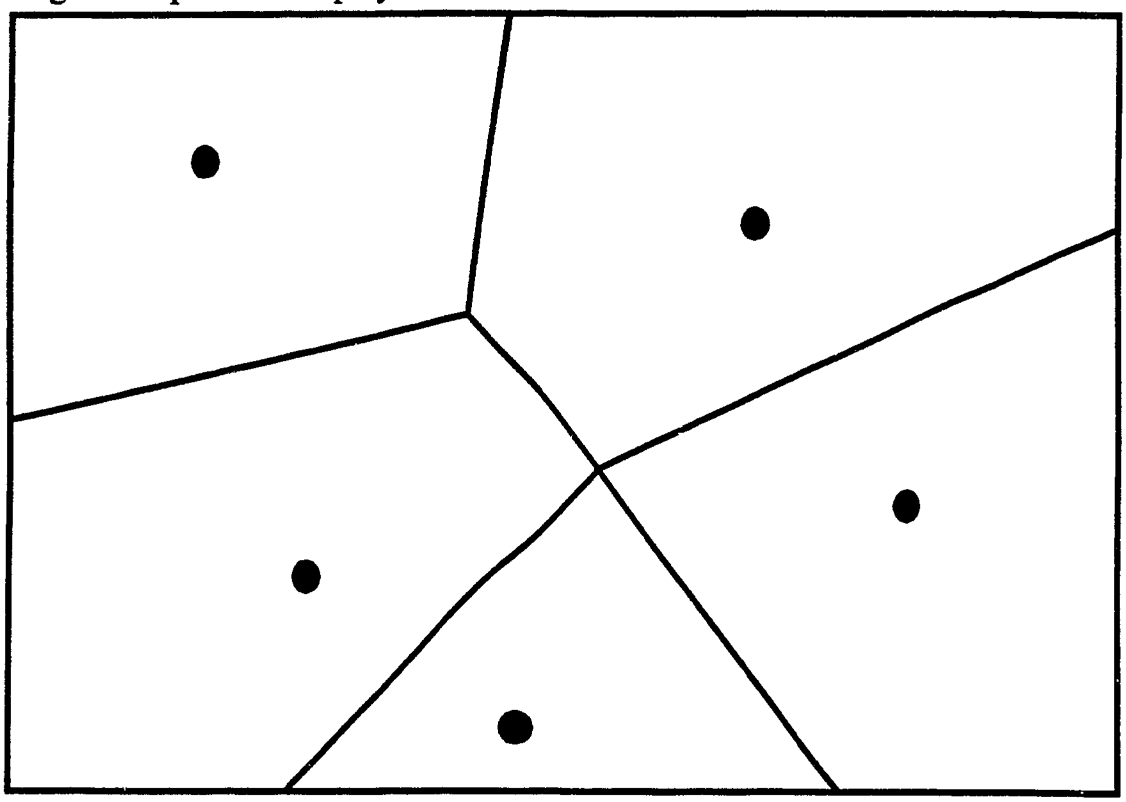
1.1 Retail Trade Area Models: A Review Of The Literature

There exists a large body of literature which is concerned with the modelling of retail market areas. Within this literature, retail trade area models have traditionally been grouped into one of two classes: the spatial monopoly approach or the market penetration approach. A number of recent publications (e.g. Craig et al. (1984), Ghosh and McLafferty (1987), Jones and Simmons (1993), Lea (1989), Lea and Menger (1990)) have all classified trade area models in a similar manner.

According to the literature, the spatial monopoly approach constructs market areas so that all of the consumers within a specified region are exclusively assigned to one facility (Jones and Simmons, 1993, 331) (see Figure 1). This approach makes the assumption that all of the customers within a given trade area will exclusively patronize the store (or centre) to which they have been deterministically assigned. Spatially monopolistic trade areas are commonly equated with the construction of order-1 Voronoi (or Thiessen) polygons (to be defined in section 2.0) for a set of store locations (e.g Ghosh and McLafferty (1987), Jones and Mock (1984), Jones and Simmons (1993)).

Several researchers have stated that when the planar, ordinary Voronoi diagram (to be defined in section 2.0) is used to model a system of trade areas, the modeller is explicitly making the assumption that all consumers will "...select the [facility] nearest to

Figure 1: Spatial Monopoly



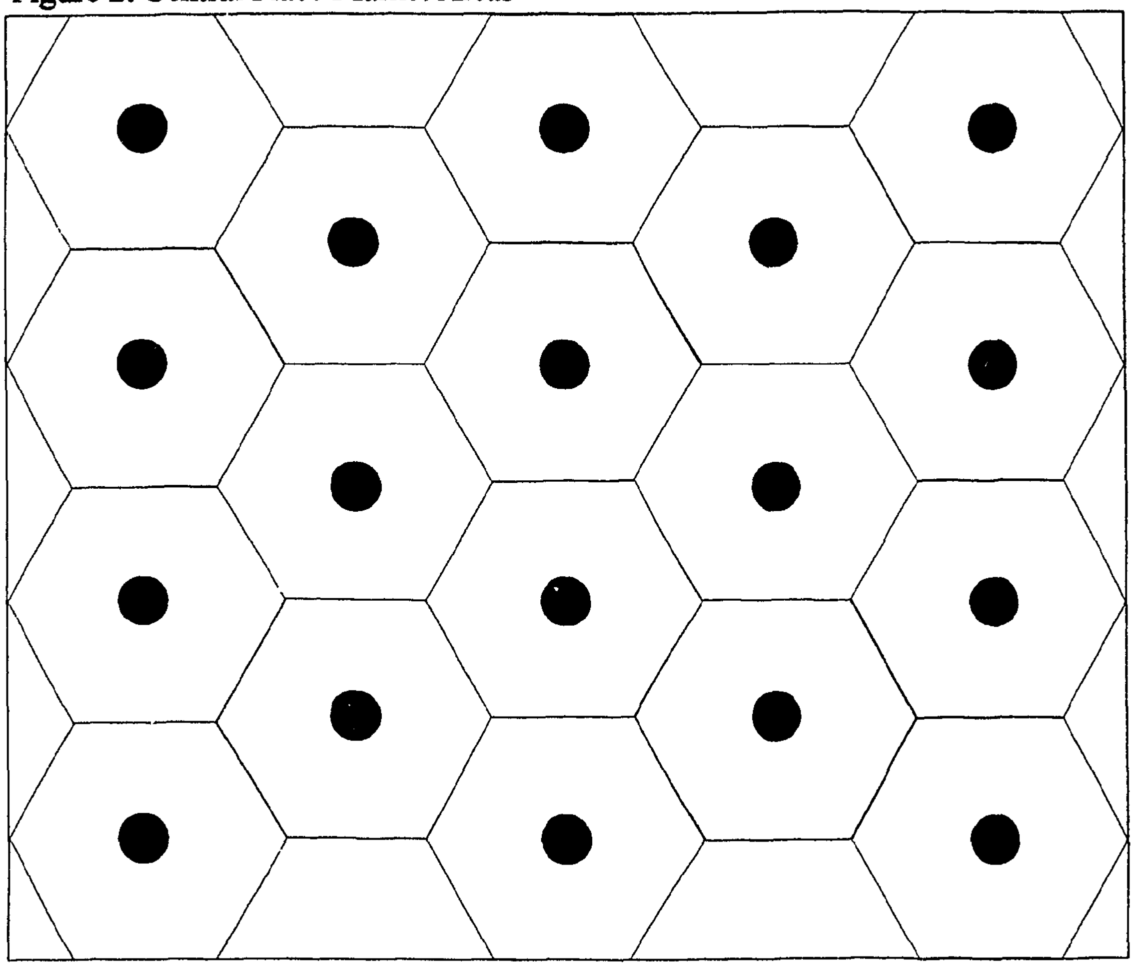
Source: Jones and Simmons, 1993, 330.

them" (Ghosh and McLafferty, 1987, 65). The planar, ordinary Voronoi diagram is a spatial tessellation that assigns all the locations in a defined region to the closest, in terms of Euclidean distance, member of a set of points; weighted Voronoi diagrams (to be defined in section 2.0) assign all the locations in a defined region to the closest, in terms of weighted distance, member of a point set. Due to this property, trade area modellers have used order-1 Voronoi diagrams as a method for the construction of spatial monopolies.

Central Place Theory (Christaller (1933), Lòsch (1940)) was one of the first trade area models to assume that consumers patronize the closest centre in terms of Euclidean distance. The hexagonal market areas of central place theory are the equivalent of planar, ordinary Voronoi polygons (see Figure 2). This approach to trade area analysis is not favoured by practitioners for the following reason.

A nearest-centre solution implies that trade areas are mutually exclusive (or non-overlapping) entities with a deterministic assignment rule. However, much research has shown that this is not realistic. Researchers feel that it is too simplistic to assume that the only objective of the consumer is to minimize transportation costs because "As choice, tastes, and other nontransportation factors increase in importance ... transportation costs [become] relatively less crucial in consumer behaviour" (Beaumont, 1987, 29). This means that consumers do not necessarily shop at the closest facility, but at the one which gives them the highest level of utility. Recent research has shown that consumers consider other variables (e.g. store attributes), in addition to distance, when deciding where to shop (e.g. Drezner (1994), Gautschi (1981), Eagle (1988), Miller (1993), Stanley and Sewall (1976), Schuler (1981), Timmermans (1982)). As Drezner (1994) points out:

Figure 2: Central Place Market Areas



> It is generally agreed that customers, through a decision making process, make a choice of the facility which maximizes their satisfaction. This choice is determined by some formula according to which customers evaluate alternative facilities' attributes weighted by their personal preferences to arrive at an overall facility attractiveness. A trade-off between distance and quality takes place (Drezner, 1994, 239).

This means that, when deciding where to shop, consumers consider the attractiveness of each alternative (based on distance and other attributes) and patronize the most attractive facility. Researchers believe that consumers will form choice sets and each alternative within an individual's choice set has a probability (of being patronized) associated with it. Thus, consumers are willing to accept the disutility of travelling further if they can increase their overall utility by acquiring better shopping opportunities at a more distant outlet. All of this implies that real world trade areas are non-mutually exclusive and probabilistic constructions.

For this reason (a mutually exclusive and deterministic outcome), the nearest-centre solution (i.e. the planar, ordinary Voronoi diagram) has not been applied to very many real world situations. Researchers have labelled this approach as a quick and dirty method, only to be used when the retailer wants to generate a general picture of his/her trade area map.

Several researchers (e.g. Hanjoul et al. (1989), Huff (1973), O'Kelly and Miller (1989), Von Hohenbalken and West (1984a)) have also modelled market areas with the use of weighted Voronoi diagrams. The majority of these models has employed the multiplicatively weighted Voronoi diagram, which is the weighted Voronoi diagram used in this thesis. The weighted Voronoi model is different from the planar, ordinary Voronoi model due to the fact that it assigns a weight of relative attractiveness (based on a set of store attributes, such as size, hours of operation, type of centre, ancillary features, etc.) to each facility. Consumers are then assumed to make a trade-off between distance and site attractiveness, and choose the store which maximizes their utility (see Figure 3).

Even though not all of the consumers patronize the closest facility in terms of Euclidean distance, past trade area models that have used weighted Voronoi diagrams have represented market areas as spatial monopolies. This means that each trade area is a mutually exclusive entity, every consumer is deterministically allocated to the nearest centre in terms of weighted distance and consumers do not have a choice set of alternatives to choose from. As a result, real world applications of the weighted Voronoi model have also been limited in number.

The market penetration approach, on the other hand, has received much attention in the academic literature and has been widely used as a practical method of retail analysis. This type of model (also known as the gravity or spatial interaction model) constructs market areas so that there is spatial variation in the proportion of customers serviced by a facility (Jones and Simmons, 1993, 339). Trade areas are defined as a series of probability zones, where the probability of a customer patronizing a certain centre decreases as the distance from the facility increases (see Figure 4).

Gravity models construct retail trade areas based on the assumption that "...the proportion of demand that any site receives from any single demand unit is inversely related to distance and positively related to the attractiveness of the site itself" (Lea and

Figure 3: Trade Areas Defined By The Multiplicatively Weighted Voronoi Diagram

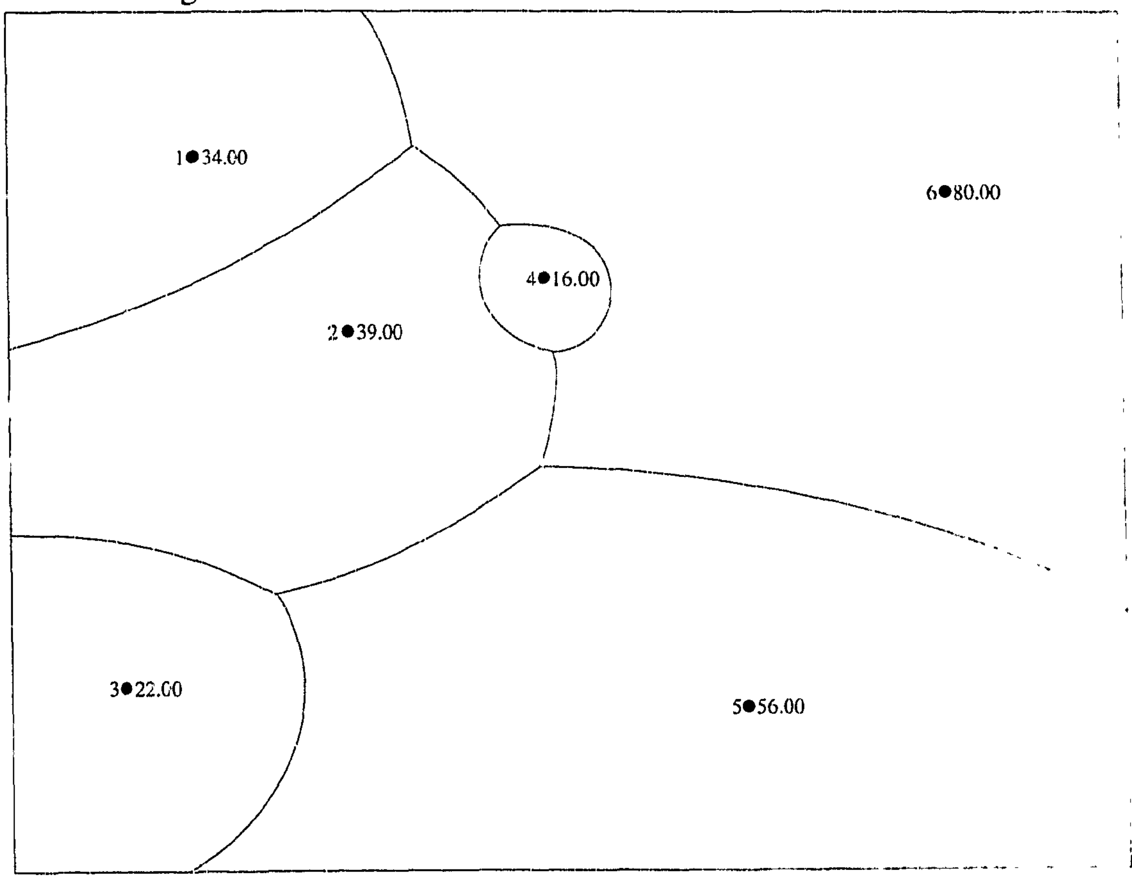
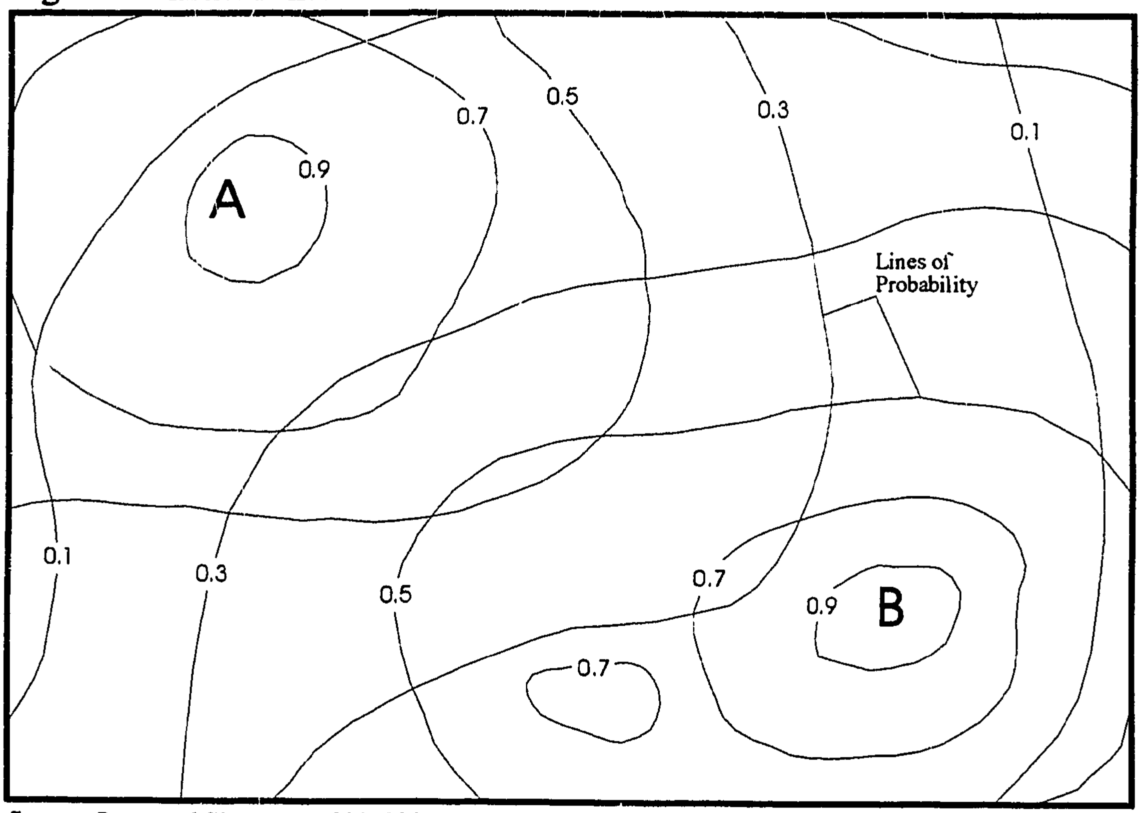


Figure 4: Market Penetration



Source: Jones and Simmons, 1993, 330.

