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Using digital elevation models to measure the surface and volumetric change of Athabasca Glacier, Alberta, Canada, 1919-1979

James Robb Reynolds
Wilfrid Laurier University

USING DIGITAL ELEVATION MODELS TO MEASURE THE SURFACE AND VOLUMETRIC CHANGE OF ATHABASCA GLACIER, ALBERTA, CANADA, 1919-1979

by

James Robb Reynolds
B.A., University of Western Ontario, 1992

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ABSTRACT

The Athabasca Glacier (52°12'N, 117°14'W), located at the Alberta-British Columbia border, was studied to quantify its volumetric, area and elevation changes below 2400 m between 1919 and 1979.

The data sources consist of maps produced using aerial and terrestrial photogrammetry in the years between 1919 and 1979. The maps were digitized and converted into raster digital elevation models (DEMs), the manipulation of which allowed values of surface and volumetric change to be calculated. These DEMs showed that between 1919 and 1979 the glacier lost 2.344×10^8 m^3 of volume and receded more than 1 km.

Each of the source maps has a precision of vertical estimation associated with the photogrammetric process used to generate it. This imprecision was quantified and used to calculate, display and compare the uncertainty of volume and elevation change measures with the calculated volume and elevation change. The magnitude of uncertainty between maps was often larger than the change measured between the maps.

Many of the maps used to generate DEMs were used in a previous study to calculate volumetric and elevation change using planimetric methods. The software package which produced the DEMs and calculated output was used to recalculate volumetric change measures using the same methodology for the period 1969-1979. The recalculated results were similar to the planimetric results for that time period.

Several other series of Canadian glacier maps exist. It is recommended that a similar study be carried out using these data.

ACKNOWLEDGEMENTS

A project of this size cannot be done in isolation. First and foremost, this thesis would never have been possible without the help of my advisor Gordon Young. His advice, help and suggestions were invaluable to me. I am also deeply indebted to him for giving me the chance to attend three meetings of the Canadian Geophysical Union, which helped in the production of this work.

Thanks also to Mike English who gave me desk space at the Cold Regions Research Centre, where the bulk of the text was written. Thanks must also be extended to the other denizens of Cold Regions: Cam, Cam, LeeAnn, Mark, Brad, Chris, Rich, Corinne, Al (''Krusty''), Paul, George and Alexi. These, my officemates, helped me from going completely insane during this time (and on the occasions that I did, were most helpful in catching flies for me to eat).

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I would like to thank my parents for the help and support that they have given over the course of this work. Without them, this degree would have been impossible to complete. Finally and sadly, I would like to thank Mr. Edward (Ted) Frederick Clarke (1922-1995). Unlike the other people I have listed here, I will not be able to thank him in person. His warmth and generous nature are missed by me and by all who knew him.

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CHAPTER ONE: INTRODUCTION

1.1 Background

Glaciers are one of the main stores of fresh water in the world today. Although more than 99% of glacial ice is stored in the Antarctic and Greenland ice caps, non-polar ice caps and valley glaciers store an estimated 180,000 km^3 of ice (Sugden and John, 1976). In addition to storing fresh water, temperate glaciers influence downstream flow by accumulating snow in the winter and by melt that occurs throughout the summer.

Glaciers are also one of the indicators of climate change. The fluctuation of a glacier is strongly influenced by annual precipitation and temperature conditions. The advance or retreat of a glacier or glaciers in a catchment has a substantial effect on downstream hydrology and land use.

Since 1959, a series of reports have been published that summarize and detail earlier and ongoing studies of individual glaciers around the world (Kasser, 1967 *et sequentia*). These reports provide a centralized listing of glacial changes, including net balance, equilibrium line altitude and accumulation area ratios. This allows the effects of possible human-induced climate change to be investigated. This worldwide database depends on records of individual glaciers. Although many Canadian glaciers have been studied in detail in the past, few are currently being studied. However, some of the information that has been collected in the past can be used to generate volumetric and surface change information for glaciers not presently in the database, broadening the knowledge base concerning modern glacial change.

1.2 Statement of Objectives

The objectives of this thesis are:

- to evaluate the accuracy of existing maps of the glacier
- to obtain measures of change from these maps
- to estimate the accuracy of this mensuration, and
- to compare the values produced using DEMs to a previous series of measurements made using planimetry

To accomplish this, contour maps of suitable scales that cover the Athabasca Glacier are converted into DEMs, which are used to calculate the surface and volumetric change of this glacier over the period of record.

Previous elevation and volumetric change calculations published for the Athabasca Glacier are based on techniques that are now obsolete. In addition, the small-scale maps, particularly the earliest map, necessarily have low levels of precision due to the small scale photography that they were produced from. Thus, this thesis deals with comparing maps made using different technologies and contrasting calculations made with different techniques.

1.3 Location of the Study Area

The Columbia Icefield, a large body of ice straddling the continental divide, contains the accumulation area of the Athabasca Glacier. The glacier is located at 52°12'N, 117°14'W, inside Jasper National Park near the Alberta-British Columbia border, in the Front Ranges of the Canadian Rocky Mountains (see figure 1.1). It flows in a north-northeast direction off the icefield over three icefalls into an alpine valley, which contains its ablation area. The highest of the icefalls is the source of the debris that covers approximately one third of the western margin of the glacier. Below the lowest icefall, the ablation area of the glacier has a relatively constant slope ($\approx 5°$) until