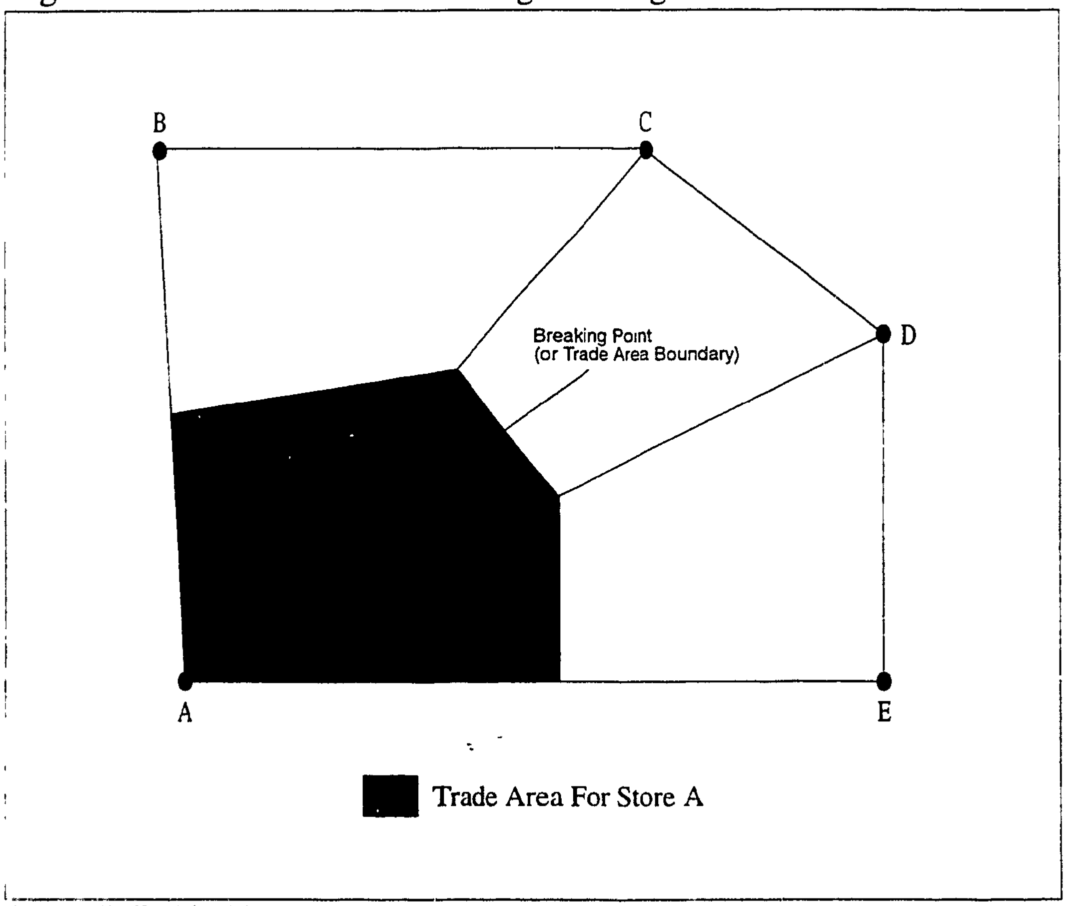
Menger, 1990, 17). As Ghosh and McLafferty (1987) point out, this assumption "...is similar to the principle of retail gravitation proposed by Reilly" (Ghosh and McLafferty, 1987, 90). Although the Reilly Law of Retail Gravitation is not considered to be a fully developed spatial interaction model, most researchers recognize it as an essential step in the development of retail gravity models. Reilly's Law, which is based on the same principles as Newton's Law of Universal Gravitation, generally states that "...two cities attract retail trade from an intermediate city or town ... approximately in direct proportion to the populations of the two cities and in inverse proportion to the square of the distances from the two cities to the intermediate town" (Huff, 1963, 81-82). The Reilly model has the following general form:

$$\frac{B_a}{B_b} = \left(\frac{P_a}{P_b}\right)\left(\frac{D_b}{D_a}\right)^2 \qquad (1)$$

where, B_a = the proportion of retail trade from the intermediate town attracted by city A;
B_b = the proportion of retail trade from the intermediate town attracted by city B;
P_a = the population of city A;
P_b = the population of city B;
D_a = the distance between the intermediate town and city A; and
D_b = the distance between the intermediate town and city B.

The breaking point between two cities (i.e. the point where both cities attract consumers in the same proportion) is calculated by setting the left-hand side of equation (1) equal to one and solving for D_a or D_b (Tinkler, 1992, 12). This breaking point has been interpreted as the trade area boundary between cities (see Figure 5). As a result, researchers have labelled Reilly's Law as being deterministic (e.g. Berry (1967), Craig et al. (1984), Ghosh

Figure 5: Trade Area Definition Using Breaking Points



Source: Huff, 1964, 36.

and McLafferty (1987)).

Reilly's model was the first trade area model to apply the principles of gravity to retail attraction. However, it was Huff (1963) who, in extending the work of Reilly, developed the first spatial interaction model. The Huff model states that the probability of a consumer (at a given location) patronizing a certain store is directly influenced by the attractiveness of the store (measured by the size of the facility) and inversely influenced by the distance between the consumer's location and the store location (measured by travel time). The general form of this model is:

$$P(C_{ij}) = \frac{\left(\frac{S_j}{T_{ij}^{\lambda}}\right)}{\sum_{j=1}^{n}\left(\frac{S_j}{T_{ij}^{\lambda}}\right)}$$

where, $P(C_{ij})$ = the probability of a consumer at location i patronizing store j;
S_j = the size of store j in square feet;
T_{ij} = the travel time needed for the consumer located at i to reach store j; and
λ = the parameter representing the effect of travel time on different kinds of shopping trips.

Although it has been over thirty years since Huff first presented his probabilistic market area model, the general formulation of the model has not changed very much. A number of researchers (e.g. Gautschi (1981), Stanley and Sewall (1976)) have improved the measure of store attraction by factoring other variables, in addition to size, into its calculation. As a result of these additional variables, the model's predictive ability has been improved (Ghosh and McLafferty, 1987, 94).

According to the literature, gravity models offer a number of significant advantages over other trade area models because:

> ...although spatial interaction models are calibrated on aggregate consumer behaviour, they are in fact based on principles of individual consumer behaviour, whereby consumers are assumed to trade off the distance of travelling to a store, or store cluster, with its attractiveness. In an aggregate consumer model, this becomes the probability that consumers in an origin zone will patronize a specific destination (Lea and Menger, 1990, 17).

The end result, researchers conclude, is that gravity models produce trade area definitions which are probabilistic, non-mutually exclusive and, as a result, more representative of the real world situation.

The literature seems to imply that there is a dichotomy of trade area models. The spatial monopoly approach has been labelled as simple and ad hoc because it creates mutually exclusive trade areas with a deterministic assignment rule. The market penetration approach, on the other hand, is seen to have more rigour because it defines trade areas as probabilistic and overlapping entities. However, there exists much evidence within the marketing literature to suggest that this separation of models is not correct.

1.2 Retail Trade Area Models: A Reinterpretation

Section 1.1 provided a general summary of how retail trade area models have been presented in the academic literature. The preceding section shows that the literature has segregated the models into two different classes. There have been very few publications which have questioned this classification. Nevertheless, the few articles which do argue against the traditional classification of models offer much evidence to suggest that this view may not be correct. These alternative ideas are now discussed.

1.2.1 Spatial Monopoly Vs. Market Penetration

A number of researchers (e.g. Beaumont (1987), Ghosh and McLafferty (1987)) have suggested that spatial monopoly models are not separate from spatial interaction models but are, in fact, a special case of the market penetration approach. As Beaumont (1987) states, "...the nearest-centre hypothesis can be thought of merely as one member of a family of more general models" (Beaumont, 1987, 31). The spatial monopoly and market penetration approaches are both based on the same fundamental tenets of consumer behaviour; both methods define trade areas based on the principle that consumers make a trade-off between centre attractiveness and distance travelled (some spatial monopoly models make the additional assumption that all facilities are equally attractive and consumers, therefore, visit the closest facility).

The main difference between spatial monopoly models and gravity models is the assumption that each makes with regard to the number of facilities that consumers visit. The spatial monopoly approach assumes that consumers visit only one centre, while market penetration models assume that customers will visit more than one centre. The difference in this assumption results in dramatically different probability functions for the two approaches. In the spatial monopoly method, the probability that a consumer patronizes a certain store is either one or zero, whereas a pure market penetration approach (i.e. consumers are able to visit any alternative that they want to) places no restrictions, beyond the normal rules of probability, on the probability function.

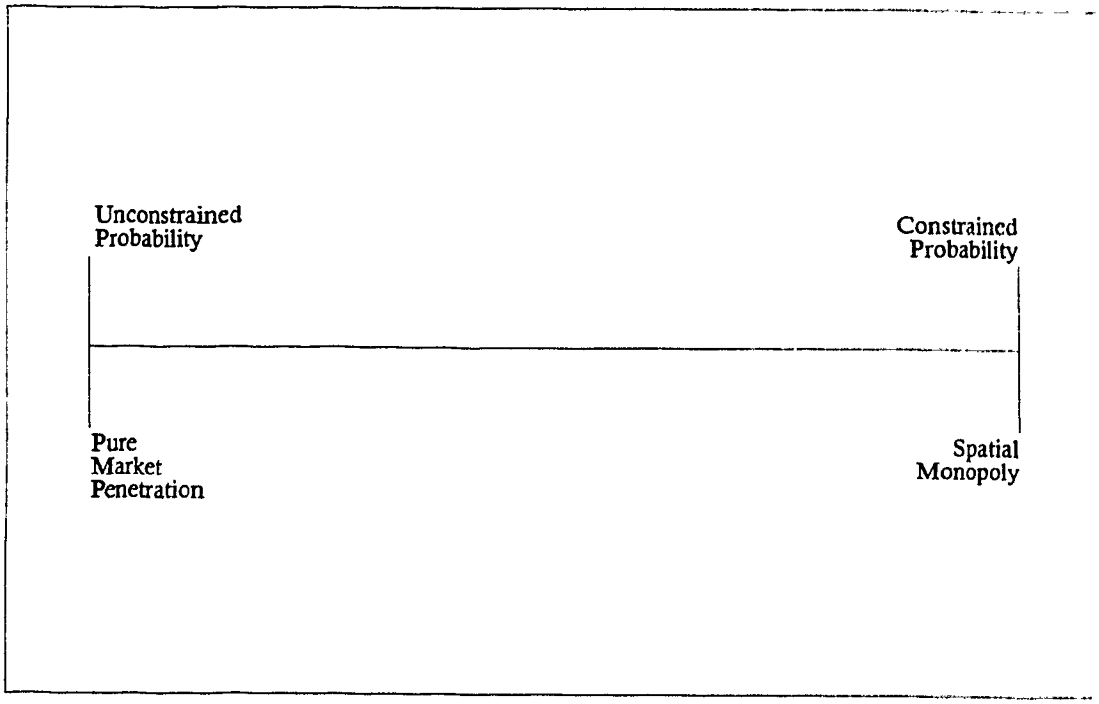
Therefore, trade area models should be reclassified into a continuum of probabilistic approaches (see Figure 6). At one end of this continuum (pure market penetration), there are no restrictions on the number of facilities that a consumer can visit and, as a result, there are no additional constraints placed on the probability function. At the other extreme of the continuum (spatial monopoly), it is assumed that consumers visit only one centre and, therefore, the probability that a consumer patronizes a certain store is constrained to 0 or 1.

1.2.2 Deterministic Vs. Probabilistic

The majority of the retail market area literature makes a clear distinction between models that are deterministic and those that are probabilistic; spatial monopoly models generate a deterministic outcome while market penetration models produce a probabilistic result. The literature also suggests that certain constructions always produce a deterministic solution (e.g. order-1 Voronoi diagrams, Reilly's Law) while other methods always generate a probabilistic outcome (e.g. spatial interaction models). This generalization is being made based on one interpretation of the trade areas produced by these approaches; if interpreted another way, the market areas produced by Voronoi diagrams and Reilly's Law (and other models which have traditionally been viewed as deterministic) become probabilistic.

The lines (i.e. bisectors) generated with Voronoi diagrams or Reilly's Law have traditionally been interpreted as trade area boundaries which consumers are not allowed to cross. This means that if a consumer falls inside a store's trade area boundary, he/she will patronize that outlet and if a consumer is located outside of a store's trade area boundary,

Figure 6: Continuum of Trade Area Models



he/she will shop at a different facility. This solution results because of the way in which the bisectors have been interpreted.

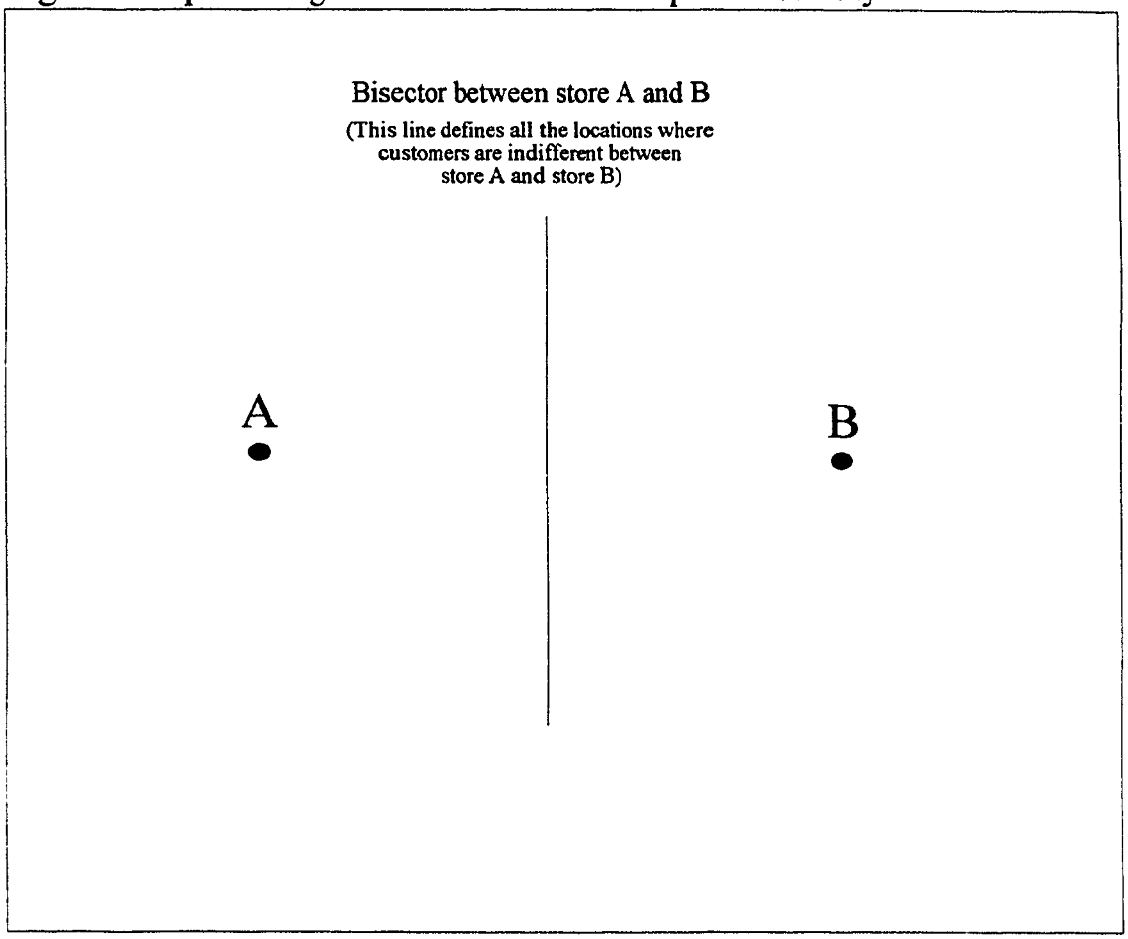
However, if the bisectors are viewed differently, these deterministic trade areas become probabilistic. A bisector between two points (e.g. stores) defines the set of locations which are equally distant from both points (either in terms of Euclidean distance or weighted distance). When thought of in this manner, the bisectors of a Voronoi diagram or Reilly's Law become lines of equal probability, where consumers are indifferent between the two options. For example, consider two stores in the Euclidean plane, store A and store B (see Figure 7). The bisector between these two facilities defines the zone of equal probability; consumers located along this line will patronize both stores with a probability of 0.5. Consumers to the left of this line are more likely to visit store A (but can shop at store B, if they wish) and consumers to the right of this line are more likely to patronize store B (but can visit store A, if they wish).

This is not the first time that this notion of interpreting "deterministic" models in a probabilistic manner has been suggested. Tinkler (1992) demonstrated that Reilly's Law is directly analogous to the probabilistic Huff model. Boots (1980) showed that the bisectors created by weighted Voronoi diagrams are equivalent to those generated by gravity models. Therefore, Voronoi diagrams can be used to create probabilistic trade area definitions.

1.2.3 Conclusion: Purpose Of This Study

The preceding argument has demonstrated that retail trade areas created with a Voronoi diagram can be interpreted in a probabilistic manner. The problem with these

Figure 7: Representing Bisectors As Lines Of Equal Probability



trade areas is that, due to the nature of the construction of order-1 Voronoi diagrams, they are frequently misconstrued as being deterministic (i.e. spatial monopolies). This problem can be solved with the use of higher-order Voronoi diagrams (to be defined in section 2.0). Higher-order Voronoi diagrams use more than one point to define the polygons in the tessellation. This means that there is a set of points associated with each polygon (or trade area) and each individual point belongs to more than one set. The end result are trade areas which are more reminiscent of those which are created by a gravity model. The research to follow introduces two "new" Voronoi diagrams: the order-k, multiplicatively weighted Voronoi diagram and the ordered, order-k, multiplicatively weighted Voronoi diagram. Both of these tessellations create trade areas which are easily interpreted as probabilistic entities.

1.3 Organization And Limitations Of The Study

The purpose of the research contained within this thesis is to construct a more realistic trade area model using the two new Voronoi diagrams. In order to accomplish this overall goal, it was necessary to define a number of smaller objectives and complete these tasks one at a time. The results presented within this thesis have been organized based on these smaller objectives. As a result, this thesis is divided into four separate, but related, sections. These sections are: technical definitions of Voronoi diagrams, the development of a new trade area model, an application of this new model and conclusions of the findings.

The first section, which comprises chapters 2 and 3, deals with the technical explanations of Voronoi diagrams (formal and informal definitions). Chapter 2 discusses the necessary mathematical preliminaries and defines the four types of Voronoi diagrams to be used in this study (i.e. planar ordinary, multiplicatively weighted, order-k and ordered, order-k). Chapter 3 defines the two new Voronoi diagrams (i.e. order-k, multiplicatively weighted and ordered, order-k, multiplicatively weighted) and the geometric properties of these tessellations. All of the above Voronoi diagrams are constructed (physically) with a computer program called VORONOI. This program was developed by Cassel (1993) and the algorithm is based on the work of Gambini (1965).

The second section, which comprises chapter 4, concentrates on the development of a new trade area model. This model utilizes the two new Voronoi diagrams defined in chapter 3. The assumptions contained within this model are developed from the current ideas about the nature of retail trade areas and consumer behaviour.

Section three, which comprises chapter 5, applies the new model to a real world data set. The data set which is used in this analysis relates to the 22 supermarkets contained within the cities of Kitchener-Waterloo. The results generated with the new model are compared to the results produced by the order-1 models.

Section four, which serves as the conclusion, comprises chapter 6. This chapter concludes the findings of the preceding sections and discuss where this study fits into the literature and how it adds to the existing body of knowledge.

This study is limited by the accessibility of retail data. As was mentioned earlier, retailing is an extremely competitive industry. Store owners are not willing to reveal important information such as total sales and overhead costs. Therefore, the data used within this study is information that is publicly accessible. As a result, it is not possible to compare the sales estimates calculated in this thesis with actual sales figures. Instead, the analysis is limited to a comparison of the trade area definitions and sales estimates generated by the different Voronoi models.

2.0 TERMS AND CONCEPTS

The purpose of this chapter is to introduce the mathematical notation that will be commonly used throughout the remainder of the discussion. The three main topics of interest are sets, matrices and Voronoi diagrams. The reader should note that the descriptions contained within this section are not exhaustive. If a more detailed explanation of any of these topics is required, a textbook on the appropriate subject (Breuer, 1958 (Sets), Bronson, 1991 (Matrices) and Okabe et al., 1992 (Voronoi Diagrams)) should be consulted.