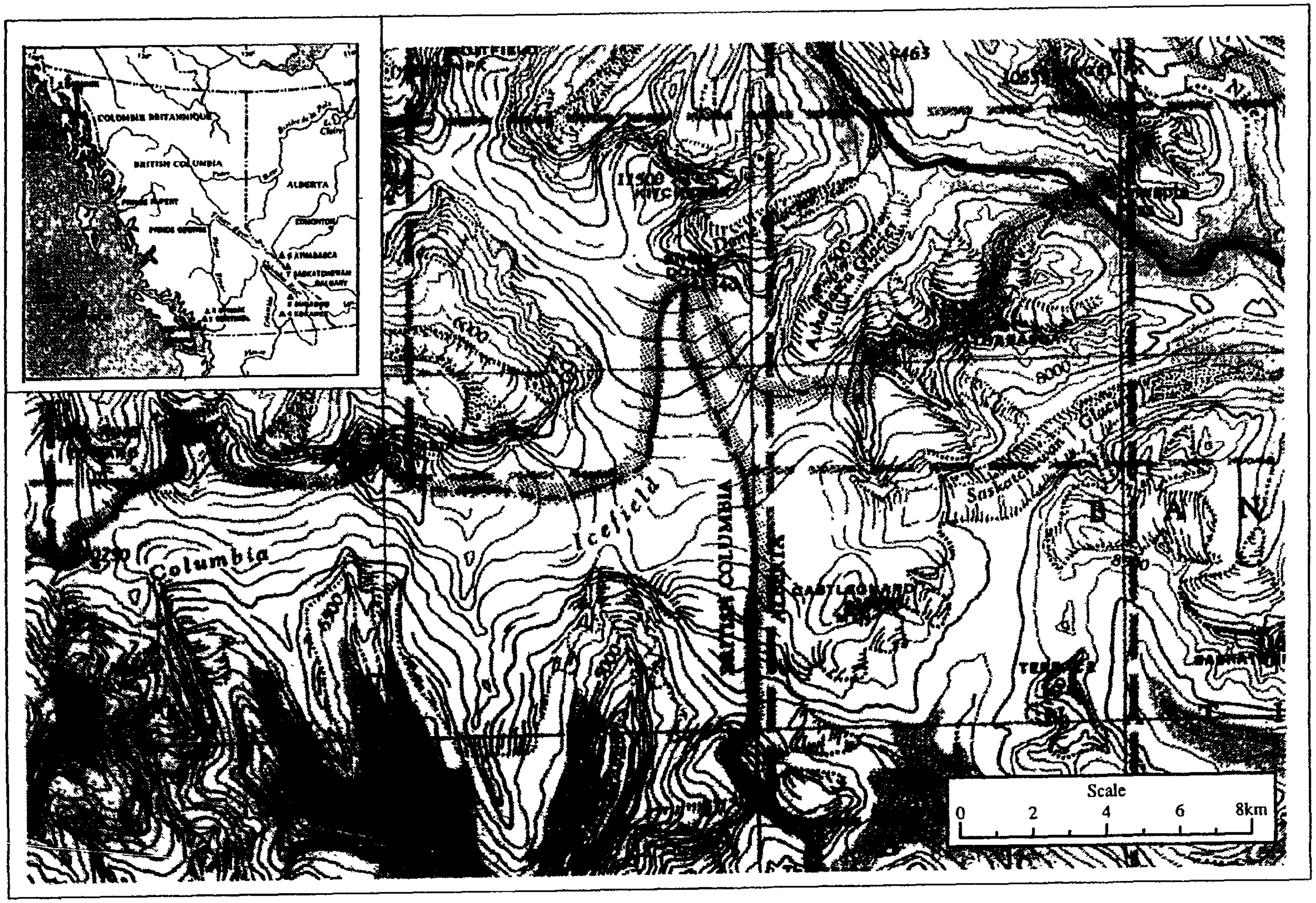


Figure 1.1: Location of Athabasca Glacier. Main map showing Columbia Icefield is a section of NTS 1:250,000 83C (Brazeau). Inset is from 1979 1:10,000 terrestrial map produced by Inland Waters.

the terminus region, where it steepens. Melt from the Athabasca Glacier flows into Sunwapta Lake at the foot of the glacier, from which it drains to the Sunwapta, Athabasca and Mackenzie Rivers with its ultimate destination the Arctic Ocean.

The highest elevation of the glacier is on the Columbia Icefield, where the ice surface is greater than 3300 metres on both the Snow Dome and Mt. Andromeda (figure 1.1). The elevation of the terminus of the glacier has changed over the period of record. In the earliest source map, from 1919, its terminus is at 1924 m; in the final 1979 map, its elevation is 1944 m (*cf* tables 4.1*a,j*). The locations of the terminus in the years of mapping are shown in figure 4.2.

Only the portion of the glacier that lies below 2400 m was studied in this thesis. This corresponds with most of the ablation area of the glacier, since, from air photos, its equilibrium line altitude (ELA) lies in the range 2450-2500 m. Below the ELA is the zone in which the greatest changes have occurred in the period of record. This portion of the glacier has been mapped more often than the accumulation area and it is the zone where the precision of measurements is the highest. The snow-covered zone above the ELA is a high albedo, low contrast area which is difficult to map with great precision using photogrammetry due to the lack of measurement points that are readily identifiable on more than one photograph.

1.4 Organization of Thesis

Following this introductory chapter, the relevant literature will be reviewed. This will include a discussion of several aspects concerning the glacier, such as its Holocene fluctuation record. It will also describe how change was measured before the introduction of DEMs, and how change is measured using DEMs.

Chapter three will discuss the methodology of the thesis. Factors discussed concerning the maps from which DEMs were produced include: the definition of borders; the accuracy of vertical estimation; and the problems of finding a common georeferencing for the different map series. Following this, *Surfer*, the primary package used for production of results in this thesis, is discussed. The method used to calculate values of elevation and volumetric change using this package is detailed. The placement of contour lines on the original maps is one of the prime determinants of the precision of volume and elevation change measures. A discussion of how the level of precision affects the uncertainty of calculation of measurement concludes the chapter.

The fourth chapter presents the results of the calculations outlined in chapter three. Results are given first for the entire ablation area of the glacier for surface and volumetric change. Following this is a comparison of previous results calculated using planimetry for clear ice surface with results produced using DEMs.

The final chapter summarizes the results presented in chapter four and presents recommendations for further research that were suggested by this thesis. A number of appendices follow the final chapter. They present material which could not be properly dealt with in the body of the thesis but is nevertheless relevant to it. These include long lists of photographs, program codes, bibliographic lists and examples of the exact methodology used for volume and area calculations.

CHAPTER TWO: BACKGROUND AND LITERATURE REVIEW

2.0 Introduction

The Athabasca Glacier is one of the most accessible glaciers in Canada, and hence one of the most studied Canadian glaciers (Luckman, 1988). This chapter will offer a brief survey of the studies that have been done to date but the emphasis will be on those dealing with glacier fluctuation. To place the topic of surface and volumetric change in a broader context, worldwide glacier fluctuations and fluctuations which have occurred in the Rocky Mountains since the end of the last ice age will be described.

The Athabasca Glacier has been described and studied since at least 1898. Before 1945, the work was largely exploratory and qualitative. Since 1945 more analytical and quantitative work has been undertaken. Much of the recent work relates to glacier fluctuation, but other features such as determination of depth (Brugman and Demuth, in preparation), flow (Paterson and Savage, 1963), englacial temperature (Paterson. 1971) and terrain irradiance (Gratton, 1991; Gratton *et al*, 1994) have been studied as well. These studies are peripheral to the topic of this thesis, and so will be noted only in passing. Kite and Reid (1977) and Brugman and Demuth (in preparation) provide a more thorough summary of these researches.

The problems of measuring surfaces which are constantly changing will be discussed. The merits of two types of photogrammetry used to produce maps of glacier surfaces will be outlined. Finally, two methods of measuring elevation and volumetric change over a period of time, planimetry and digital elevation models, will be described and compared.

2.1 Glacial Fluctuation

Glacier fluctuation is influenced by many factors, including general climate, local climate, mass and energy exchange, net mass balance and local topography (Paterson, 1981). The most important of these factors affecting long-term fluctuation is the general climate: the average conditions over a large area containing the glacier.