2.1 Sets

A set can be defined as "...a collection of definite distinct objects of our perception or thought, which are called elements of the set." (Breuer, 1958, 4). Sets are usually represented by uppercase letters and the elements of the set by lowercase letters. In general, a set of n elements, C, is represented by the following notation

$$C = \{x_1, x_2, \ldots, x_n\}. \tag{2}$$

If the element x belongs to the set C, we write $x \in C$; if x does not belong to C, we write $x \notin C$. A set with at least one element is called a non-empty set and is represented as per equation (2); a set with no elements is referred to as an empty set and is written as $\varnothing$.

Number systems can be represented as sets. In terms of this discussion, there are two number systems that will be defined. First, the set of all real numbers is represented by $\mathbb{R}$. Second, $\mathbb{I}_\infty$ is used to denote the set of all natural numbers and $\mathbb{I}_n$ is used to represent the set of all natural numbers between 1 and n, where $n < \infty$.

Some sets require that the elements contained within them meet certain conditions. Sets of this nature are written as

$$C = \{x \mid \text{conditions}\}.$$

An example of this might be

$$C = \{x \mid x \text{ is a vowel in the English language}\}$$

which would be formally written as

$$C = \{a,e,i,o,u\}. \tag{3}$$

The set C is said to be a subset of set D if every element in C is also an element in D (i.e. $x \in D$ for all $x \in C$). This is represented as $C \subset D$. For example, if $D = \{x \mid x$ is a letter in the English language$\}$ then $C \subset D$, if C is defined by equation (3).

There are three types of operations which can be performed on pairs of sets: union, intersection and complement. The union of two sets is the set of all elements that belongs to at least one of the two sets (see Figure 8a). The union of set C and set D is written as

$$C \cup D = \{x \mid x \in C \text{ or } x \in D\}.$$

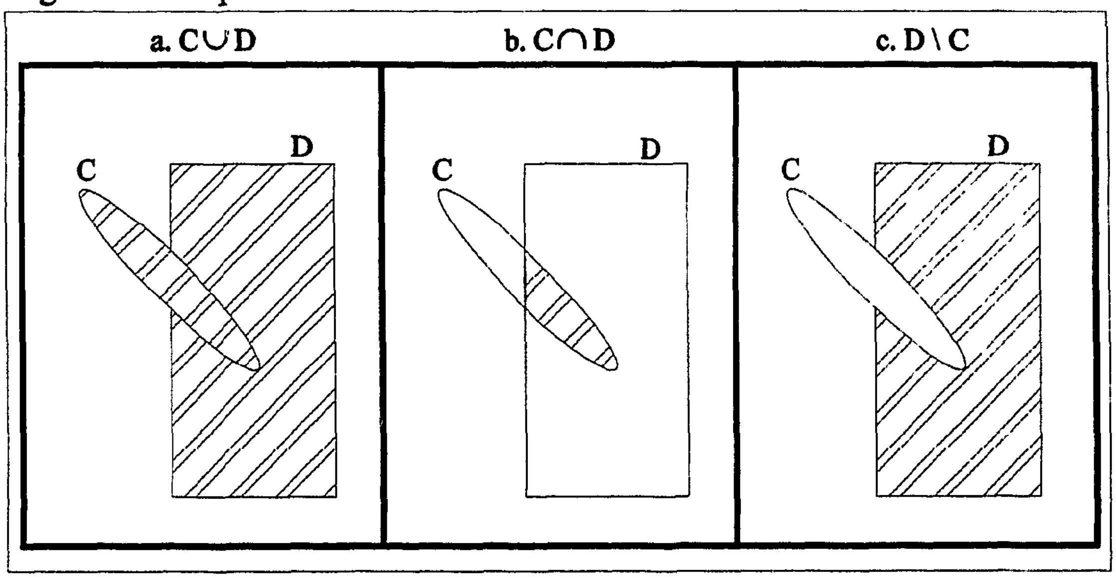
The intersection of two sets is the set of all elements that belongs to both sets (see Figure 8b). The intersection of set C and set D is represented by

$$C \cap D = \{x \mid x \in C \text{ and } x \in D\}.$$

The complement of two sets can be explained in the following way. If set C is a subset of set D, then the complement of C in D is the set of elements in D which do not belong to C (see Figure 8c). This is written as

$$D \backslash C = \{x \mid x \in D \text{ and } x \notin C\}.$$

Figure 8: Set Operations



2.2 Matrices

A matrix is a rectangular (including square) collection of elements (e.g. numbers) arranged in a series of rows and columns (Bronson, 1991, 1). A matrix is conventionally represented by a bold face uppercase letter, while the elements are represented by lowercase letters. If a matrix, **B**, has dimensions $m \times n$, this means that it contains m rows and n columns, and it is represented in the following manner

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}.$$

A column vector is a $m \times 1$ matrix and is written in the following way

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}.$$

A row vector is a $1 \times n$ matrix and is represented by

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}.$$

2.3 Voronoi Diagrams

There are four types of Voronoi diagrams referred to throughout this discussion: the planar ordinary, multiplicatively weighted, order-k and ordered, order-k Voronoi diagrams. The definition of these diagrams will include a verbal description and a formal mathematical representation. It should be noted that the following definitions are similar in form to those found in Okabe et al. (1992) and that much more extensive derivations and proofs can be found there.

2.3.1 The Planar, Ordinary Voronoi Diagram

The planar, ordinary Voronoi diagram (or POVD) can be explained in the following way. Given a finite set of two or more distinct points in the Euclidean plane (the filled circles in Figure 9), space is partitioned in such a way that all of the locations in the plane are assigned to the closest member of the point set with respect to Euclidean distance (Okabe et al., 1992, 66) (see Figure 9). The resulting tessellation is referred to as the POVD and the regions contained within the Voronoi diagram are called ordinary Voronoi polygons (Okabe et al., 1992, 66).

This definition can be stated formally in the following way.

n = a finite number of distinct points in the Euclidean plane, where $2 \leq n < \infty$
$P = \{p_1,\ldots,p_n\}$ represents the set of points
the location of the points are represented with Cartesian coordinates
$(x_{11},x_{12}),\ldots,(x_{n1},x_{n2})$ or location vectors $\mathbf{x}_1,\ldots,\mathbf{x}_n$, where $\mathbf{x}_i \neq \mathbf{x}_j$

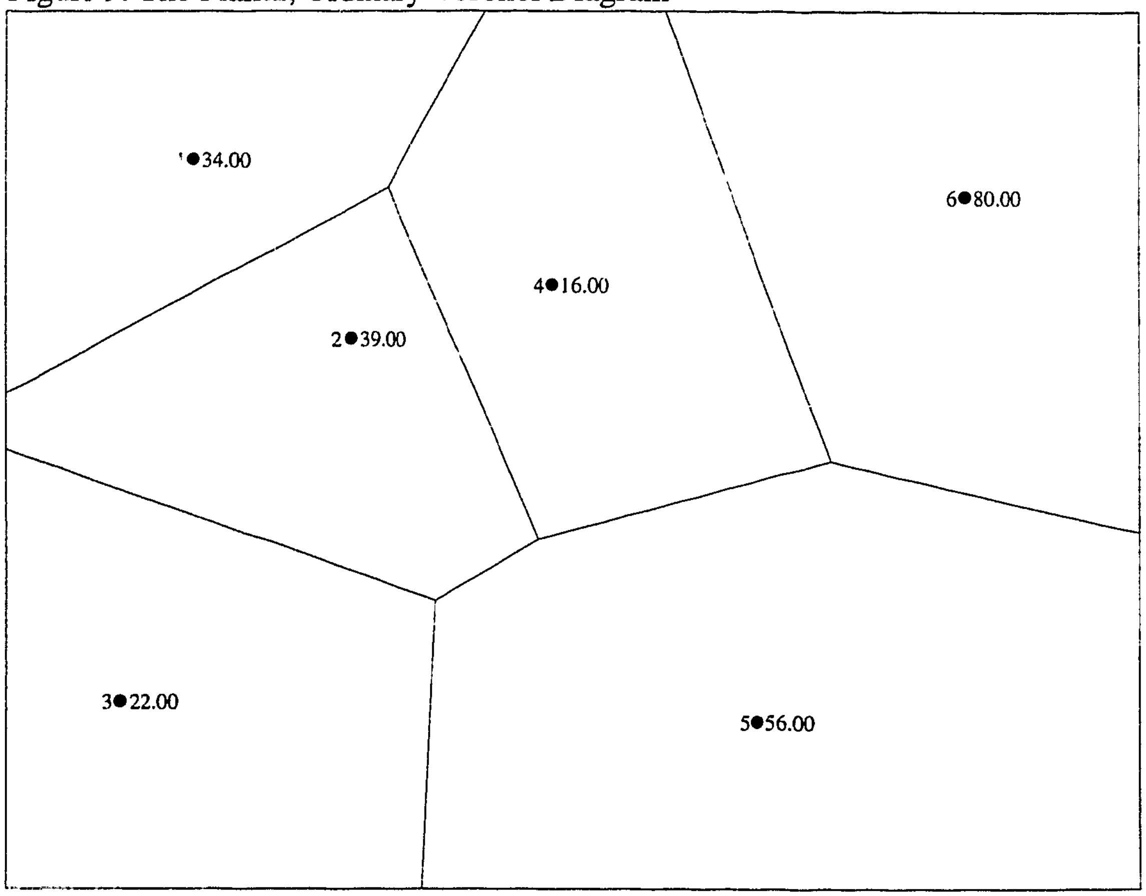
Thus, if p is a point in the plane with a location vector $\mathbf{x}$, then the Euclidean distance from p to p_i is

$$d(p,p_i) = \|\mathbf{x}-\mathbf{x}_i\| = \sqrt{[(\mathbf{x}_1-\mathbf{x}_{i1})^2 + (\mathbf{x}_2-\mathbf{x}_{i2})^2]}.$$

If p_i is the closest point to p or p_i is one of the closest points to p, the following relationship holds:

$$\|\mathbf{x}-\mathbf{x}_i\| \leq \|\mathbf{x}-\mathbf{x}_j\| \text{ for } j \neq i,\ j \in I_n.$$

Figure 9: The Planar, Ordinary Voronoi Diagram



Thus, the POVD can be written mathematically in the following way.

$$V(p_i) = \{\mathbf{x} | \ \|\mathbf{x}-\mathbf{x}_i\| \leq \|\mathbf{x}-\mathbf{x}_j\| \text{ for } j \neq i, j \in I_n\}$$

represents the ordinary Voronoi polygon associated with p_i, and the POVD of P is given by

$$\mathfrak{V} = \{V(p_i),\ldots,V(p_n)\}.$$

Locations that are equidistant from two points form the edges of the Voronoi regions and locations that are equidistant from three or more points are the vertices of the Voronoi edges. The Voronoi edges are the perpendicular bisectors of straight line segments between neighbouring points and are defined by

$$b(p_i,p_j) = \{\mathbf{x} | \ \|\mathbf{x}-\mathbf{x}_i\| = \|\mathbf{x}-\mathbf{x}_j\|\}, j \neq i.$$

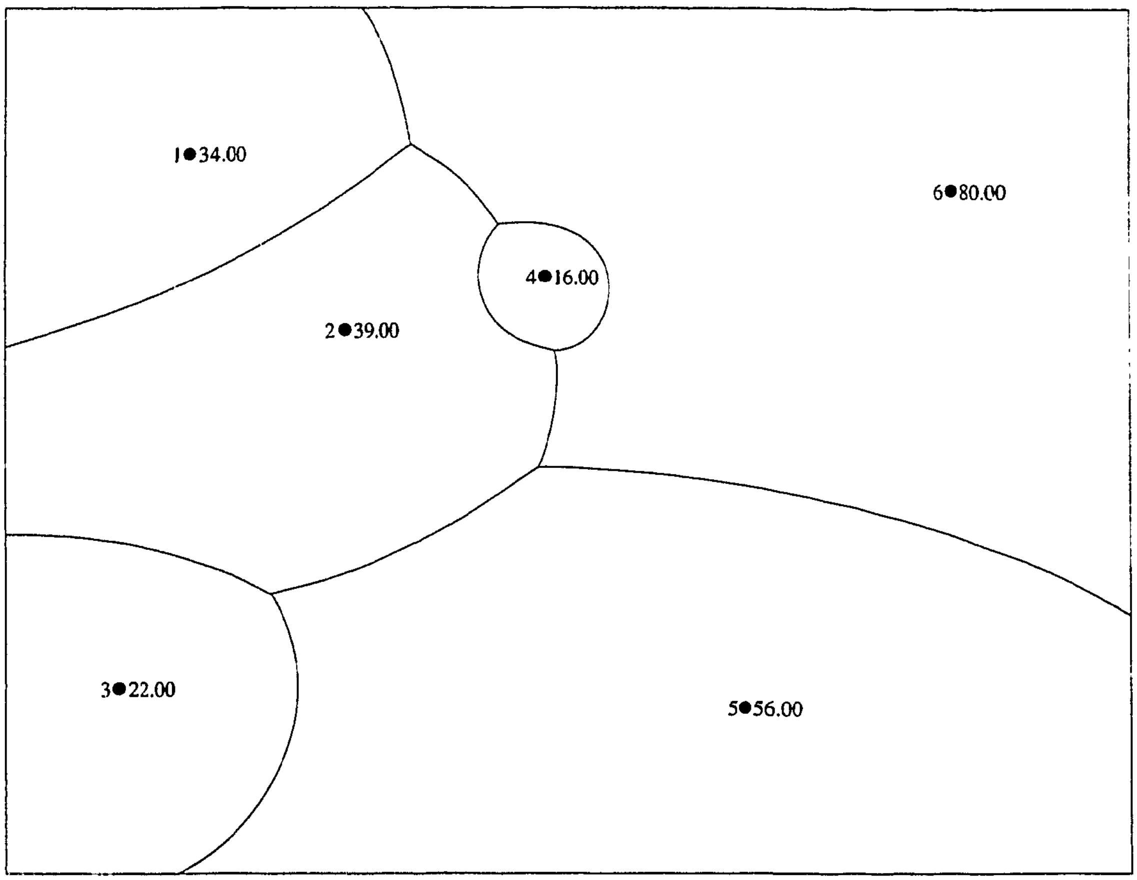
2.3.2 The Multiplicatively Weighted Voronoi Diagram

The ideas developed in the previous section can easily be extended to the multiplicatively weighted Voronoi diagram (or MWVD). With the POVD, all of the members of the point set are identical (except for their locations) (Okabe et al., 1992, 128). However, more often than not, it is the case that the attributes associated with the given point set are not identical for each member of the set (e.g. supermarkets in a city vary in size). Therefore, each point can be weighted based on the attributes associated with the points (the number to the right of the filled circles Figure 10), and the Voronoi diagram can be drawn according to "weighted distances." For the MWVD these weighted distances can be written formally as

$$d(p,p_i)_{mw} = (1/w_i)\|\mathbf{x}-\mathbf{x}_i\|, \ w_i > 0, \text{ where } w_i \text{ is the weight assigned to } p_i.$$

This is referred to as the multiplicatively weighted distance. These distances are used to

Figure 10: The Multiplicatively Weighted Voronoi Diagram



find the multiplicatively weighted Voronoi region. Due to differential weighting between points, the bisectors are no longer straight lines. In the MWVD the bisectors become circles and arcs, and are given by

$$b(p,p_i) = \{\mathbf{x} |\ \|\mathbf{x}-([w_i^2/(w_i^2-w_j^2)]\mathbf{x}_j + [w_j^2/(w_i^2-w_j^2)]\mathbf{x}_i)\| = (w_i w_j)/(w_i^2-w_j^2)\|\mathbf{x}_j-\mathbf{x}_i\|\},$$
$$w_i \neq w_j,\ i \neq j.$$

If the weights between two points are equal ($w_i = w_j$) then the bisector degenerates to a straight line (i.e. the perpendicular bisector between p_i and p_j) and if all of the weights for a point set are the same, the MWVD reduces to the POVD (Okabe et al., 1992, 131).

The multiplicatively weighted Voronoi region can be defined mathematically as

$$V(p_i)_{mw} = \{\mathbf{x} |\ (1/w_i)\|\mathbf{x}-\mathbf{x}_i\| \leq (1/w_j)\|\mathbf{x}-\mathbf{x}_j\| \text{ for } i \neq j,\ j \in I_n\}$$

and the set given by

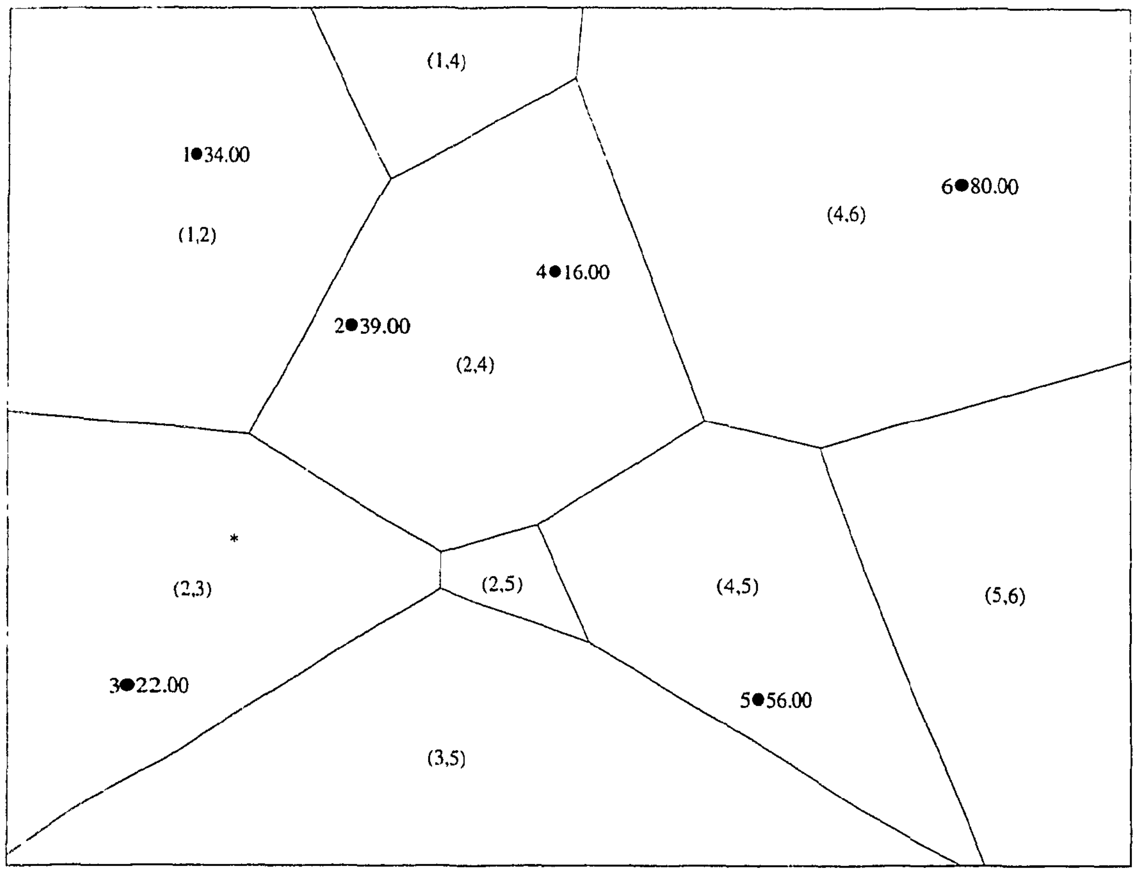
$$\mathfrak{V}_{mw} = \{V(p_i)_{mw},\ldots,V(p_n)_{mw}\}$$

is called the MWVD generated by P (see Figure 10).