Climate varies on many scales, both temporal and spatial. Warm, dry periods such as the Hypsithermal, and cooler, wetter periods such as the Neoglacial, can occur lasting several thousand years. These features appear to be governed by long-term cycles in the Earth's orbit (Luckman, 1990). Within the Neoglacial, various periods of glacier advance followed by retreat occurred. Finally, significant changes can occur in less than a hundred years, as is shown by the record of glacial fluctuation in this century (Haeberli *et al.*, 1989; IAHS, 1994). The elevation and volumetric change of the Athabasca glacier that have occurred in this century take place in the context of ongoing, longer-term glacier fluctuation, both worldwide and local.

2.1.1 Current Global Glacier Fluctuations

Global glacier fluctuations occur at different rates in different parts of the world. The glaciers of some regions can advance, while the glaciers of other regions retreat. Since the mid-1970s, the majority of glaciers in the Alps have been receding, while many Norwegian glaciers have advanced (IAHS, 1994). In this case, it is the increased precipitation that Scandinavia is receiving in contrast to the Alps that is fuelling the advance of the Scandinavian glaciers.

Long observational records of the position of glacial termini exist for various glaciated areas around the world, including glaciers from the Alps, Norway, Iceland and

Pakistan. However, the written record of glacier fluctuation in the Rocky Mountains is always less than a century old. Other methods must be used to determine how the glaciers of the Rocky Mountains have changed in earlier times.

2.1.2 Rocky Mountains

The history of climatic change and glacier fluctuation in the Rockies and at the Athabasca Glacier are acquired in two ways: directly from the instrumental record and indirectly from the proxy record. Proxy climate is inferred from such sources as tree-rings, macrofossils, pollen deposition and sediment records. These sources are not uniformly available, and have varying levels of temporal resolution. However, the proxy climate record over the entire Rocky Mountain area is highly intercorrelated, with glacier advances in the region having similar timing and comparable magnitudes (Luckman, 1993). Thus, proxy data from different locations in the Rocky Mountains can be used as a generalized climate signal for a particular area.

The Holocene in the Rocky Mountains began with deglaciation at the end of the Pleistocene. Radiocarbon dates from main outlet valleys show that glaciers were retreating by 13,000 - 11,000 BP (Reasoner and Rutter, 1987). Various limiting radiocarbon dates throughout the Rockies indicate that glaciers had retreated to within Little Ice Age maximum limits between 11,400 (Reasoner and Rutter, 1987) and 9,600 $\pm$ 305 BP (Beaudoin and King, 1990).

The Holocene in the Rockies may be divided into two periods: the Hypsithermal and the Neoglacial. The Hypsithermal, the earlier period, was characterized by temperatures that were warmer and drier than the present. By 8800 BP, the treeline was higher than it is at present (Luckman, 1990). During that period, treeline elevation fluctuated, but generally remained higher than current levels (Luckman and Kearney 1986). As late as 5300 BP, temperatures were greater than 1°C warmer than they are at present. During this time, glaciers were substantially smaller than they are now (Osborn and Luckman, 1988).

The transition to the Neoglacial, a generally cooler and wetter period than the Hypsithermal, occurred between 5000 and 4000 BP (Beaudoin and King, 1990). During the Neoglacial, there have been at least three major periods of glacier advance: ca. 4000 BP, 3000-2500 BP, and AD 1100-1850, with the most recent advances being the most extensive (Luckman *et al.*, 1993; Luckman, 1993).

Tree-ring studies at several glaciers in the Rocky Mountains indicate that there is a strong relationship between the magnitude and timing of glacier fluctuation and tree-ring records throughout the region (Luckman, 1993). Long tree-ring records from the region (Reynolds, 1992; Luckman *et al.*, 1992) indicate several episodes of curtailment of growth which are assumed to be caused by local climatic deterioration. Some of these episodes may be related to glacier fluctuation. The two most recent advances, in the early eighteenth and mid nineteenth centuries, were the most significant for regional glaciers. All regional glaciers have outer or readvance moraines dating to the mid nineteenth century; approximately one third of glaciers have outermost moraines dating to the first quarter of the eighteenth century (Luckman, 1993).

2.1.3 The Athabasca Glacier since 1843-44

The Athabasca Glacier, located on the Alberta side of the British Columbia-Alberta border, reached its maximum Holocene extent in AD 1843-1844. Moraines from its previous maximum extent, dated to AD 1714, exist in very limited areas, suggesting that its early eighteenth century advance was of the same or slightly lesser extent (Luckman, 1988). Since its nineteenth century maximum extent, the Athabasca has retreated substantially.

Although the glacier is about 80 km from the Kicking Horse Pass and Yellowhead Pass, used for the fur trade during the nineteenth century, the first recorded visit to the glacier occurred in 1898. The first visitors who left a written record were Collie and Wooley, associated with the mountaineering expedition of Stutfield and Collie. The primary interest of this expedition was mountaineering, not glacial observation (Stutfield and Collie, 1903). Despite this bias, some information on the position of the glacier can be obtained from their work. Collie produced a map of their route, a portion of which is reproduced in figure 2.1 (Collie, 1903). Although it is only a sketch map, it can be seen that the Athabasca Glacier had not receded very far from its Little Ice Age maximum position. The termini of the Athabasca and the adjacent Dome glaciers are still coalesced.

The next recorded visits to the glacier occurred in 1906-1908 and were made by Schaffer (Schaffer, 1908; Schaffer, 1911). Schaffer was primarily a tourist, and her work reflects this. Its greatest scientific value consists of the photographs she took of the glacier which enabled later workers to reconstruct its recession. These photographs show that in the years between her visits and those of Stutfield and Collie, the Athabasca Glacier continued to recede. Its terminus was no longer coalesced with the terminus of the Dome Glacier when she visited the site. Between the glacial maximum in 1844 and the Schaffer visits of 1906-08, the glacier receded at a rate of about 3 m per year (Luckman, 1988).

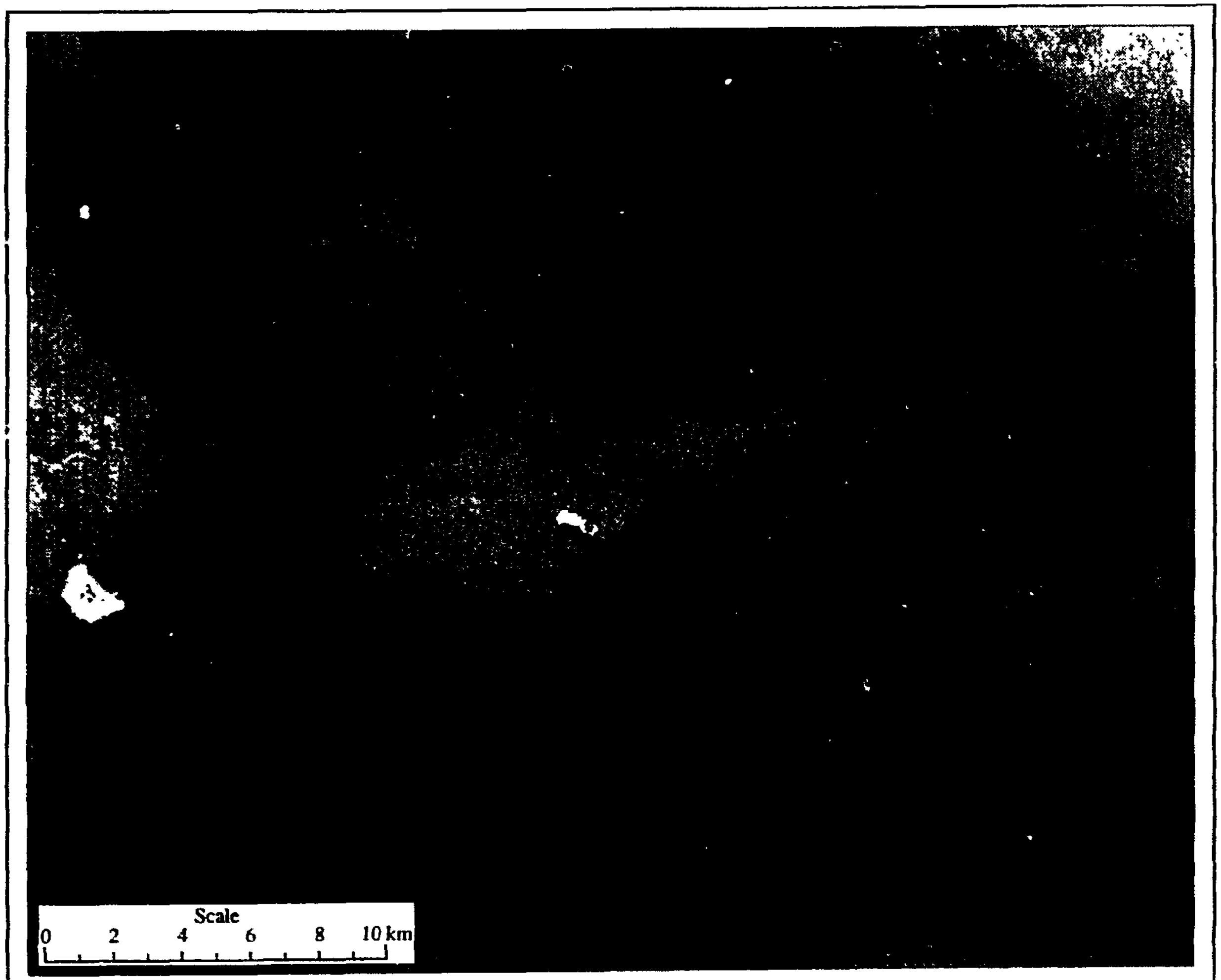


Figure 2.1: Section of "Sketch map of the Canadian Rocky Mountains" by J. Norman Collie (1903). Scale of original 1:500,000. Elevations in feet. Termini of Athabasca and Dome Glaciers are still coalesced.

The Alberta-British Columbia Boundary Commission provided the next record of the glacier. The tasks of the commission were to delimit the interprovincial border and to prepare contour maps of the region (Cautley *et al.*, 1924). Since the Athabasca Glacier lies near the border, the report and atlas produced by this commission provided the first quantitative information about the glacier's surface (Cautley and Wheeler, 1924*a,b*). A portion of the map they produced covering the Athabasca Glacier is reproduced in figure 3.1.

The National Geographic Society also sponsored an expedition to the area. The article that was published (Freeman, 1925) was a description of experiences and difficulties on the trail in the back country in the 1920s. Many photos were published in this article, and many more were taken, but few were of the Athabasca Glacier.

The remainder of the pre-1945 materials dealing with the glacier consisted of various photos taken of the glacier, either by interested private parties such as Byron Harmon and J.M. Thorington or such bodies as the Alpine Club of Canada (Wheeler, 1920). Some of the early photography is reprinted in Kite and Reid (1977). Luckman (1986) published a more complete list of old photographs of the glacier. The photographs indicate that from 1906 to the 1940s, recession accelerated. In the period 1938-1950, air photos and terminus measurements indicate that the glacier was receding at a rate of approximately 30 m/year (Luckman, 1988). Appendix 1 contains a list of early photographs of the Athabasca Glacier which are found in the Whyte Museum of the Canadian Rockies, the major repository for archival photography of the area. This list includes the photos referred to above.

After 1945, the Athabasca Glacier was studied more systematically. The research relied more on collection and analysis of quantitative data such as surface and terminus surveys, ice flow and depth analyses, and temperature studies (Kite and Reid, 1977).

The Water Resources Branch began an annual, then bi-annual study in 1945 on several western glaciers including the Athabasca. These studies concerned outlining the toe of the glacier, its recession, determination of the flow from the stream at the terminus of the glacier, and measurement of the rate of surface velocity of the toe (McFarlane, 1945 *et sequentia*; see appendix 2). Although the WSC only studied the recession of the clear ice portion of the terminus, it still provides a valuable record of retreat. These records show that recession was very swift during the 1940s and 1950s, averaging more than 27 m/yr in the period 1945-1960. In the 1960s and 1970s, recession slowed substantially. From 1960-1970, recession averaged about 8 m/yr. From 1970-1980, recession was 4.4 m/yr (Water Survey of Canada, 1982). Appendix 2 contains a bibliographic listing of the WSC reports.

The WSC bi-annual studies of the recession of the Athabasca Glacier ended in 1980 with almost no recession reported between 1977 and 1979. It was during the time of slow retreat that one of the small glaciers on Mts. Andromeda/Athabasca, which shares the Athabasca Glacier's catchment, recorded a slight readvance: the resulting readvance moraine is visible when comparing 1979 and 1992 air photos. The retreat of the Athabasca began accelerating in the 1980s. Recent observations of the Athabasca Glacier report that it is currently receding at rates comparable to those recorded in the 1940s and 1950s: 20 metres per year or more (Kucera, 1993; Brugman, personal communication, 1994).