2.3.3 The Order-k Voronoi Diagram

The order-k Voronoi diagram (or OKVD) belongs to a special subset called "higher-order" Voronoi diagrams. Higher order Voronoi diagrams assign locations, not to the nearest point, but, to a set of nearest points. For example, suppose that a set of points are placed in the Euclidean plane (the filled circles in Figure 11). Consider all of the Euclidean distances from a specified location (the asterisk) in the plane to all the members of the point set. If an order-2 Voronoi diagram is to be drawn then the objective is to find the members of the point set that are the first and second nearest points to this location. As can be seen in Figure 11, the first and second nearest points to the asterisk are 2 and 3.

Figure 11: The Order-2 Voronoi Diagram



As a result we assign this location to the set $\{2,3\}$. Generally, if a set of the first and second nearest points from a location is $\{p_i,p_j\}$, we assign the location to $\{p_i,p_j\}$ (Okabe et al., 1992, 142-143). This condition holds for sets of three points (order-3), four points (order-4) and so on. Note that we are not concerned with which point is the first nearest in $\{p_i,p_j\}$; p_i may be the first or second nearest point (Okabe et al., 1992, 143). This means that $\{p_i,p_j\} = \{p_j,p_i\}$.

The OKVD can be written mathematically in the following way (this will be developed for the order-2 case first and then extended to the order-k situation).

> n = a finite number of distinct points, where $2 \leq n < \infty$
> $P = \{p_1,\ldots,p_n\}$ represents the set of points
> $A^{(2)}(P) = \{P_1^{(2)},\ldots,P_i^{(2)},\ldots,P_m^{(2)}\}$, where $P_i^{(2)} = \{p_{i1},p_{i2}\}$, $p_{i1},p_{i2} \in P$ and $m = {}_nC_2$
> $d(p,p_{ij})$ represents the Euclidean distance from p to p_{ij}

The points p_{i1} and p_{i2} are the first and second nearest points to p if, and only if, the distances from p to p_{i1} and p_{i2} are less than or equal to the distances from p to all of the other members of the point set (Okabe et al., 1992, 144). Thus, the set of points assigned to $\{p_{i1},p_{i2}\}$ can be written as follows.

$$V(P_i^{(2)}) = \{p| \{d(p,p_{i1}) \leq d(p,p_j)\} \text{ and } \{d(p,p_{i2}) \leq d(p,p_j)\}, \text{ for } p_j \in P\backslash P_i^{(2)}\}$$

represents the order-2 Voronoi region associated with $P_i^{(2)}$ and can be alternatively written as

$$V(P_i^{(2)}) = \{p| \max\{d(p,p_h) \mid p_h \in P_i^{(2)}\} \leq \min\{d(p,p_j) \mid p_j \in P\backslash P_i^{(2)}\}\}.$$

Extending this definition to the order-k situation is as follows.

$$V(P_i^{(k)}) = \{p| \max\{d(p,p_h) \mid p_h \in P_i^{(k)}\} \leq \min\{d(p,p_j) \mid p_j \in P\backslash P_i^{(k)}\}\}$$

represents the order-k Voronoi region associated with $P_i^{(k)}$ and the set given by

$\mathfrak{V}^{(k)} = \{V(P_1^{(k)}),\ldots,V(P_m^{(k)})\}$

is called the OKVD generated by P (see Figure 11).

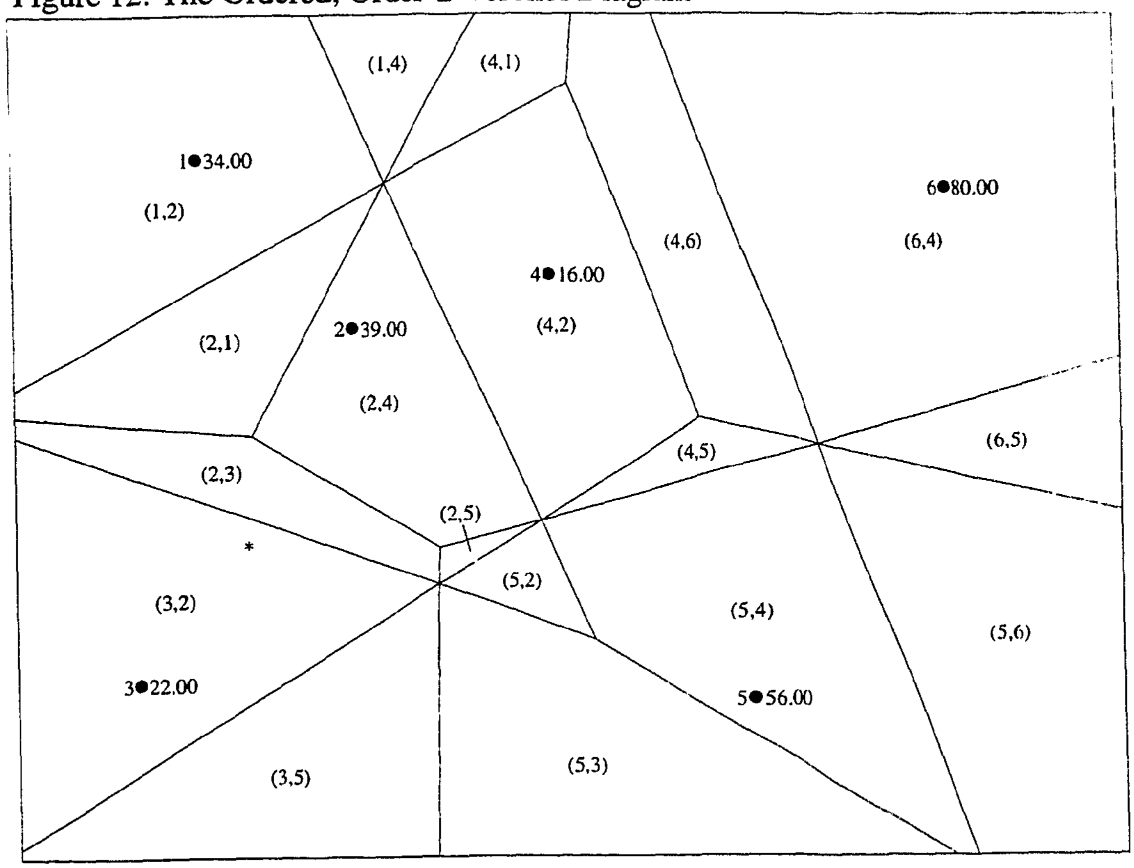
2.3.4 The Ordered, Order-k Voronoi Diagram

The ordered, order-k Voronoi diagram (OOKVD) is also a higher-order Voronoi diagram and is closely related to the OKVD. The difference is that in the OOKVD, the order of the points (i.e. which point is the closest) matters (recall, in the OKVD we were not concerned with the order of the points). For example, if the objective is to construct the OO2VD for a given point set, then space is partitioned in such a way so that all the locations on the plane are assigned to the first and second nearest members of the point set, where the order of these points matters (see Figure 12). Consider all of the Euclidean distances from a specified location (the asterisk) in the plane to all the members of the point set. As can be seen in Figure 12, the two closest points to this location are 3 and 2. This location is therefore assigned to the set {3,2}, because 3 is the first closest point to this location and 2 is the second nearest point. Generally, if p_i is the first closest point to a particular location and p_j is the second closest point to the same location, that location is assigned to the set $\{p_i,p_j\}$. Note that because the order of the points matters, $\{p_i,p_j\} \neq \{p_j,p_i\}$.

This definition can be written formally in the following way (this will be developed for the ordered, order-2 situation first and then extended to the ordered, order-k case).

n = a finite number of distinct points
$P = \{p_1,\ldots,p_n\}$ represents the set of points
$A^{<2>}(P) = \{P_1^{<2>},\ldots,P_i^{<2>},\ldots,P_m^{<2>}\}$, where $P_i^{<2>} = (p_{i1},p_{i2})$, $p_{i1},p_{i2} \in P$
$m = n(n-1)$

Figure 12: The Ordered, Order-2 Voronoi Diagram



The OO2 Voronoi polygon associated with $P_i^{<2>}$ can be written as

$$V(P_i^{<2>}) = \{p \mid d(p,p_{i1}) \leq d(p,p_{i2}) \leq d(p,p_j),\ p_j \in P \backslash \{p_{i1},p_{i2}\}\}.$$

The extension to the OOK situation is as follows. The OOK Voronoi polygon associated with $P_i^{<k>}$ can be written as

$$V(P_i^{<k>}) = \{p \mid d(p,p_{i1}) \leq d(p,p_{i2}) \leq \ldots \leq d(p,p_{ik}) \leq d(p,p_j),\ p_j \in P \backslash \{p_{i1},\ldots,p_{ik}\}\}$$

and the OOKVD constructed from P can be represented by

$$\mathfrak{V}^{<k>} = \{V(P_1^{<k>}),\ldots,V(P_m^{<k>})\}.$$

2.4 Conclusion

This chapter has provided the reader with background on the basic terms and concepts used within this thesis. The next step is to define and construct examples of the "new" Voronoi diagrams.

3.0 The New Voronoi Diagrams

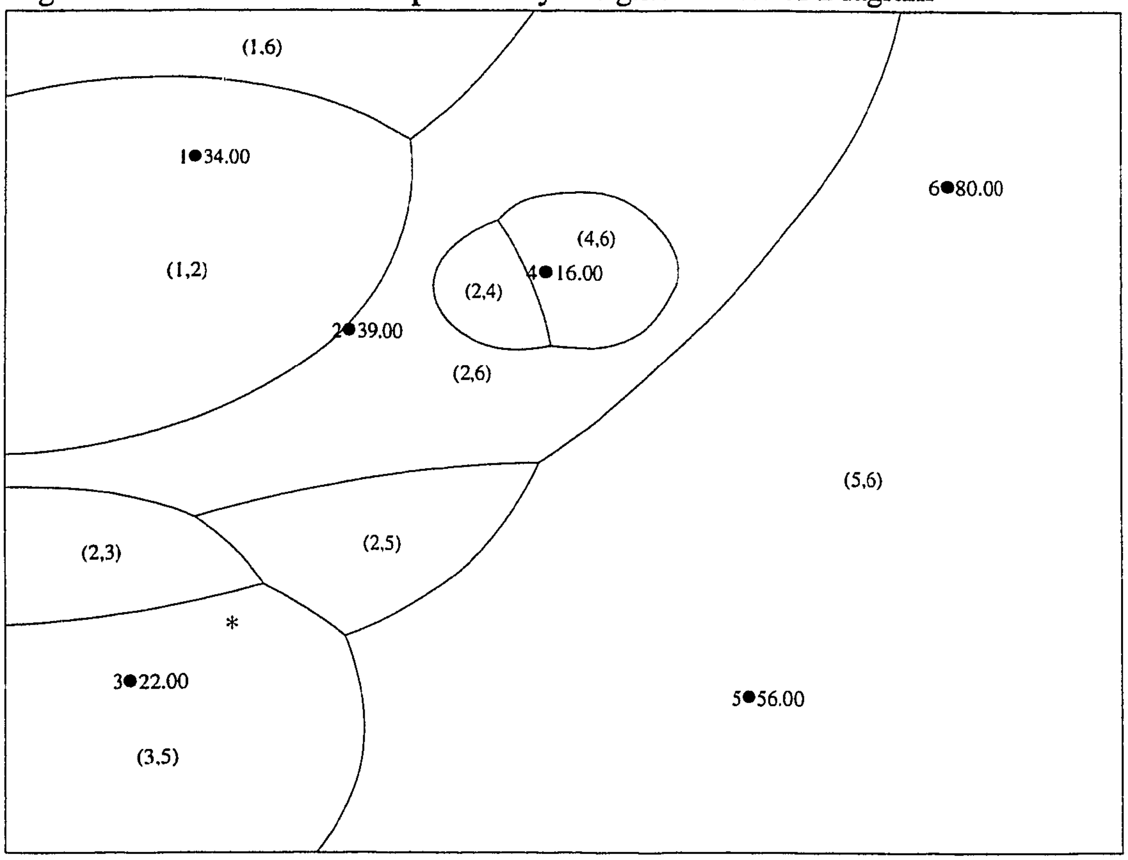
This chapter develops the theory behind the order-k, multiplicatively weighted Voronoi diagram (OKMWVD) and the ordered, order-k, multiplicatively weighted Voronoi diagram (OOKMWVD) by presenting informal and formal definitions of these new tessellations and their geometric properties. This chapter is concluded with a discussion of the characteristics of the OKMWVD and OOKMWVD which are directly related to trade areas.

3.1 The Order-k, Multiplicatively Weighted Voronoi Diagram

The OKMWVD is, essentially, a unification of the MWVD and the OKVD. Due to this union, the definition of the OKMWVD is very similar to those of its two parents. The OKMWVD can be classified as a "higher-order" Voronoi diagram, and, like its unweighted counterpart, the OKMWVD assigns each location to a set of k-nearest points. However, unlike the OKVD, the OKMWVD assigns locations to the k-nearest points in terms of weighted distance. Therefore, if an O2MWVD was to be drawn for a set of points, then the objective would be to find, for every location in the plane, the first and second nearest points in terms of weighted distance (see Figure 13). Generally, if a set of first and second nearest points, in terms of weighted distance, from a location is $\{p_i,p_j\}$, we assign that location to $\{p_i,p_j\}$. Note that we are not concerned with which point is the first nearest in $\{p_i,p_j\}$. This means that $\{p_i,p_j\} = \{p_j,p_i\}$.

The OKMWVD can be written mathematically in the following manner (this will be developed for the order-2 case first and then extended to the order-k situation).

Figure 13: The Order-2 Multiplicatively Weighted Voronoi Diagram



n = a finite number of distinct points, where $2 \le n < \infty$
$P = \{p_1,\ldots,p_n\}$ represents the set of points
$A^{(2)}(P) = \{P_1^{(2)},\ldots,P_i^{(2)},\ldots,P_m^{(2)}\}$, where $P_i^{(2)} = \{p_{i1},p_{i2}\}$, $p_{i1},p_{i2} \in P$ and $m = {}_nC_2$

$d(p,p_{ij})_{mw} = (1/w_{ij})\|\mathbf{x}-\mathbf{x}_{ij}\|$, $w_{ij} > 0$ is the weighted distance from p to p_{ij}

The points p_{i1} and p_{i2} are the first and second nearest points to p if, and only if, the weighted distances from p to p_{i1} and p_{i2} are less than or equal to the weighted distances between p and all of the other members of the point set. Therefore, the set of locations assigned to $\{p_{i1},p_{i2}\}$ can be written as follows:

$$V(P_i^{(2)})_{mw} = \{p|\{d(p,p_{i1})_{mw} \le d(p,p_j)_{mw}\}\text{and}\{d(p,p_{i2})_{mw} \le d(p,p_j)_{mw}\},\ \text{for } p_j \in P \backslash P_i^{(2)}\}$$

represents the O2MW Voronoi region associated with $P_i^{(2)}$ and can be alternatively written as

$$V(P_i^{(2)})_{mw} = \{p|\ \max\{d(p,p_h)_{mw} \mid p_h \in P_i^{(2)}\} \le \min\{d(p,p_j)_{mw} \mid p_j \in P \backslash P_i^{(2)}\}\}.$$

Extending this definition to the OKMW situation is as follows.

$$V(P_i^{(k)})_{mw} = \{p|\ \max\{d(p,p_h)_{mw} \mid p_h \in P_i^{(k)}\} \le \min\{d(p,p_j)_{mw} \mid p_j \in P \backslash P_i^{(k)}\}\}$$

represents the OKMW Voronoi region associated with $P_i^{(k)}$ and the set given by

$$\mathfrak{I}^{(k)}{}_{mw} = \{V(P_1^{(k)})_{mw},\ldots,V(P_m^{(k)})_{mw}\}$$

is called the OKMWVD generated by P.

3.2 Geometric Properties Of The OKMWVD

Now that the OKMWVD has been formally defined, it is appropriate to discuss the geometric properties of this tessellation. For simplicity, these properties will be discussed in terms of the order-2 situation, but can easily be extended to the order-k case.

Property OKMW1: Convexity

An OKMW Voronoi region need not be convex or connected, and it may have a hole(s) (see Figure 14). An OKMW Voronoi region $V(p_i,p_j)$ is convex if, and only if, the minimum weights of the neighbouring OKMW Voronoi regions are not smaller than the minimum weight of $V(p_i,p_j)$. This can be written formally in the following way.

Let $V(p_i,p_j)$ be an O2MW Voronoi region with weights w_i and w_j, and where $w_i \leq w_j$. Also, let P_n be the set of O2MW Voronoi regions which are the neighbours of $V(p_i,p_j)$ and where w_k is the smallest weight associated with these regions. Therefore, $V(p_i,p_j)$ will be convex if, and only if, $w_i \leq w_k$.

Property OKMW2: Unboundedness

As with the order-1 case, OKMW Voronoi regions associated with the largest weight(s) are the only polygons that can be unbounded (see Figure 14). The conditions for a OKMW Voronoi region to be infinite can be written as two separate cases.

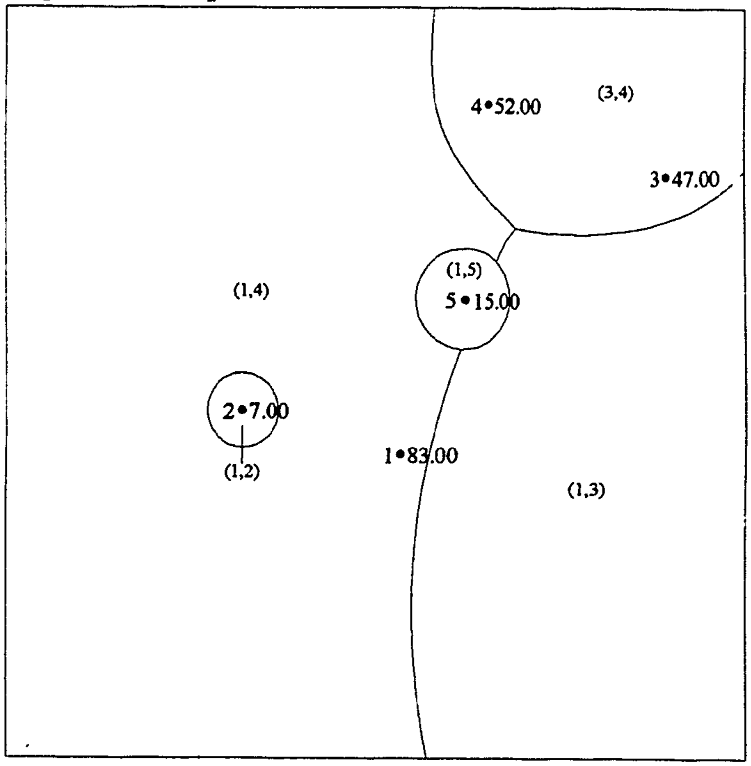
Case 1: One unbounded polygon in the O1MW Voronoi diagram

Let P_u be the set of infinite MW Voronoi polygons in the O1MWVD.
Let I_{P_u} be the number of points in P_u.
Therefore, $I_{P_u} = 1$.
Let p_i be the point in P_u with weight w_i.
Let $w_{max} = \max_j\{w_j, j \in I_n\}$ and P_{max} be the subset of P given by $P_{max} = \{p_j | w_j = w_{max}\}$.