Figure 2.2*a-f* consists of six aerial photographs of the Athabasca Glacier which

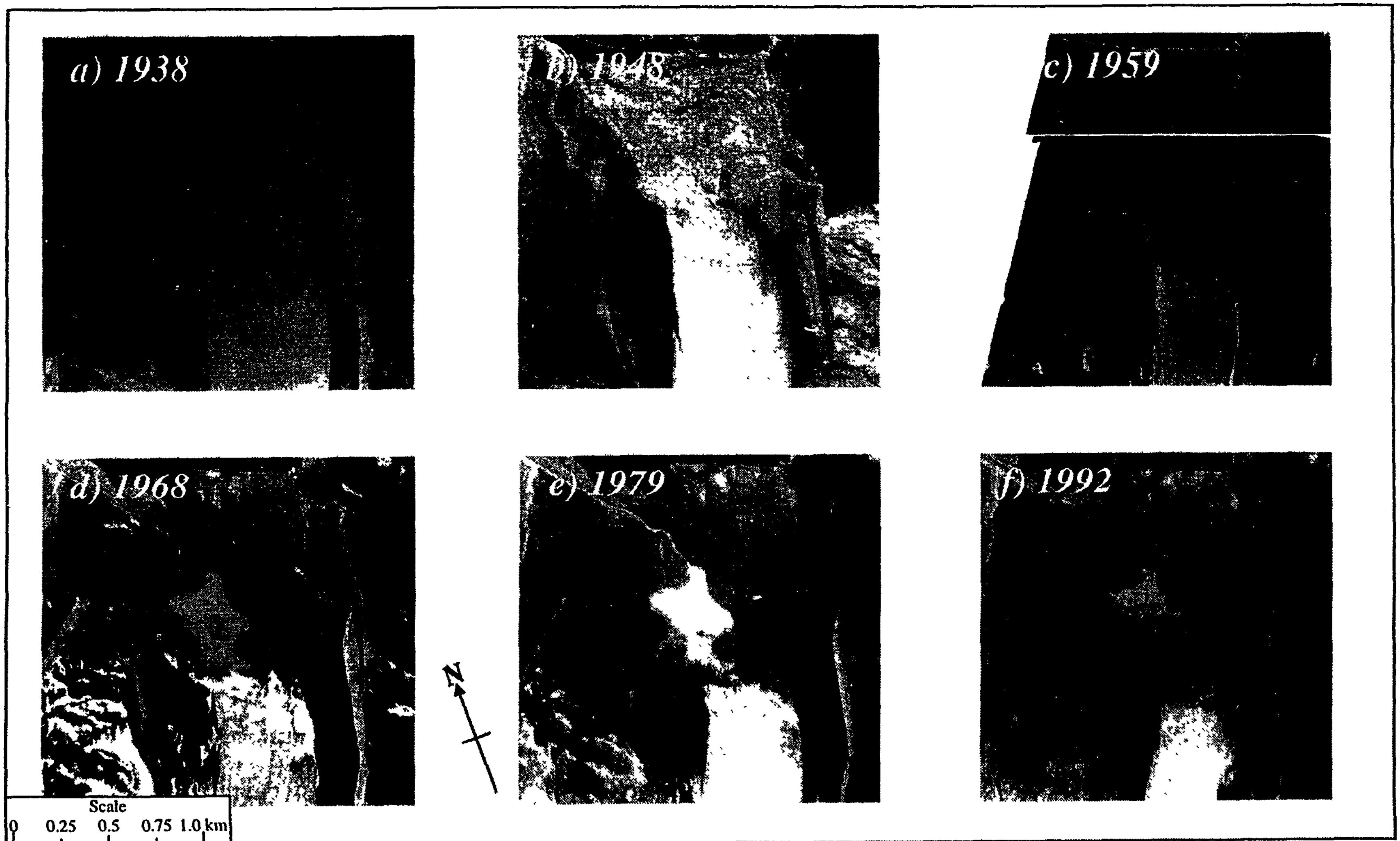


Figure 2.2: Air photos of the Athabasca Glacier, 1938-1992. Photos are at the same scale and orientation, and show the same approximate area of the terminus of the glacier.

were taken between 1938 and 1992. The photos are of approximately the same scale and orientation. They show the same area of the terminus of the glacier, and thus provide a qualitative portrayal of the recession of the glacier over the time of the air photo record. Appendix 3 contains a listing of many of the air photos of the Athabasca Glacier taken since 1938.

2.2 Measuring Glacier Surfaces

Glacier surfaces are constantly changing. The study of glacier change requires accurate information about the surface of the glacier at different times. One such source of information comes from appropriately-scaled contour maps of glaciers and their surrounding terrain. Suitable time intervals between maps allow changes to be measured (Haumann, 1960). The contour maps used in this study were drawn using data gathered through various kinds of photogrammetry.

2.2.1 Photogrammetry

Photogrammetry is defined as '...the art, science and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring and interpreting photographic images and patterns of electro-magnetic radiant energy and other phenomena...' (Slama, 1980 in Lane *et al.*, 1993). The photograph is a very data-rich method of measurement. Everything visible on a photo can be measured. Photogrammetry has another advantage over ground surveys in that aside from the time required to survey ground control points it is an instantaneous method of data acquisition, whereas measurements using ground surveys are more time-intensive.

Two varieties of photogrammetry will be discussed: stereoscopic and non-stereoscopic. Chapter three discusses the levels of accuracy associated with each method and how this affects measurement of elevation and volumetric change.

It should be noted that other methods of data acquisition using remote sensing exist, notably radar and satellite mapping and surface surveys. Information on the glacier surface produced using these methods were not available for use in this thesis however, and so will not be discussed.

2.2.1.1 Non-Stereoscopic

Terrestrial non-stereoscopic photogrammetry was used in the early years of this century to map the British Columbia-Alberta border. This variety of photogrammetry is more suited to exploratory work than to detailed mapping since measurement requires only a camera and a transit theodolite. For the 1919 survey, selected stations of known location were occupied and photographs were taken from them. The orientation of the camera was measured using the transit theodolite. Then the positions of the stations were determined by triangulation from ground-based surveying (Cautley *et al.*, 1924). With the position of the camera and the orientation of the optical axis (a line perpendicular to the plane of the photographic film which passes through the camera lens) known for both photos, points which could be identified on both photographs could be located precisely in space.

Elevations and positions of objects recorded on the photography were determined by measuring angles of elevation or depression and by measuring displacement from the centre line for each point on the various photographs and applying the necessary corrections for curvature and refraction. This process is known as trigonometric levelling (Cautley *et al.*, 1924).

The primary drawback of this method is its lack of ground control. Without ground control, slight errors in determining the station position and the orientation of the optical axis can cause a high degree of uncertainty in determining the ground position of points on the photographs. With ground control, such errors can be detected. Without them, there is no way to guard against this error.

2.2.1.2 Stereoscopic

The other maps used in this study were produced using stereoscopic photogrammetry. This type of photogrammetry measures elevations by measuring photographic parallax: the amount of displacement that an object on one photograph has compared with the same object on another photograph having a parallel optical axis (Lillesand and Kiefer, 1987). Unlike non-stereoscopic photogrammetry, ground control points (GCPs) must appear on the photos. GCPs are points whose location and elevation have been determined precisely by surveying either before or after the photos were taken.

Stereo photogrammetry can be either aerial or terrestrial. Photographs acquired for aerial photogrammetry have a vertical optical axis and are taken from an airplane flying at the desired altitude. This produces photos at the scale dictated by the altitude and focal length of the camera (Lillesand and Kiefer, 1987). Such photos, taken of glaciers or other high-relief features, can produce maps which display the subject of study and the terrain surrounding it to good advantage (Konecny, 1966).