Therefore, $V(p_i)$ is infinite if, and only if, $p_i \in P_{max}$ and p_i is on the boundary of $CH(P_{max})$.

Extending this to the order-k case is as follows.

Figure 14: Properties Of The O2MWVD



Polygons (1,2), (1,5) and (3,4) are examples of convex Voronoi regions, while polygons (1,3) and (1,4) are both non-convex.

Polygon (1,4) is unbounded and has a hole. The hole results because polygon (1,2) is contained within region (1,4).

Polygons (1,3) and (1,4) share a disconnected edge.

Let P_k be the set of points whose MW Voronoi polygons are the neighbours of $V(p_i)$.
Let $w_{kmax} = \max_k\{w_k, k \in P_k\}$ and P_{kmax} be the subset of P_k given by $P_{kmax} = \{p_k | p_k \in P_k; w_k = w_{kmax}\}$.

Therefore, $V(p_i,p_j)$ is infinite if, and only if, $p_j \in P_{kmax}$ and p_j is on the boundary of

$CH(P_{kmax})$.

Case 2: More than one unbounded polygon in the O1MW Voronoi diagram

$I_{Pu} \geq 2$.
Let P_k, p_i and p_j be defined as in Case 1.

Therefore, $V(p_i,p_j)$ is infinite if, and only if, $p_j \in P_u$.

Property OKMW3: Voronoi Edges

Two OKMW Voronoi regions may share disconnected edges (see Figure 14). An edge is a circular arc if, and only if, the pairs of weights of the OKMW Voronoi regions sharing the edge are different; an edge is a straight line if, and only if, the pairs of weights of the OKMW Voronoi regions sharing the edge are the same.

Property OKMW4: Dominance Regions

Let wd = weighted distance.

$$V(P_i^{(k)}) = \{p \mid \max_{p_h}(wd(p,p_h) \mid p_h \in P_i^{(k)}) \leq \min_{p_j}(wd(p,p_j) \mid p_j \in P_j^{(k)} \setminus P_i^{(k)})\}.$$

$$Dom(P_i^{(k)},P_j^{(k)}) = \{p \mid \max_{p_h}(wd(p,p_h) \mid p_h \in P_i^{(k)}) \leq \min_{p_j}(wd(p,p_j) \mid p_j \in P_j^{(k)} \setminus P_i^{(k)})\}.$$

Therefore, $Dom(P_i^{(k)},P_j^{(k)}) \neq \varnothing$ and

$$\bigcup_{i=1,j=1}^{n} V(P_i^{(k)},P_j^{(k)}) = S.$$

This means that $Dom(P_i^{(k)},P_j^{(k)})$ is a well-behaving dominance region (creates a collectively exhaustive and mutually exclusive (except for boundaries) tessellation of S). Since $Dom(P_i^{(k)},P_j^{(k)}) \cup Dom(P_k,P_n) \neq S$, $Dom(P_i^{(k)},P_j^{(k)})$ is not regular.

Property OKMW5: Areal Influence

The set $V(P_i^{(k)})$ may be empty, a point, or an area. The weight attached to the points means that each point will have some area of influence. The only way that a point could create an empty set is if its weight is equal to zero.

Property OKMW6: Number of Points

A non-empty OKMW Voronoi region can contain 0,..., or k points of P.

Property OKMW7: Number of Edges and Vertices

Let N_e = Number of order-1 edges.
Let $N_p^{(2)}$ = Number of order-2 polygons.

$N_e = N_p^{(2)}$ (Count disconnected edges once).

Let $N_v^{(k)}$, $N_e^{(k)}$, $N_f^{(k)}$ and $N_u^{(k)}$ be the number of vertices, edges, finite and infinite order-k Voronoi polygons, respectively.

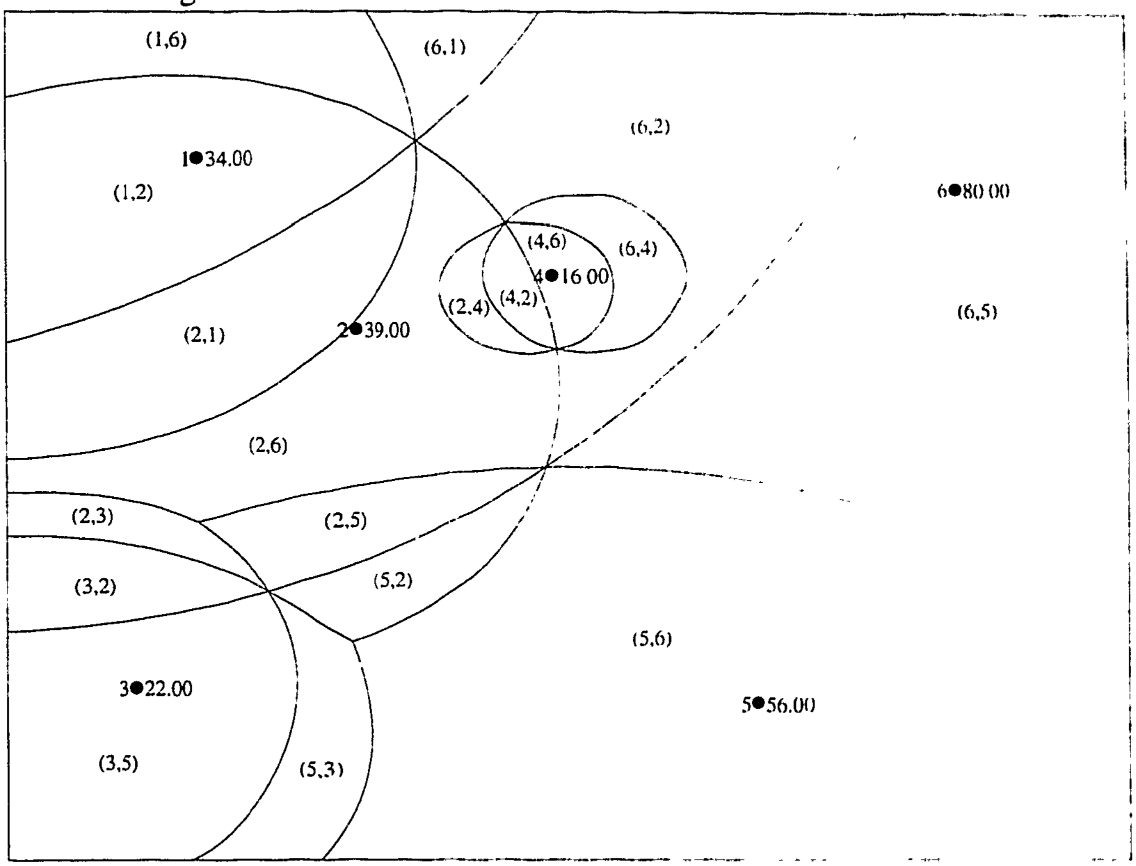
$N_v^{(k)} = 2(N_f^{(k)} - 1) - N_u^{(k)}$.

$N_e^{(k)} \leq 5(N_f^{(k)}) + (N_u^{(k)} + 6)$.

3.3 The Ordered, Order-k, Multiplicatively Weighted Voronoi Diagram

The OOKMWVD is closely related to the OOKVD. The difference between the two tessellations is that the OOKVD partitions space based on Euclidean distance, while the OOKMWVD divides space in terms of weighted distance. Thus, the OOKMWVD assigns each location in the plane to the k-nearest members of a given point set in terms of weighted distance. For example, if an OO2MWVD is to be constructed for a given point set, then space is partitioned in such a way so that all the locations in the plane are assigned to the first and second nearest members of the point set in terms of weighted distance, where it matters which point is the first closest to each location (see Figure 15). Generally, if p_i is the first nearest point to a certain location and p_j is the second closest

Figure 15: The Ordered, Order-2 Multiplicatively Weighted Voronoi Diagram



point to the same location, that location is assigned to the set $\{p_i, p_j\}$. Note that because the order of the points matters, $\{p_i, p_j\} \neq \{p_j, p_i\}$.

This definition can be written formally in the following manner (this will be developed for the order-2 case first and then extended to the order-k situation). The points p_{i1} and p_{i2} are the first and second closest points to p if, and only if, the weighted distances from p to p_{i1} and p_{i2} are less than or equal to the weighted distances between p and all of the other members of the point set. The point p_{i1} is the first closest point to p if, and only if, the weighted distance between p and p_{i1} is less than or equal to the weighted distance from p to p_{i2}. Therefore, the set of locations assigned to $\{p_{i1}, p_{i2}\}$ can be written in the following way.

n = a finite number of distinct points, where $2 \leq n \leq \infty$
$P = \{p_1, \ldots, p_n\}$ represents the set of points
$A^{<2>}(P) = \{P_1^{<2>}, \ldots, P_i^{<2>}, \ldots, P_m^{<2>}\}$, where $P_i^{<2>} = (p_{i1}, p_{i2})$, $p_{i1}, p_{i2} \in P$
$m = n(n-1)$

$$V(P_i^{<2>})_{mw} = \{p \mid \{d(p,p_{i1})_{mw} \leq d(p,p_{i2})_{mw} \leq d(p,p_j)_{mw},\ p_j \in P \backslash \{p_{i1}, p_{i2}\}\}$$

defines the OO2MW Voronoi region associated with $P_i^{<2>}$. This can be extended to the order-k situation as follows.

$$V(P_i^{<k>})_{mw} = \{p \mid d(p,p_{i1})_{mw} \leq d(p,p_{i2})_{mw} \leq \ldots \leq d(p,p_{ik})_{mw} \leq d(p,p_j)_{mw},\ p_j \in P \backslash \{p_{i1}, \ldots, p_{ik}\}\}$$

is called the OOKMW Voronoi region associated with $P_i^{<k>}$ and the set given by

$$\mathfrak{V}^{<k>}_{mw} = \{V(P_1^{<k>})_{mw}, \ldots, V(P_m^{<k>})_{mw}\}$$

is called the OOKMWVD associated with P.

3.4 Geometric Properties Of The OOKMWVD

Having defined the OOKMWVD in section 3.3, we must now observe the geometric properties of the tessellation. As with the OKMWVD, these attributes will be discussed with reference to the order-2 case, but can easily be extended to the order-k situation.

Property OOKMW1: Convexity

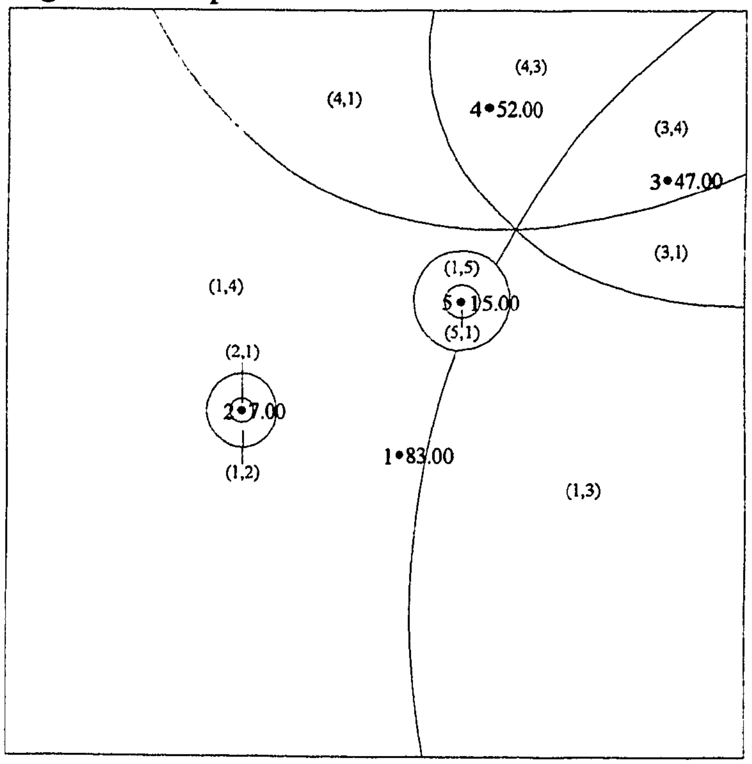
An OOKMW Voronoi region need not be convex or connected, and it may have a hole(s) (see Figure 16). An OOKMW Voronoi region $V(p_i,p_j)$ is convex if, and only if, the minimum weight of the neighbouring OOKMW Voronoi regions is not smaller than the first order weight of $V(p_i,p_j)$ and the first order weight of $V(p_i,p_j)$ is not greater than the second order weight. This can be written formally in the following manner.

Let $V(p_i,p_j)$ be an OO2MW Voronoi region with weights w_i and w_j and where $w_i \leq w_j$. Also, let P_n be a set of OO2MW Voronoi regions which are the neighbours of $V(p_i,p_j)$ and where w_k is the smallest weight associated with these polygons. Therefore, $V(p_i,p_j)$ will be convex if, and only if, $w_i \leq w_k$ (where $V(p_i,p_j) \neq V(p_j,p_i)$).

Property OOKMW2: Unboundedness

In order for an OOKMW Voronoi region to be infinite it must be unbounded at the OKMW level by one of the two cases stated in Property OKMW2. Beyond this, in order to be unbounded, an OOKMW Voronoi polygon must meet one of the two following criteria.

Figure 16: Properties Of The OO2MWVD



Polygons (2,1), (5,1) and (3,4) are examples of convex Voronoi regions, while all of the other polygons are non-convex.

Polygon (1,4) remains as the only unbounded Voronoi region.

Polygons (1,2), (1,5) and (1,4) all have holes.

Polygons (1,3) and (1,4) share a disconnected edge.

Case 1: Two unequal weights ($w_i \neq w_j$)

Let $V(p_i,p_j)^{(k)}$ be an unbounded OKMW Voronoi region.
Let $V(p_i,p_j)^{<k>}$ and $V(p_j,p_i)^{<k>}$ be the OOKMW Voronoi regions created by the partitioning of $V(p_i,p_j)^{(k)}$, where $w_i<w_j$.

Therefore, the partitioning of $V(p_i,p_j)^{(k)}$ will result in one unbounded polygon, which in this case would be $V(p_j,p_i)^{<k>}$ (see Figure 16).

Case 2: Two equal weights ($w_i = w_j$)

Let $V(p_i,p_j)^{(k)}$, $V(p_i,p_j)^{<k>}$ and $V(p_j,p_i)^{<k>}$ be defined as in Case 1, with the exception that $w_i = w_j$.

Therefore, the partitioning of $V(p_i,p_j)^{(k)}$ will result in both $V(p_i,p_j)^{<k>}$ and $V(p_j,p_i)^{<k>}$ being infinite.

Property OOKMW3: Voronoi Edges

Two OOKMW Voronoi regions may share disconnected edges (see Figure 16). An edge is a circular arc if, and only if, the weights of the OOKMW Voronoi regions sharing the edge are different ($V(p_i,p_j) \neq V(p_j,p_i)$); an edge is a straight line if, and only if, the weights of the OOKMW Voronoi regions sharing the edge are the same.

Property OOKMW4: Dominance Regions

$Dom(P_i^{<k>},P_j^{<k>})$ is a well-behaving dominance region that is not regular (see Property OKMW4 for equations).

Property OOKMW5: Areal Influence

The set $V(p_i,p_j)^{<k>}$ may be empty, a point or an area (refer to Property OKMW5 for explanation).

Property OOKMW6: Number of Points

An OOKMW Voronoi region can contain 0,...,or k-1 points of P.

Property OOKMW7: Number of Edges

Let $N_v^{<2>}$ and $N_e^{<2>}$ be the number of OO2MW edges and vertices, respectively.
Let $N_v^{(2)}$ and $N_e^{(2)}$ be the number of O2MW edges and vertices, respectively.
Let N_e be the number of O1MW edges.

$N_v^{<2>} = N_v^{(2)}$.

$N_e^{<2>} = N_e^{(2)} + N_e$.

3.5 Characteristics Of The New Voronoi Diagrams

The two new Voronoi diagrams defined in the preceding sections provide a framework which allows for meaningful trade area analysis. These Voronoi diagrams have a number of characteristics that, in aggregate, produce trade areas that are realistic and easily interpreted. These characteristics will now be discussed.

The first feature of the two new Voronoi diagrams that leads to the creation of more representative trade areas is the weight attached to each member of the point set. This weight represents the attractiveness of each point. Hence, the larger the number of variables used in calculating the weights, the more accurate this measure of attractiveness will be. A wide range of attributes, in addition to distance (or travel cost) can be used to establish a weighting scheme. Some of these variables could include: the selection of goods (measured by store size), store image (measured by the age of the store), parking availability (measured by the number of parking spaces) and distance to other functions (due to multi-purpose shopping). It is important to develop an appropriate weighting scheme as consumers do consider the "other" attributes when deciding where to shop.

The second important characteristic of these Voronoi diagrams is that each location is assigned to a set of points. This means that consumers have a choice set of alternative stores to shop at (the bracketed numbers in the diagrams define the choice set of the consumers within each polygon). The order of the diagram defines the number of stores in the consumers choice set. For example, in Figure 13, the choice set for the consumers located at the asterisk (*) is facility 3 and facility 5. A consumers choice set typically includes two or three stores. This assignment rule means that the probabilistic and overlapping nature of these trade areas is easier to see. This eliminates the problem of the trade areas being interpreted as spatial monopolies.

The OKMWVD and OOKMWVD provide the retail analyst with an effective method of modelling trade areas. The characteristics discussed above make the market areas created by these tessellations more realistic and easier to interpret. These features create trade area definitions that are directly comparable to those generated by the family of spatial interaction models.

4.0 The New Voronoi Trade Area Model

Now that the OKMWVD and OOKMWVD have been defined, a trade area model that incorporates the characteristics of these tessellations can be developed. The model presented here is developed for the order-2 situation, but could easily be adapted for higher order cases. Once the model has been introduced, it will be applied to a real world data set and the results will be compared to those that would have been obtained using the other Voronoi diagrams (i.e. the POVD and MWVD).

4.1 The Model

The model that has been developed for the OKMWVD and the OOKMWVD can be described in the following manner.

1. A number of stores (represented as points), n, of the same type (e.g. supermarkets) are located on a finite planar region, S (e.g. a city).

2. Customers travel to one or more of these stores to acquire a particular bundle of goods, m.

3. The price of m is assumed to be the same at all stores.

4. An individual store, i, is assigned a weight, w_i ($w_i > 0$), based on its relative attractiveness to consumers. This level of attractiveness is determined by one or more attributes (e.g. size, hours of operation, age, etc.). It is assumed that all consumers evaluate these store characteristics identically, so that the attractiveness of a centre is the same for all consumers.

5. The accessibility of store i for customer j, A_{ij}, is directly related to the attractiveness of store i, w_i, and is inversely related to the distance travelled by j to shop at i, d_{ij}. Thus,

$$A_{ij} = w_i/d_{ij}.$$

Thus, the store which is the most accessible for customer j is the one for which

d_{ij}/w_i is minimized ($\forall_i = 1,n$).

6. Consumers have a choice set which is defined by the order of the Voronoi diagram used to represent the market areas. In this case, consumers have a choice set of two stores, which means that they limit their shopping for bundle m to the two most accessible facilities, A_1 and A_2.

7a. When purchasing bundle m, if the customers are indifferent between A_1 or A_2, then the appropriate diagram to use would be the O2MWVD.

7b. When purchasing bundle m, if consumers show a preference for the store with the higher level of accessibility, then the appropriate diagram to use would be the OO2MWVD.

4.2 Assigning Probabilities To Facilities

The model defined in section 4.1 describes a situation where consumers patronize stores in a probabilistic manner. Therefore, once the analyst has decided which diagram (e.g. OKMWVD or OOKMWVD) best represents their particular situation, then probabilities can be assigned to each facility.

4.2.1 Defining Probabilities For The OKMWVD

The OKMWVD is used to calculate trade areas in situations where consumers are indifferent between the options in their choice set. This means that consumers will patronize all of the facilities in their choice set with equal probabilities. Therefore, if the OKMWVD is used to define trade areas for a set of stores, then consumers will patronize each of the stores within their choice set with a probability of 1/k, where k is the number of options in the consumer choice set (defined by the order of the diagram).

4.2.2 Defining Probabilities For The OOKMWVD

The OOKMWVD, on the other hand, is used to model trade areas when customers show a preference for the most accessible facility. This means that consumers will patronize the different facilities in their choice set with varying levels of probability. These probabilities can be assigned endogenously or exogenously (the application of this model, in section 5.0, will define probabilities exogenously).

Probabilities can be defined endogenously with the use of the weights assigned to the facilities. For example, consider two facilities A_1 and A_2, where A_1 is the most accessible alternative, and w_1 and w_2 are the weights associated with the two stores. Thus, the probability of a customer patronizing A_1 is

$$P = 0.5 + ([w_1/(w_1+w_2)] \times 0.5)$$

and the probability of a consumer visiting A_2 is equal to (1-P). This format will insure that the most accessible facility (i.e. A_1) will be assigned a probability that falls between 0.5 and 1.0. When assigning probabilities endogenously, a simple ratio between the weights of the facilities should not be used. This will result in the highest weighted facility always having the highest probability of being patronized (whether it is the most accessible facility or not).

If probabilities are defined exogenously, a sensitivity analysis can be performed by varying the probability levels and determining the impact that this has on the sales estimates of each facility.

4.3 Using Trade Areas To Generate Sales Estimates

The trade areas generated by the above model can be used to calculate sales estimates for each of the facilities in a particular set of stores. In order to do this, we assume a uniform distribution of sales potential (i.e. customers and income). The equations necessary to calculate these figures will be developed for the order-1, order-2 and ordered, order-2 scenarios.

Order-1 Situation:

In all order-1 situations (i.e. POVD and MWVD) the volume of sales that any facility receives is directly proportional to the size of its trade area.

Order-2 Situation:

In the order-2 case, we assume that consumers are indifferent between all of the options in their choice set and patronize each facility with a probability of 1/2. Therefore, the sales of centre i can be estimated by

$$TS_i = \left(\frac{1}{2}\right) \sum_{j,j \neq i} |V^{(2)}(p_i,p_j)|, \text{ where } |V^{(2)}(p_i,p_j)| \text{ is the area of } V^{(2)}(p_i,p_j).$$

Ordered, Order-2 Situation:

In the ordered, order-2 scenario, the assumption is being made that consumers will patronize both of the facilities in their choice set, but show a preference for the most accessible centre. Therefore, if the probability of going to the nearest facility is P (1>P>1/2), the probability of visiting the second nearest centre is (1-P). If i is the most accessible facility, then the sales of centre i can be estimated by

$$TS_i = \left[(P) \sum_{j,j \neq i} |V^{<2>}(p_i,p_j)|\right] + \left[(1-P) \sum_{j,j \neq i} |V^{<2>}(p_j,p_i)|\right].$$

5.0 MODEL APPLICATION: SUPERMARKETS IN KITCHENER-WATERLOO

5.1 Study Area And Data

The region that will serve as the study area for a demonstration of the model described in the section 4.0 is Kitchener-Waterloo (see Figure 17 for a map of the area). The retail facilities used in the following analysis are the supermarkets in the Kitchener-Waterloo area, as they stood at the end of 1995. Figure 17 shows the locations of all the supermarkets located within Kitchener-Waterloo and Table 1 provides a listing of these stores. The reader will notice that one store (Knob Hill Farms) is located in Cambridge. It was decided that the attractiveness of this store was strong enough to draw customers from Kitchener-Waterloo.

The data used in this study were obtained from a number of sources. First, a listing of the supermarkets and their locations (i.e. street address) was provided by IPCF Properties (the Real Estate Branch of National Grocers). Second, these store locations were digitized into Terrasoft using 1:25,000 Topographic Map Sheets, and X,Y coordinates were then calculated for each store (see Table 2 for X,Y coordinates). Third, a survey of each store was conducted in order to obtain variables to be used in a weighting scheme. This procedure included visiting each facility to observe store attributes and using published materials (i.e. 1995 Directory of Shopping Centres and Vernon's Kitchener and Waterloo City Directory) to obtain data on store size, centre size and store age. In the end, a total of eight variables were collected for each store (see

Figure 17: Map Of Kitchener-Waterloo

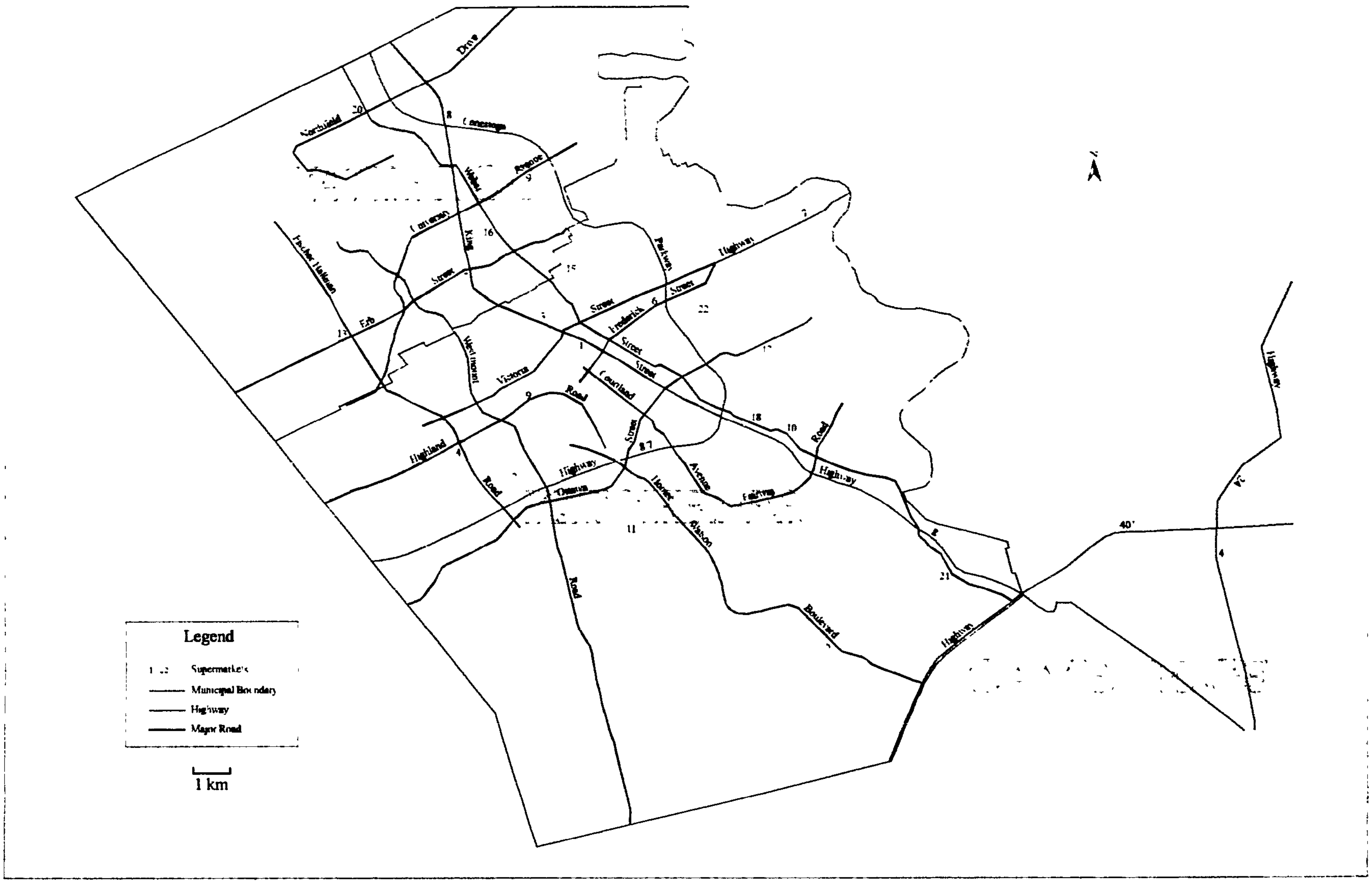


Table 1: Supermarkets in Kitchener-Waterloo

STORE ID	BANNER
1	FOODTOWN
2	FOODTOWN
3	CENTRAL MEAT MKT.
4	KNOB HILL FARMS
5	ZEHRS
6	ZEHRS
7	ZEHRS
8	ZEHRS
9	ZEHRS
10	ZEHRS
11	ZEHRS
12	ZEHRS
13	ZEHRS
14	ZEHRS
15	DUTCH BOY
16	DUTCH BOY
17	DUTCH BOY
18	DUTCH BOY
19	DUTCH BOY
20	DUTCH BOY
21	PRICE CLUB
22	DIPIETRO'S FOOD MKT.

Table 2: Supermarket Locations

STORE	BANNER	ADDRESS	X COORD	Y COORD
1	FOODTOWN	214 KING ST. W.	540993.92	4810806 89
2	FOODTOWN	412 GREENBROOK DR.	539516.52	4807914.00
3	CENTRAL MEAT MKT.	760 KING ST. W.	539969.81	4811286 97
4	KNOB HILL FARMS	HWY. 401 & HWY. 24	554428.86	4806181.60
5	ZEHRS	123 PIONEER DR.	545986.96	4804613.03
6	ZEHRS	395 FREDERICK ST.	542548 39	4811533 72
7	ZEHRS	75 KING ST. S.	538617.18	4812110.73
8	ZEHRS	550 KING ST. W.	538156.01	4815580.26
9	ZEHRS	315 LINCOLN RD.	539850.01	4814391.66
10	ZEHRS	1375 WEBER ST. E.	545271.71	4808781.37
11	ZEHRS	700 STRASBURG RD.	542089.69	4806840.67
12	ZEHRS	1005 OTTAWA ST. N.	544899.46	4810746 52
13	ZEHRS	450 ERB ST. W.	536090.02	4810941.24
14	ZEHRS	875 HIGHLAND RD. W.	538344.81	4808553.51
15	DUTCH BOY	351 MARGARET AVE.	540694.44	4812339.48
16	DUTCH BOY	94 BRIDGEPORT ST. E.	539266.39	4812987.23
17	DUTCH BOY	720 WESTMOUNT RD. E.	540379.10	4807210 69
18	DUTCH BOY	1111 WEBER ST. E.	544630.98	4808974 38
19	DUTCH BOY	274 HIGHLAND RD. W.	539786.81	4809534.15
20	DUTCH BOY	585 WEBER ST. N.	536361.03	4815762.16
21	PRICE CLUB	4438 KING ST. E.	548865.28	4805880.05
22	DIPIETRO'S FOOD MKT.	501 KRUG ST	543547.25	4811441.05

Table 3 for a description of the variables collected). The results of this survey have been summarized in Table 4.

The weight (i.e. the index of attraction) for each facility was calculated in the following manner. First, the value of each variable for each individual case in Table 4 was standardized by dividing the value by its corresponding variable total. The end result was that each case was converted to a percentage of its respective variable (see Table 5).

Second, a principal components analysis (PCA) was performed on the transformed variables (see Table 6 for the results of the PCA). PCA, essentially, is a data reduction technique which identifies clusters of interrelated variables and combines these variable clusters into a smaller number of factors. The primary task of PCA is to define the principal axes for a number of variables (similar to a regression procedure).

The PCA extracted four factors which combine to explain 90.2% of the total variance within the data set. The first factor extracted explains 32.4% of the total variance in the original variables. This means that Factor 1 does not account for all of the correlation in the original data set. Therefore, the PCA extracted a second factor which explains 28.9% of the total variation in the original data. Factor 2 does not include the parts of the variables related to the first factor and, as a result, is not correlated with Factor 1. The PCA also extracted Factor 3 and Factor 4, which respectively account for 21.0% and 7.9% of the total variation in the original data. No other factors were extracted because they did not produce significant increases in the total variance explained.

Table 3: Variables Collected In Supermarket Survey

Variable	Description
SIZE	Size of the store in square feet.
CTRSIZE	Size of the centre (i.e. mall or plaza) in square feet. If the store is stand alone, then SIZE = CTRSIZE.
AGE	Number of years since the store opened.
TYPE	Number of stores located in the centre.
ANCHOR	Number of other anchor stores located in the centre.
HOURS	Total number of hours store is open during a one week period.
ANCIL	Number of ancillary features located in store (i.e. bank machine, debit card, post office).
INSTORE	Number of features related specifically to grocery shopping that store offers (i.e. bulk food, bakery, pharmacist, fish counter, wine shop, flower shop, small appliances, clothing, hardware, jewellery, furniture, restaurant, hair stylist).

The relationship of each variable with the four factors can be seen in Table 6b. These numbers are called factor loadings and represent the correlations between the original variables and the new factors. As can be seen in Table 6b, the PCA identified ANCHOR (Factor 1), SIZE (Factor 2), HOURS (Factor 3) and ANCIL (Factor 4) as the variables which best summarize the four factors. Therefore, these four variables were used to represent the entire data set. In order to make sure that each variable was

Table 4: Kitchener-Waterloo Supermarket Survey Results (Raw Data)

STORE	BANNER	SIZE	CTRSIZE	AGE	TYPE	ANCHOR	HOURS	ANCIL	INSTORE
1	FOODTOWN	11700.00	11700.00	40.00	1.00	0.00	70.00	0.00	1.00
2	FOODTOWN	18793.00	67800.00	32.00	16.00	0.00	78.50	0.00	1.00
3	CENTRAL MEAT MKT.	24000.00	24000.00	34.00	1.00	0.00	72.50	2.00	1.00
4	KNOB HILL FARMS	325000.00	325000.00	5.00	1.00	0.00	77.00	1.00	8.00
5	ZEHRS	22300.00	57682.00	18.00	14.00	0.00	83.00	2.00	2.00
6	ZEHRS	20000.00	175000.00	40.00	41.00	1.00	80.00	2.00	1.00
7	ZEHRS	28859.00	228243.00	34.00	42.00	1.00	80.50	2.00	2.00
8	ZEHRS	36636.00	375780.00	17.00	93.00	2.00	83.00	2.00	2.00
9	ZEHRS	45606.00	54685.00	21.00	9.00	0.00	93.00	3.00	2.00
10	ZEHRS	45514.00	78176.00	6.00	13.00	0.00	94.00	2.00	4.00
11	ZEHRS	41348.00	159500.00	16.00	27.00	2.00	93.00	3.00	5.00
12	ZEHRS	37000.00	225000.00	26.00	38.00	2.00	92.00	2.00	2.00
13	ZEHRS	43508.00	79864.00	10.00	18.00	0.00	93.00	2.00	4.00
14	ZEHRS	65533.00	209956.00	8.00	40.00	1.00	94.00	2.00	5.00
15	DUTCH BOY	19386.00	19386.00	32.00	1.00	0.00	105.00	2.00	5.00
16	DUTCH BOY	39500.00	192000.00	27.00	26.00	1.00	105.00	2.00	4.00
17	DUTCH BOY	36000.00	70300.00	15.00	11.00	0.00	105.00	2.00	4.00
18	DUTCH BOY	30000.00	30000.00	19.00	1.00	0.00	105.00	2.00	5.00
19	DUTCH BOY	25000.00	90816.00	26.00	15.00	0.00	105.00	2.00	6.00
20	DUTCH BOY	30000.00	50280.00	8.00	11.00	0.00	105.00	2.00	4.00
21	PRICE CLUB	110000.00	110000.00	2.00	1.00	0.00	65.00	2.00	7.00
22	DIPIETRO'S FOOD MKT.	15000.00	53287.00	0.00	16.00	0.00	85.00	2.00	3.00

Table 5: Kitchener-Waterloo Supermarket Survey Results (Percentages)

STORE	BANNER	SIZE	CTRSIZE	AGE	TYPE	ANCHOR	HOURS	ANCIL	INSTORE
1	FOODTOWN	0.0109	0.0044	0.0917	0.0023	0.0000	0.0357	0.0000	0.0128
2	FOODTOWN	0.0176	0.0252	0.0734	0.0367	0.0000	0.0400	0.0000	0.0128
3	CENTRAL MEAT MKT.	0.0224	0.0089	0.0780	0.0023	0 0000	0.0369	0.0488	0.0128
4	KNOB HILL FARMS	0.3035	0.1209	0.0115	0.0023	0.0000	0.0392	0.0244	0.1026
5	ZEHRS	0.0208	0.0215	0.0413	0.0321	0.0000	0.0423	0.0488	0.0256
6	ZEHRS	0.0187	0.0651	0.0917	0.0940	0.1000	0.0407	0.0488	0.0128
7	ZEHRS	0.0270	0.0849	0.0780	0.0963	0.1000	0.0410	0.0488	0.0256
8	ZEHRS	0.0342	0.1398	0.0390	0.2133	0.2000	0.0423	0.0488	0.0256
9	ZEHRS	0.0426	0.0203	0.0482	0.0206	0.0000	0.0474	0.0732	0.0256
10	ZEHRS	0.0425	0.0291	0.0138	0.0298	0.0000	0.0479	0.0488	0.0513
11	ZEHRS	0.0386	0.0593	0.0367	0.0619	0.2000	0.0474	0.0732	0.0641
12	ZEHRS	0.0346	0.0837	0.0596	0.0872	0 2000	0.0469	0.0488	0.0256
13	ZEHRS	0.0406	0.0297	0.0229	0.0413	0.0000	0.0474	0.0488	0 0513
14	ZEHRS	0.0612	0.0781	0.0184	0.0917	0.1000	0.0479	0.0488	0.0641
15	DUTCH BOY	0.0181	0.0072	0.0734	0.0023	0.0000	0.0535	0.0488	0.0641
16	DUTCH BOY	0.0369	0.0714	0.0619	0.0596	0.1000	0.0535	0.0488	0.0513
17	DUTCH BOY	0.0336	0.0262	0.0344	0.0252	0.0000	0.0535	0.0488	0.0513
18	DUTCH BOY	0.0280	0.0112	0.0436	0.0023	0.0000	0.0535	0.0488	0 0641
19	DUTCH BOY	0.0234	0.0338	0.0596	0.0344	0.0000	0.0535	0.0488	0.0769
20	DUTCH BOY	0.0280	0.0187	0.0184	0.0252	0.0000	0.0535	0.0488	0.0513
21	PRICE CLUB	0.1027	0.0409	0.0046	0.0023	0.0000	0.0331	0.0488	0.0897
22	DIPIETRO'S FOOD MKT.	0.0140	0.0198	0.0000	0.0367	0.0000	0.0433	0.0488	0.0385

Table 6a: Results Of PCA (Explained Variance)

Factor	Proportion Of Variance Explained	Cumulative Explained Variance
1	0.324	0.324
2	0.289	0.613
3	0.210	0.823
4	0.079	0.902

Table 6b: Results Of PCA (Rotated Factor Matrix)

Variable	Factor 1	Factor 2	Factor 3	Factor 4
SIZE	0.06218	0.92148*	-0.23490	-0.10825
CTRSIZE	0.88463	0.42244	-0.12626	-0.06144
AGE	0.10984	-0.59662	0.08790	-0.68577
TYPE	0.91961	-0.20767	-0.04658	0.09665
ANCHOR	0.92286*	-0.13740	0.08014	0.08004
HOURS	-0.04435	-0.00623	0.95509*	0.17381
ANCIL	0.20958	-0.09033	0.37161	0.80220*
INSTORE	-0.14507	-0.85798	0.31568	0.21474

* Variables which best summarize the respective factors.

weighted properly, the third step was to multiply each case (for the four selected variables) by the proportion of variance that its corresponding factor was responsible for explaining. These four figures were then added up for each case and the resulting number represents the weight of relative attractiveness for each store (see Table 7).