Aerial photogrammetry has its disadvantages, however. Due to atmospheric turbulence, the airplane is subject to pitch, roll and yaw, which change the orientation of the photograph and cause the optical axis to vary from the vertical. These problems are amenable to geometric correction by measuring the relative locations of ground control points. Another disadvantage of aerial photogrammetry is that it is effectively impossible to reoccupy the photographic stations used (Konecny, 1963), making the production of a time series of photographs of a similar scale and orientation difficult.

A more serious disadvantage of aerial photogrammetry in high-relief terrain is that the airplane often cannot fly as close to the subject of study as is desirable. To avoid the complex winds which characterize high-relief zones (Barry, 1992), the plane must fly at a relatively high elevation. Given a camera with a standard focal length (152.4 mm), the scale of photography is reduced, which places limits on mapping capabilities (Ghosh, 1988).

In terrestrial photogrammetry, pictures are taken from a high location overlooking the object of study. The cameras still have parallel optical axes, but the axes are oblique rather than vertical or near-vertical. Since the photos used in terrestrial photogrammetry are taken at low angles, much of the surface of high relief terrain such as that surrounding the Athabasca Glacier is not in line of sight of both photo stations. Areas which appear on only one photo cannot be mapped. Thus, extensive portions of maps made of high-terrain areas using terrestrial photogrammetry taken from one pair of stations may contain many blank areas.

Despite these limitations, terrestrial photogrammetry has many advantages. It is far less expensive than aerial photogrammetry and less susceptible to inclement weather (Reid, 1972). The location of the photographic stations can be known precisely. The stations can be marked and reoccupied to produce a time series of photos having identical orientation that were taken from the same site. In addition, terrestrial photo stations can be much closer to their subject than aerial photo stations, with the associated advantages of larger photographic scale (Konecny, 1966).

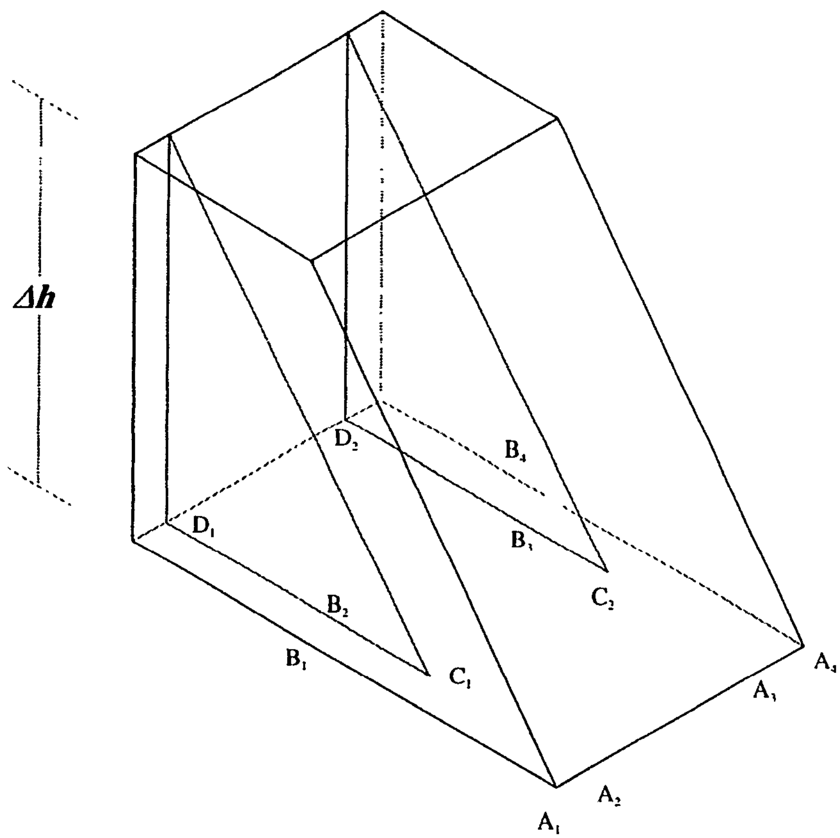
2.2.2 Measurement of Change prior to Digital Elevation Models

Topographical maps are an empirical representation of the surface of an area. Although it is possible to measure both elevation and volumetric change from contour maps, it is not very convenient to do so (Burrough, 1986). Despite the problems, the volumetric and elevation change of the Athabasca Glacier have been measured using data from topographic maps. The methods used are discussed below.

2.2.2.1 Volumetric Change

Brandenburger and Bull (1966) describe and outline four methods to approximate volumetric change between maps: measuring volume change as a function of the estimated elevation change; the Finsterwalder method; the Haumann method; and a method developed by Davey and identified in Brandenburger and Bull (1966). Each of these methods depends on the close comparison of different contour maps and were developed in the era before the existence of fast and convenient computing power. Volume change was measured by planimetrically determining areas and changes in areas between contour lines and using several equations to convert measures of area to measures of volume. This method requires painstaking, labour intensive measurement by human operators.

In the previous volumetric change studies on the Athabasca Glacier, Reid and Charbonneau (1981) used the Haumann method, a slightly modified version of the Finsterwalder method (Haumann, 1960), to calculate volumetric change. The calculations required to produce volumetric change values are complex and indirect. Figure 2.3 illustrates how change is calculated using this method.



Equations

$F_1 = A_2A_3B_3B_2$ $\quad \Delta F_1 = A_2A_3C_2C_1$

$F_2 = C_1C_2D_2D_1$ $\quad \Delta F_2 = B_2B_3D_2D_1$

$F_1'' = A_1A_4B_4B_1$ $\quad F_m = \frac{(F_2 + F_1'')}{2}$

dV = volumetric change

dh = height loss $\quad$ Δh = contour interval

$$dh = \left(\frac{\Delta F_1 + \Delta F_2}{F_1 + F_2}\right)\Delta h$$

$$dV = F_m\, dh$$

Figure 2.3: Calculation of volumetric change using the Haumann (1960) method.

Appendix 4 contains a mathematical comparison of the calculation of volumetric difference. Two flat-sided polyhedrons were generated, and the volumetric difference between them was calculated using geometric methods and by using the Haumann method. It can be seen from the proof presented in that appendix that the Haumann method produces results identical to the geometrically calculated volumetric change only when the widths of the two polyhedrons compared are identical. When the widths are not the same, the results produced by the Haumann method are substantially different from the results produced by geometric calculation. The Haumann method cannot properly measure the volumetric difference resulting from a change of width.

In addition, although it was stated in Brandenburger and Bull (1966) that the Haumann method assumed that glacier surfaces remained parallel from one map to the next, this was not confirmed by the proof in Appendix 4. This showed that whether the different surfaces were parallel or not is irrelevant. The essential control of accuracy of measurement is the relative width of the polyhedrons being measured.