The reader should note that the weighting scheme adopted in this analysis is only one of many different approaches that could have been utilized, and that the results of the following analysis could vary under a different weighting scheme. The weighting scheme used in this study was chosen only after a number of different approaches were attempted (e.g. factor scores, direct additive) and it was found that the results of the different procedures were very similar. The adopted weighting scheme was chosen over the others because of its relative ease of implementation.

An interesting feature of the supermarket industry in Kitchener-Waterloo is the dominance that the Zehrs chain exerts over its competitors. The Zehrs chain accounts for 45.5% (10 of 22) of the supermarkets within Kitchener-Waterloo. The supremacy of Zehrs can also be seen by sorting the weighting scheme in Table 7 from highest to lowest (see Table 8). As can be seen in Table 8, Zehrs stores account for 7 of the 10 most attractive stores and 4 of the top 5. This clearly shows the strength of the Zehrs chain within the Kitchener-Waterloo market.

Table 7: Weighting Scheme

STORE ID	ANCHOR	SIZE	HOURS	ANCIL	(1) ANCHOR x 0.324*	(2) SIZE x 0.289*	(3) HOURS x 0.210*	(4) ANCIL x 0.079*	WEIGHT (1+2+3+4) x 1000
1	0.000	0.011	0.036	0.000	0.00000	0.00315	0.00750	0.00000	10.65
2	0.000	0.018	0.040	0.000	0.00000	0.00509	0.00840	0.00000	13.49
3	0.000	0.022	0.037	0.049	0.00000	0.00647	0.00775	0.00386	18.08
4	0.000	0.304	0.039	0.024	0.00000	0.08771	0.00823	0.00193	97.87
5	0.000	0.021	0.042	0.049	0.00000	0.00601	0.00888	0.00386	18.75
6	0.100	0.019	0.041	0.049	0.03240	0.00540	0.00855	0.00386	50.21
7	0.100	0.027	0.041	0.049	0.03240	0.00780	0.00861	0.00386	52.67
8	0.200	0.034	0.042	0.049	0.06480	0.00988	0.00888	0.00386	87.42
9	0.000	0.043	0.047	0.073	0.00000	0.01231	0.00995	0.00578	28.05
10	0.000	0.043	0.048	0.049	0.00000	0.01228	0.01006	0.00386	26.20
11	0.200	0.039	0.047	0.073	0.06480	0.01116	0.00995	0.00578	91.69
12	0.200	0.035	0.047	0.049	0.06480	0.01000	0.00985	0.00386	88.50
13	0.000	0.041	0.047	0.049	0.00000	0.01173	0.00995	0.00386	25.54
14	0.100	0.061	0.048	0.049	0.03240	0.01769	0.01006	0.00386	64.00
15	0.000	0.018	0.054	0.049	0.00000	0.00523	0.01124	0.00386	20.32
16	0.100	0.037	0.054	0.049	0.03240	0.01066	0.01124	0.00386	58.15
17	0.000	0.034	0.054	0.049	0.00000	0.00971	0.01124	0.00386	24.80
18	0.000	0.028	0.054	0.049	0.00000	0.00809	0.01124	0.00386	23.18
19	0.000	0.023	0.054	0.049	0.00000	0.00676	0.01124	0.00386	21.85
20	0.000	0.028	0.054	0.049	0.00000	0.00809	0.01124	0.00386	23.18
21	0.000	0.103	0.033	0.049	0.00000	0.02968	0.00695	0.00386	40.49
22	0.000	0.014	0.043	0.049	0.00000	0.00405	0.00909	0.00386	16.99

* Proportions of variance explained from Table 6a.

Table 8: Sorted Weighting Scheme

STORE ID	BANNER	WEIGHT
4	KNOB HILL FARMS	97.87
11	ZEHRS	91.69
12	ZEHRS	88.50
8	ZEHRS	87.42
14	ZEHRS	64.00
16	DUTCH BOY	58.15
7	ZEHRS	52.67
6	ZEHRS	50.21
21	PRICE CLUB	40.49
9	ZEHRS	28.05
10	ZEHRS	26.20
13	ZEHRS	25.54
17	DUTCH BOY	24.80
20	DUTCH BOY	23.18
18	DUTCH BOY	23.18
19	DUTCH BOY	21.85
15	DUTCH BOY	20.32
5	ZEHRS	18.75
3	CENTRAL MEAT MKT.	18.08
22	DIPIETRO'S FOOD MKT.	16.99
2	FOODTOWN	13.49
1	FOODTOWN	10.65

The dominant position of Zehrs in the Kitchener-Waterloo supermarket industry is also supported by a recent newspaper article. The November 11, 1995 Kitchener-Waterloo Record article entitled "Supermarket Shakedown," contained a NADBank survey which asked 825 Kitchener-Waterloo residents to identify the supermarkets that they patronized during the last month. The results of this survey have been reproduced in Table 9. The vast majority of the respondents (83%) said that they patronized Zehrs during the previous month. Dutch Boy, which finished second in this survey, was mentioned by only 41% of the respondents. These results demonstrate the dominant position of Zehrs within the Kitchener-Waterloo region. Further, since all of the Superfresh stores in Kitchener-Waterloo have now closed, Zehrs could, potentially, be patronized by the entire market.

The NADBank survey also lends support to using the order-2 model for the Kitchener-Waterloo data. If all of the percentages in Table 9 are added up and divided by 100, the number that results is 1.66. This number represents the average number of supermarkets that people in Kitchener-Waterloo visit to buy their groceries. This means that shoppers in Kitchener-Waterloo visit between 1 and 2 stores to purchase their groceries. Therefore, the order-2 models will provide the appropriate trade areas for the supermarkets located within Kitchener-Waterloo.

Table 9: NADBank Survey Results

Supermarket Chain	Percentage Of Consumers	No. Of Stores (1995)*
Zehrs	83	13
Dutch Boy	41	6
Superfresh	17	4^
Central Meat Mkt.	16	1
Knob Hill Farms	9	1

Source: Kitchener-Waterloo Record, "Supermarket Shakedown."
* This survey includes Cambridge stores as well as Kitchener-Waterloo.
^ Superfresh has closed its four stores since this survey was conducted.

5.2 Results

5.2.1 Trade Area Configurations

The trade areas that the new model generates for the Kitchener-Waterloo supermarket data can be seen in Figure 18 and Figure 19. The filled circles represent the locations of the supermarkets within Kitchener-Waterloo. These locations have been labelled from 1 to 22 (these numbers correspond to the identification labels in Table 1). The bracketed numbers represent the choice set of the consumers located within each trade area.

Figure 18 shows the trade areas defined through the O2MW model. As can be seen in Figure 18, the trade areas overlap and are scaled according to the weights

Figure 18: Supermarket Trade Areas In Kitchener-Waterloo - The O2MWVD

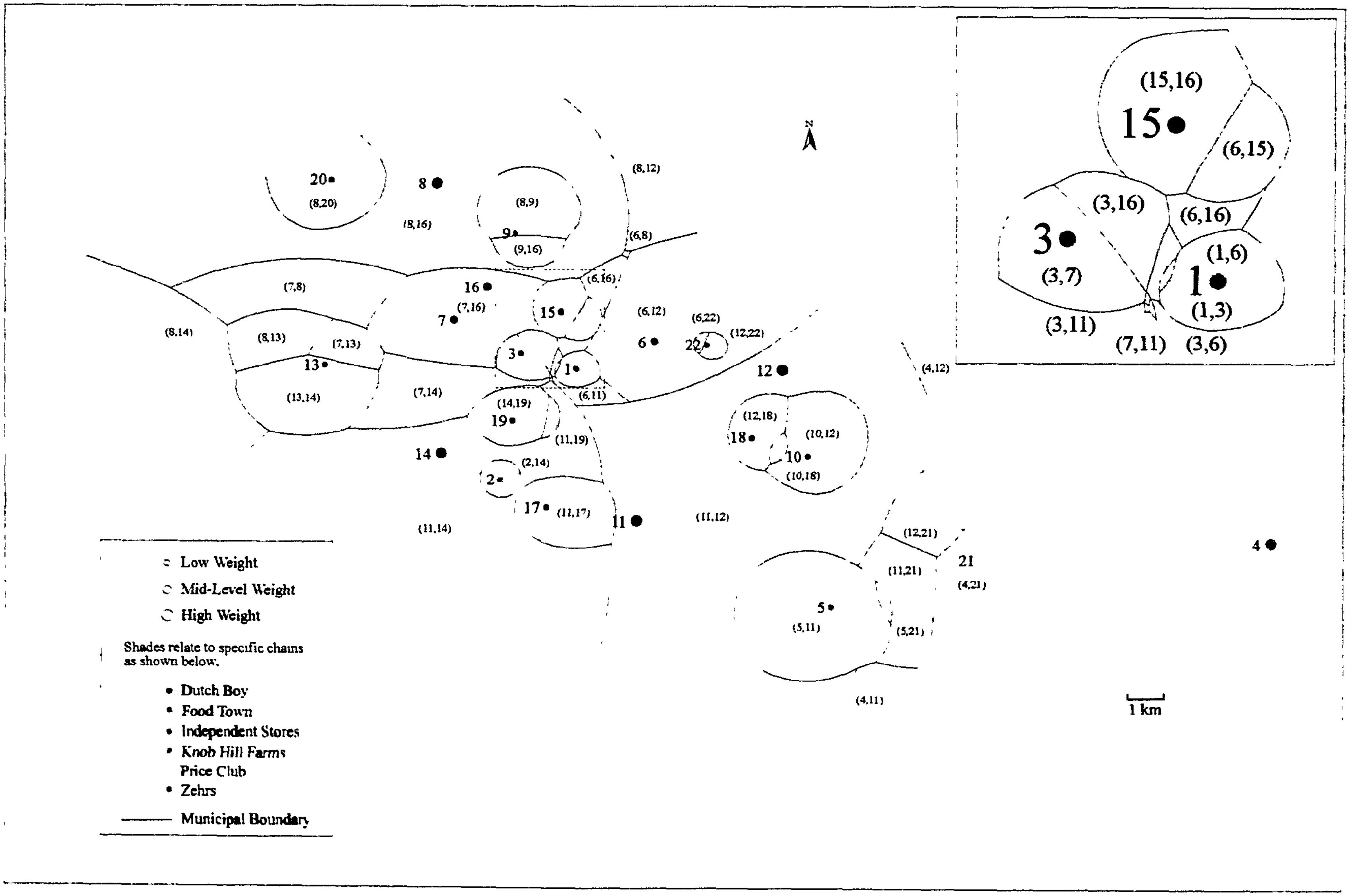
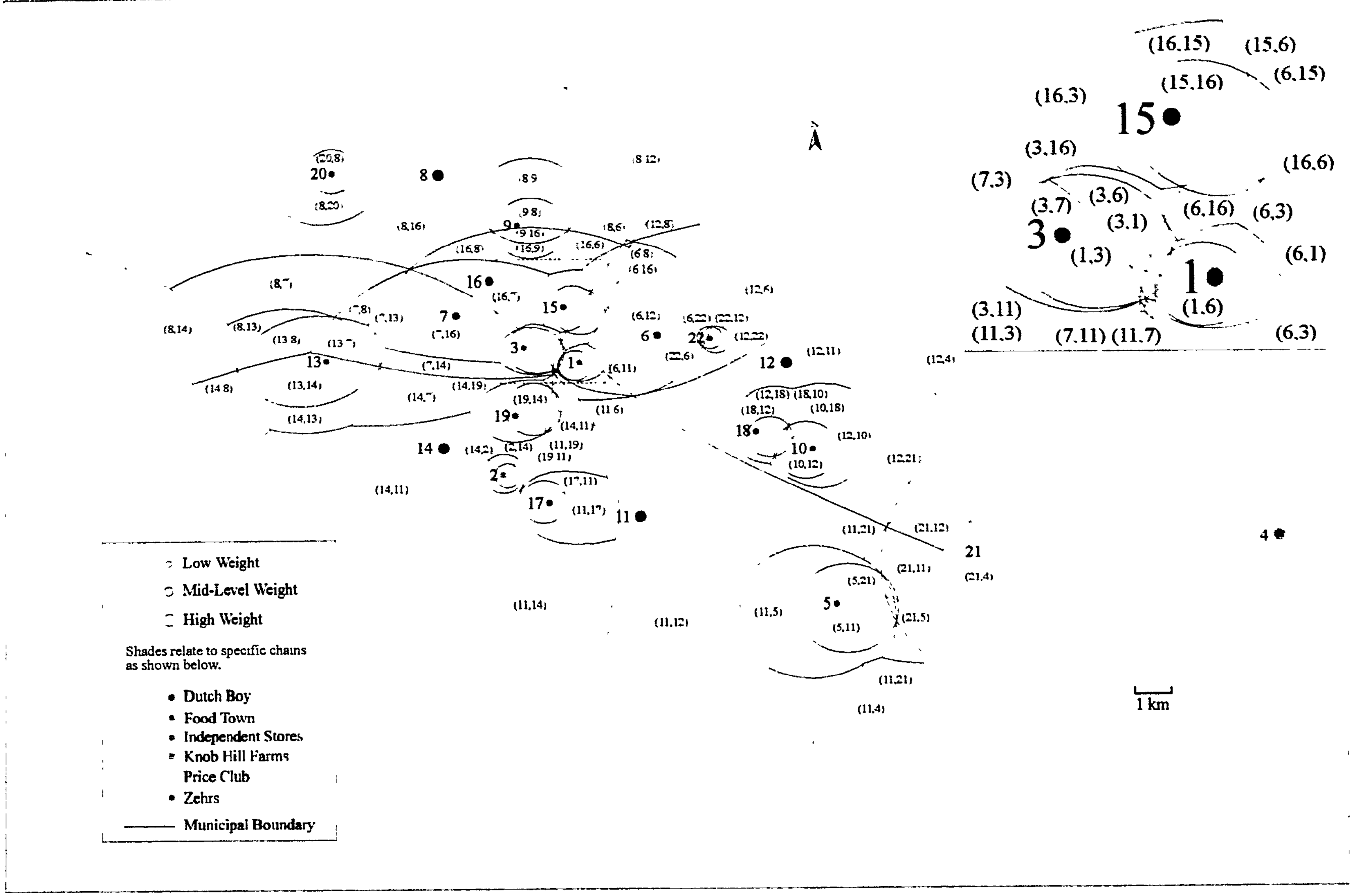


Figure 19: Supermarket Trade Areas In Kitchener-Waterloo - The OO2MWVD



assigned to their respective facilities. Also, consumers are assigned a set of alternatives to select from (in this case, the choice set consists of two stores). The probability of a consumer selecting one of the alternatives in their choice set is equal to 0.5 (recall, in the O2MW model consumers are indifferent between the alternatives in their choice set).

Figure 19 displays the results of the OO2MW model. These trade areas are very similar to those defined in Figure 18, except now consumers show a preference for the alternatives in their choice set. This means that the probabilities associated with the options in a consumer's choice set are no longer equal; consumers are now assumed to patronize the most accessible facility with a higher level of probability.

A visual inspection of both Figure 18 and Figure 19 reveals the dominant position of Zehrs within the Kitchener-Waterloo market. The trade areas that are associated with Zehrs stores account for a very large percentage of the land area that falls within the municipal boundaries of Kitchener-Waterloo. Figures 18 and 19 also show that Knob Hill Farms captures a very small proportion of the Kitchener-Waterloo market. As can be seen in both Figures 18 and 19, only a very small portion of Knob Hill Farms' trade area lies within the boundaries of Kitchener-Waterloo (also see Table 9). Knob Hill Farms does not attract very many consumers from Kitchener-Waterloo because of its location. Although Knob Hill Farms is the highest weighted store, the distance involved in travelling to Cambridge deters most residents of Kitchener-Waterloo from patronizing this facility on a regular basis.

These results can be compared to the trade area definitions that would have been obtained if one of the other Voronoi models (i.e. the POVD or MWVD) had been used. Figure 20 shows the trade areas defined by the POVD. This picture differs greatly from the previous two. The POVD constructs bisectors based on distance only. As a result, the trade areas defined in Figure 20 are disproportionately scaled. Due to the fact that the POVD is constructed based on distance only, the trade area definitions in Figure 20 are highly dependent upon the spatial configuration of stores. As a result, the trade areas in Figure 20 tend to under emphasize the influence of the more attractive stores (i.e. Zehrs) and over emphasize the drawing power of the less attractive stores. This trade area definition results in a situation where Zehrs is still represented as the dominant supermarket chain within Kitchener-Waterloo, but this position has been grossly underestimated (compared to the more realistic results contained within Figures 18 and 19, and Tables 8 and 9). The other weakness of the POVD as a trade area model, as was mentioned earlier, is the nature of its construction. A person who is unfamiliar with the POVD may interpret the bisectors as trade area boundaries. As a result, the trade areas created by the POVD can easily be perceived as spatial monopolies (which may or may not be the case). For these reasons, Figure 20 should only be used by experienced retail analysts for rough estimates of a store's trade area.

The market areas that are generated with the MWVD can be seen in Figure 21. These market areas are more appropriately scaled, as they factor the attractiveness of the facilities into the construction of the tessellation. The trade areas defined in Figure 21

Figure 20: Supermarket Trade Areas In Kitchener-Waterloo - The POVD

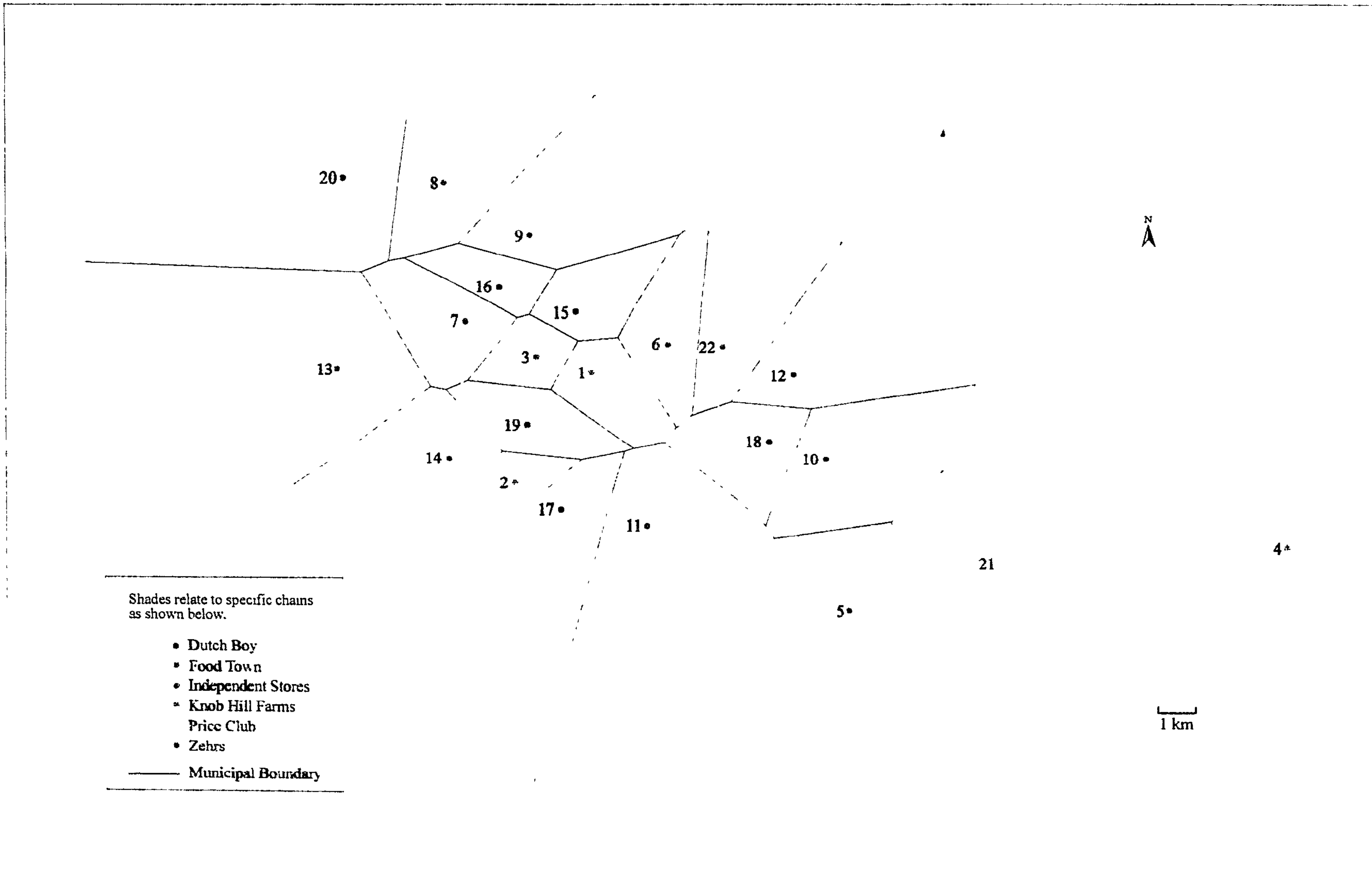
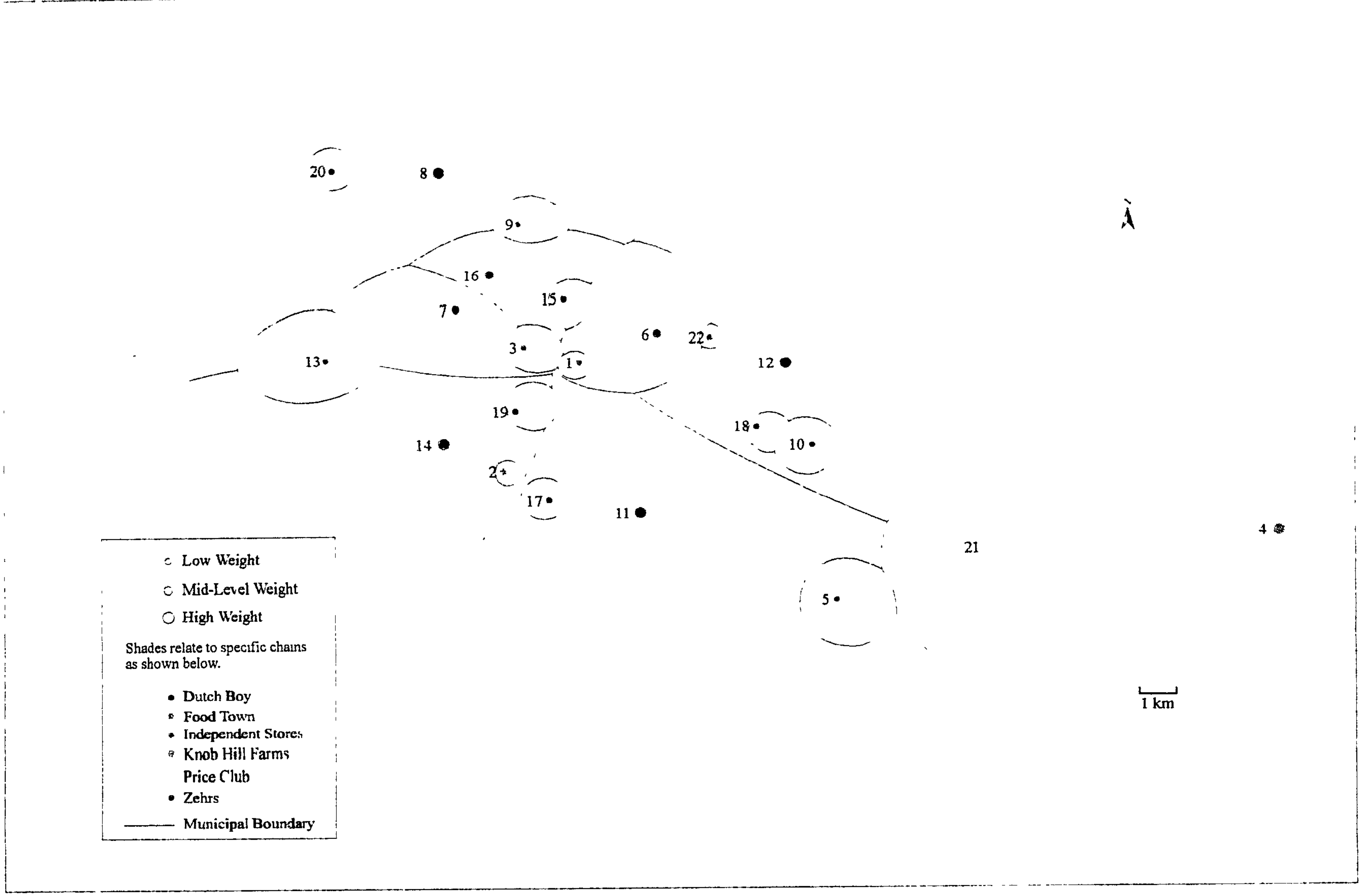


Figure 21: Supermarket Trade Areas In Kitchener-Waterloo - The MWVD



present a situation similar to the picture created by Figures 18 and 19, and Tables 8 and 9; Zehrs is clearly the most dominant supermarket chain within the Kitchener-Waterloo market. These trade areas, however, can still be easily misinterpreted as spatial monopolies. Therefore, Figure 21 can be used by experienced retail analysts for more accurate estimates of a store's trade area.

5.2.2 Sales Estimates

The trade area configurations defined in section 5.2.1 can be used to calculate sales estimates for the stores in the data set. In this analysis, only a small subset of the stores will be considered. For each of the models described in the preceding section, sales estimates were calculated for three of the supermarkets within Kitchener-Waterloo. The three stores that were used in this analysis were 14 (Zehrs), 21 (Price Club) and 22 (Dipietro's). The criteria used to select these stores was the weighting scheme. The weight was used as the selection criteria so that a comparison of how the different models estimate sales for a store with a high weight (store 14), a facility with a mid-level weight (store 21) and a store with a low weight (store 22) could be drawn.

There were two steps involved in this analysis. First, it was necessary to calculate the size of the three trade areas for each of the four models used in section 5.2.1. This was accomplished by digitizing the appropriate bisectors into Terrasoft and using the area calculation function to find the area of each polygon that falls within the municipal boundaries of Kitchener-Waterloo (see Table 10). These figures were then used to calculate the size of each store's trade area (see Table 11). In the POVD and MWVD

Table 10: Polygon Sizes (For The Four Models)

MODEL	POLYGON	SIZE OF POLYGON (Sq. km)	MODEL	POLYGON	SIZE OF POLYGON (Sq. km)
POVD	14	10.408	OO2MWVD	(14,13)	1.871
	21	5.396	(con't)	(7,14)	1.112
	22	5.837		(14,7)	2.760
MWVD	14	21.768		(14,19)	0.942
	21	6 359		(19,14)	1.082
	22	0.222		(11,14)	15.735
O2MWVD	(14,8)	5.052		(14,11)	14.708
	(14,13)	4.291		(2,14)	0.282
	(14,7)	3.872		(14,2)	0.254
	(14,19)	2.025		(4,21)	0.000
	(14,11)	30.442		(21,4)	2.750
	(14,2)	0.536		(5,21)	0.151
	(21,4)	2.750		(21,5)	0 152
	(21,5)	0.302		(11,21)	0.658
	(21,11)	3.168		(21,11)	2.510
	(21,12)	1.350		(12,21)	0.340
	(22,6)	0.101		(21,12)	1.011
	(22,12)	0.400		(6,22)	0.046
OO2MWVD	(8,14)	4.009		(22,6)	0.055
	(14,8)	1.043		(12,22)	0.233
	(13,14)	2.149		(22,12)	0.167

Table 11: Trade Area Sizes (For The Four Models)

MODEL	STORE	TRADE AREA SIZE (Sq. km)
POVD	14	10.408
	21	5.396
	22	5.837
MWVD	14	21.768
	21	6.359
	22	0.222
O2MWVD	14	46.217
	21	7.570
	22	0.501
OO2MWVD	14	46.217
	21	7.570
	22	0.501

models, the size of a store's trade area is equivalent to the size of the polygon constructed for that store. In the OKMWVD and OOKMWVD models, the physical size of a store's trade area is found by adding up the size of all of the territories associated with that store.

The second step required converting the trade area sizes in Table 11 into sales estimates for each store (see Table 12). This procedure was carried out according to the methods described in section 4.3. In the POVD and MWVD, the volume of sales assigned to any facility is consistent with the size of the trade area computed for that store. In the O2MWVD, the level of sales assigned to any centre is proportional to half of the size of

Table 12: Sales Estimates (For The Four Models)

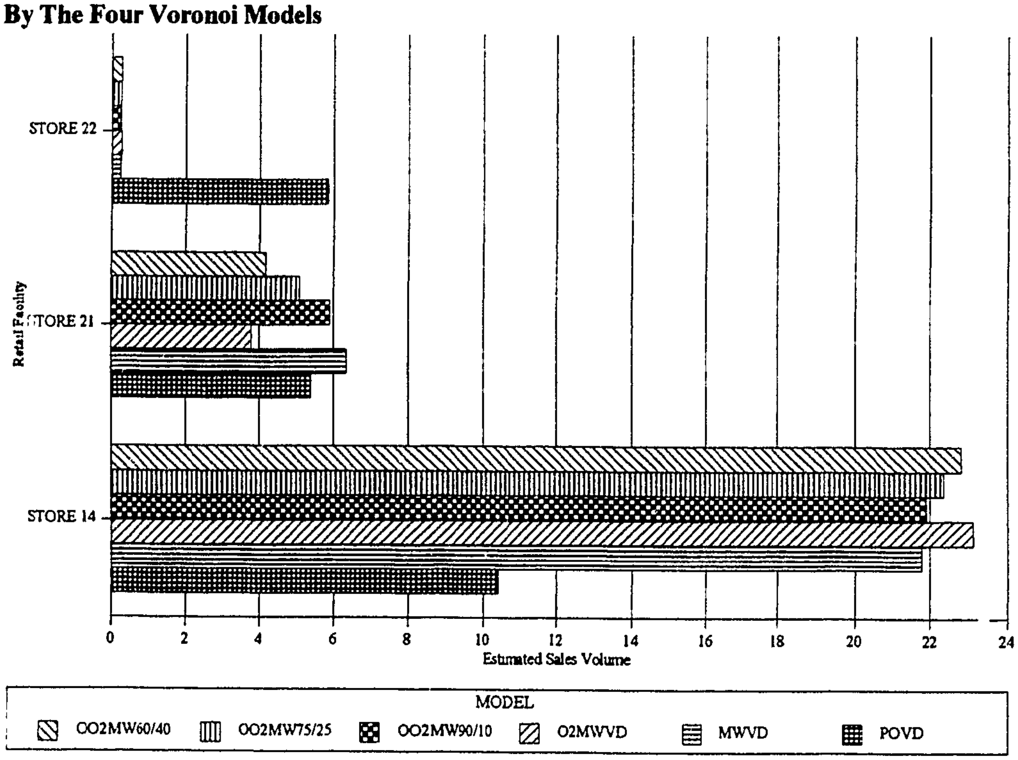
STORE	POVD	MWVD	O2MWVD	OO2MWVD (0.9/0.1)*	OO2MWVD (0.75/0.25)*	OO2MWVD (0.6/0.4)*
	SALES ESTIMATES					
14	10.408	21.768	23.109	21.884	22.343	22.803
21	5.396	6.359	3.785	5.894	5.103	4.178
22	5.837	0.222	0.250	0.228	0.236	0.245

* The probabilities for the OO2MWVD model were defined exogenously. Three different cases have been defined in this study. The probabilities were defined so that the closest facility was always assigned the higher level of probability.

the trade area defined for that outlet. The estimation of sales in the OO2MWVD model is more complex than in the previous models. The first step in this procedure is to multiply the size of all the polygons assigned to a facility by the probability that a consumer located in that area will patronize that store. The second step is to then add up all of these products, which results in the sales estimate for the facility under study. As was mentioned earlier, the probabilities assigned to the facilities in the OO2MWVD can be exogenously determined. This allows for a sensitivity analysis to be performed on the sales estimates.

As can be seen in Table 12, the four Voronoi models all generate different estimates of sales volume for the three stores being studied. The specific patterns of how each model estimates sales volumes are easier to see in graphical form (see Figure 22). Figure 22 has been set up so that all of the estimates produced for each store have been grouped together.

Figure 22: Sales Volumes Estimated By The Four Voronoi Models



One pattern that is very noticeable involves the predictive ability of the POVD. As can be seen in Figure 22, when compared to the estimates produced by the other models, the POVD generates sales estimates which are very erratic. The estimates that the POVD produces for store 14 and store 22 are out of line with the estimates generated by the weighted models. This pattern results due to the fact that the POVD ignores store attractiveness when constructing trade areas.

The POVD calculates the volume of sales in an egalitarian way. As was mentioned earlier, the POVD treats all stores as being equally attractive and only factors distance into the construction of the trade areas. These conditions result in a situation where the influence of the more attractive facilities (store 14) is under-emphasized and the drawing power of the less attractive stores (store 22) is over-emphasized. This situation generally leads to unusually low sales estimates for highly weighted stores and unusually high estimates for facilities with a low weight. Due to the volatile nature of the predictive ability of the POVD, it will not be considered in the remainder of the analysis.

In a comparison of the weighted models only, it can be seen in Table 12 and Figure 22 that the extremes of these estimates are always defined by the MWVD and O2MWVD. The OO2MWVD calculates sales estimates which fall between those of the other two models and increase/decrease as the ratio between the probabilities approaches 1.

These results indicate that the marketing strategy of the retailer has a dramatic impact upon the level of sales generated by his/her store. Store 21 is able to maximize its sales volume by concentrating its marketing efforts on the customers that immediately surround the facility. Stores 14 and 22, on the other hand, can maximize their sales volumes by taking a different approach. If stores 14 and 22 concentrate their marketing efforts solely on their primary market (i.e. the customers that immediately surround their facility), they are putting an unnecessary limit on their sales potential. These two facilities are able to maximize their sales volumes in a situation where the probability split with their neighbours is 0.5/0.5 (as it is in the O2MWVD). This means that stores 14 and 22 can make themselves better off by focusing their marketing efforts on a larger number of customers and, essentially, sharing their primary trade area with surrounding stores.

5.3 Conclusions

This application has demonstrated the superiority of the new model over the order-1 Voronoi models. The new model has the ability to create probabilistic trade areas which are easily interpreted and accurately scaled. The new model also allows the researcher to define a number of different scenarios in order to find the marketing strategy which maximizes the sales volume of the store under study. This makes the model very versatile; it can be adapted to fit any situation. These characteristics make the OKMWVD and OOKMWVD appealing as methods of trade area analysis.

6.0 CONCLUSIONS

The research contained within this thesis has shown that retail trade area models, traditionally, have been classified into a dichotomy of approaches. Past researchers have labelled trade area models either as a spatial monopoly or market penetration technique. However, this author makes the argument that it is not the model which falls into one of these two classes, but rather the way in which the individual chooses to interpret the results. This means that one person could interpret a specific model as a spatial monopoly model, while another person could perceive the same model as a market penetration model.

Voronoi diagrams, when interpreted as retail trade areas, have traditionally been classified as spatial monopolies. However, the preceding research has shown that retail trade areas defined with Voronoi diagrams can be interpreted in a probabilistic manner. One major flaw with order-1 Voronoi diagrams is the nature of their construction; because order-1 Voronoi diagrams only have one point associated with each polygon, the bisectors are often interpreted as the boundaries of spatially monopolistic trade areas. This author has argued, however, that the bisectors in a Voronoi diagram should be interpreted as lines of equal probability and not rigid boundaries.

Two new Voronoi diagrams (the OKMWVD and OOKMWVD) were introduced as alternatives to the order-1 Voronoi models. The higher-order Voronoi diagrams, because they define polygons for sets of points, create trade areas that are easily interpreted as overlapping and probabilistic constructions.

The two new Voronoi diagrams were used as the basis for the development of a retail trade area model which assigns choice sets to all consumers. This new model was applied to a real world data set and these results were compared to those generated by the order-1 models.

The conclusions of this research are as follows. First, the two higher-order Voronoi models generate trade area definitions that are more accurate and easier to interpret than those defined by the order-1 models. The POVD creates trade areas that are disproportionately scaled and both of the order-1 models can easily be interpreted as spatial monopolies. The OKMWVD and OOKMWVD eliminate both of these problems.

Second, the higher-order models are more versatile in the calculation of sales volume estimates compared to the order-1 models. The higher-order models allow the retailer to define a number of different situations in order to find the marketing strategy under which sales volumes are maximized.

The research contained within this thesis has shown that Voronoi diagrams can be used as an effective method of trade area analysis. There are, however, several areas that should be addressed by future research. First, the research contained within this thesis utilized Euclidean distance. However, trade area definitions are sensitive to changes in how distances are measured. Therefore, research into how the OKMWVD and OOKMWVD behave under different distance measures, such as Manhattan, is needed. Second, the weighting scheme utilized within this research is multiplicative. Voronoi diagrams can also be weighted additively and compoundly. Hence, research into these different weighting schemes is also needed. Third, the order used in this study was $k = 2$. Other values of k may be more appropriate in different situations. Finally, this thesis has made some strong arguments against the traditional classification of retail trade area models. As a result, much research is needed with regard to model interpretation. Every researcher who uses retail trade area models should be aware that the way in which they choose to interpret the models has a significant impact on the results they produce. Future studies should compare the results that models produce under different assumptions.

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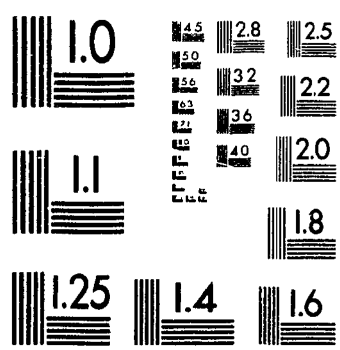
2 of/de 2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT



PRECISION℠ RESOLUTION TARGETS

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