Thus, the Haumann method is unsuitable for measuring volumetric change of glaciers whose lateral cross-sections vary markedly from one study period to the next. Its use by Reid and Charbonneau (1981) for volumetric studies on the Athabasca Glacier was justifiable since in the period of their study the ablation zone, with the exception of the terminus, did not have a substantial change in lateral dimension.

2.2.2.1.1 Previous Calculation of the Volumetric Change of Athabasca Glacier

Between the years 1959 and 1979, a series of ten large-scale maps of the ablation area of the Athabasca Glacier were produced by the Water Resources Branch and its successor, the Inland Waters Branch. A listing of the bibliographic references of the reports appears in appendix 5. The bibliographic references of the maps that appeared in that report series appear in appendix 6. Volumetric change between these maps was calculated using the Haumann method (Reid and Charbonneau, 1981) for 25-foot elevation zones (before 1969) and 10-metre elevation zones (after 1969). Some difficulties with these calculations should be noted. Only the volumetric change of the clear-ice portion of the glacier was calculated. Debris-covered ice amounting to one third of its surface area was not included. In addition, the elevation up the glacier over which volumetric change was measured varied with each pair of years, ranging from as low as 2270 metres to as high as 2360 metres. The result of this is that the total volumetric change figures published do not measure the same thing from one pair of measurements to the next.

The results of these calculations were published in Reid and Charbonneau (1981); a summary of this information appears in Table 2.1. Table 2.1 is a modified version of Table 13 in Reid and Charbonneau (1981). In it, minor summation errors made in that Inland Waters report are corrected. In addition, both the sum total volumetric change from 1959 to 1979 and the elevation up the glacier over which volumetric change was measured in each interval is shown.

Table 2.1: Athabasca Glacier-- Summary of volumetric change in the period 1959-1979 reported in Glacier Surveys in Alberta

Interval	Total volumetric change (m^3) (,000)	Volumetric change per year ($m^3 \cdot yr^{-1}$) (,000)	Average elevation change ($m \cdot yr^{-1}$)	Elevation to which measurements made (m)
1959-62	-1,781	-594	-0.23	2347
1962-65	-3,107	-1,036	-0.45	2286
1965-67	+3,141	+1,570	+0.69	2286
1967-69	+667	+333	+0.15	2286
1969-71	-16,760	-8,382	-3.86	2270
1971-73	+6,186	+3,093	+1.25	2350
1973-75	-6,129	-3,065	-1.22	2360
1975-77	-5,946	-2,973	-1.20	2360
1977-79	-2,791	-1,396	-0.55	2360
1959-79	-26,520	-1,326	-	-

- modified from Reid and Charbonneau, 1981

2.2.2.2 Measurement of Elevation Change

Without computers capable of producing and managing significant amounts of data, it is also difficult to measure elevation change from maps. It is a highly labour-intensive process in which subjective judgement is unavoidable.

The following method has been used to calculate elevation change manually in many studies. The maps between which elevation change is to be measured are prepared by superimposing on them uniformly spaced rectangular grids which cover the portion of the glacier to be measured. The grids are placed such that the intersections of grid lines (the nodes) occupy identical positions on both maps. This task is made more difficult since maps made at different times are often made with different scales (Davey in Brandenburger and Bull, 1966; Young *et al.*, 1978; Jianming, 1984; Haakensen, 1988).

Once this is done, the elevation at each grid node is subjectively interpolated from surrounding contour lines and nodes of identical location on different maps are compared. The difference in elevation between nodes of identical location on different maps is the amount of elevation change at that node.

The problems of measuring elevation change this way are substantial and stem largely from the necessary close participation of the human operator in the process of calculation and measurement. The close participation of the operator limits the calculation of elevation change in two primary ways. The need to estimate elevations subjectively at grid nodes by eye obviously introduces substantial error. Secondly, since the grid is measured manually, it is necessarily coarse. If the grid is fine, then the number of nodes to be checked would become impossibly large for a human operator to calculate in any reasonable length of time. Thus, finely detailed variations on a surface would not necessarily be discovered by this method due to the coarseness of the measure.

Despite its weaknesses, this method has been used in glacier studies before, most notably to measure volumetric change as derived from elevation change (Davey in Brandenburger and Bull, 1966); to compare elevation change with hydrological outputs (Haakensen, 1988); to measure short term surface variation (Jianming, 1984); and to compare maps made of the same feature at the same time using different methods (Young *et al.*, 1978)

Two examples exist of previous elevation change calculation that were made on the Athabasca Glacier. A series of elevation change calculations was made in the Inland Waters report series for 10 m (after 1969) or 25-foot (before 1969) elevation zones. In addition, elevation differences recorded between three measurements made of the glacier in 1977 were studied in Young *et al.* (1978). The results of these studies are detailed in the following subsections.

2.2.2.2.1 The Inland Waters calculation of elevation change

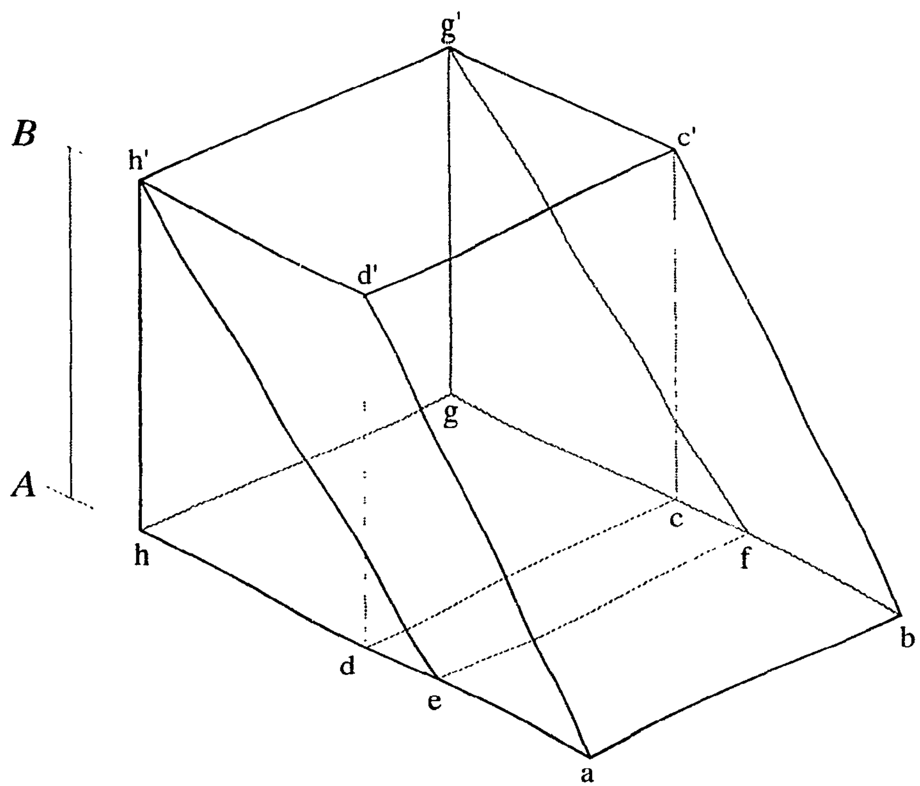
The Inland Waters reports calculated volumetric change and area between pairs of identical contour lines on two maps. The mean average elevation change in an elevation zone was calculated from these values:

$$\Delta h = \frac{\Delta V}{mean\ area}$$

This calculation produces values of elevation change that only approximate empirically derived results. The difficulty is that these calculations attempt to produce a vertical measure from horizontal data. Figure 2.4 demonstrates this. Volumetric change is measured in a horizontal elevation zone at right angles to elevation change. The calculation of elevation change assumes that the surfaces from different years that pass through the elevation zone being measured have a relatively constant slope. Marked changes in the slope of either surface immediately below the elevation zone being measured will render the horizontally- based elevation change measures invalid. As a result of these factors, elevation change as presented in the Inland Waters reports must be looked upon as being no more than an approximation.

2.2.2.2.2 Elevation difference comparison between 1977 measurements

In 1977, the Athabasca Glacier was mapped using three methods. On August 13,



In elevation zone A-B two surfaces

abc'd' and ***efg'h'***

have planar area ***abcd*** and ***efgh*** and volumetric change in that elevation zone of $\boldsymbol{dV_{AB}}$ determined by the difference in the volumes of the polyhedrons.

Elevation change (Δh) is calculated in Glacier Surveys in Alberta to be

$$\Delta h = \frac{\text{Volumetric change}}{\text{mean area}}$$

$$= \frac{\boldsymbol{dV_{AB}}}{(\boldsymbol{abcd} + \boldsymbol{efgh})/2}$$

$$= \frac{2(\boldsymbol{dV_{AB}})}{(\boldsymbol{abcd} + \boldsymbol{efgh})}$$

Figure 2.4: Calculation of elevation change in Glacier Surveys in Alberta

terrestrial photography was taken for the Inland Water series of 1:10,000 maps. On August 16, aerial photos were flown. The aerial photography became the basis for two methods of measurement: a map having a scale of 1:50,000, produced using standard photogrammetric methods, and an orthophotomap used to produce a DEM with a grid-spacing of roughly nine metres.

Young *et al.* (1978) placed a grid with a 100 metre interval over the entire glacier on the two conventional maps and estimated elevations at the resulting grid nodes as discussed in section 2.2.2.2 above. Elevation data for corresponding points from the orthophoto-based DEM was also produced.

When elevations at all the corresponding points were compared, it was seen that the mean average differences between maps were greatest in snow-covered areas and in areas of high slope angle. The authors stated that none of the three methods of representing the surface was obviously superior to the others in terms of product quality. In terms of cost of production, however, the orthophotomap was substantially less expensive and faster to make than the other two.

2.2.3 Digital Elevation Models and the Measurement of Change

In the previous section it was noted that the main obstacle to the measurement of change was that the data representing the glacier surface, contour maps, is in a format that does not lend itself readily to the calculation of volumetric and elevation change (Burrough, 1986). Elevation and volumetric change are much more readily calculated using digital elevation models (DEMs).

A DEM is a representation of spatial data generated by a specialized program which is capable of storing, processing and displaying spatially-related information

(Rentsch *et al.*, 1990). A DEM represents continuous variation of elevation or some other spatial variable over space digitally (Burrough, 1986). Data for DEMs can be in any chosen coordinate system: Latitude and Longitude, Universal Transverse Mercator or Cartesian (x,y,z). Data for DEMs can be acquired in three ways: digitizing existing contour maps; field surveys using tacheometry; or by photogrammetric measurement (Ebner, 1987).

There are two different classes of data structure for DEMs: Triangular Irregular Network (TIN) models and raster (grid square) models (Ebner, 1987). In TIN models, the measured points represent the nodes of the network. The nodes are connected with a series of straight lines. The spaces between the nodes are represented as a series of triangular planes whose edges are the lines connecting the nodes. Elevation values for spaces between the nodes are represented by solving for z the equation of the plane defined by the three surrounding nodal points.

Raster models are built of meshes which form a square grid in the x-y plane. Nodes are the intersections of the grid lines. Their values are derived by interpolation from surrounding reference points (Burrough, 1986). Raster models are more commonly used because of their regular data structure (Ebner, 1987).

With terrain data represented in digital format, elevation and volumetric change is more readily calculated. Raster-based DEMs calculate elevation change using the method described in section 2.2.2.2.2: placing a regular mesh over different representations of the same area and calculating elevation differences at the corresponding nodes. With DEMs however, both interpolation and measurement are made by the computer package, thus rendering visual estimation unnecessary and removing the need for a coarse grid structure which were the main drawbacks associated with the manual calculation of elevation change.

The calculation of volumetric change is also greatly improved by the use of DEMs. DEMs allow generated surfaces representing the same area at different times to be compared directly, rather than using indirect measures such as the Haumann method. Reinhardt and Rentsch (1988) discussed the relative utility of DEMs and contour models to measure volumetric change. They concluded that DEMs are much more efficient timewise than contour models and produce results of similar accuracy.

Chapter three describes the methods used to calculate elevation and volume change using the DEMs generated from the data used.

2.2.3.1 Use in Previous Studies

DEMs can be used in many ways to analyze and generate information from base data. Some of these uses are unique to the study of glaciers; others are common to all uses of DEMs.

DEMs are primarily methods of storing, displaying and processing continuously varying spatial data (Rentsch *et al.*, 1990). The ability to store and process data for an area leads to other, more complex uses. DEMs are able to display the data stored by way of contour maps with optional height intervals. With two or more DEMs of the same location, the magnitude of volume and elevation change between the DEM surfaces can be calculated in total and annual terms. If elevation change is heterogenous over the study area, then the variation in magnitude of elevation change over space can be calculated and displayed as a surface of change, rather than a surface of elevation (Ebner, 1987; Rentsch *et al.*, 1990). Brugman and Demuth (in preparation) include examples of these surfaces of change generated from comparison of survey data and remote sensing information for recent changes of the Athabasca Glacier. Chapter four includes figures and tables which illustrate these uses for the data of this thesis.

Various other calculations can be made and displayed using DEMs. These include: terrain profiles for the production of orthophotos; perspective views and visibility maps; slope and aspect information; ice movement vectors; and statistical analysis and comparison between several DEMs (Rentsch *et al.*, 1990).

CHAPTER THREE: DATA SOURCES AND ASSESSMENT

3.0 Introduction

This chapter is divided into three parts. The first discusses the paper maps that are the sources of data for this study of change. The second part discusses the computer package which was used to generate Digital Elevation Models (DEMs). The third section deals with converting the paper maps into DEMs, and how the DEMs so generated are compared to produce quantitative surface and volumetric change figures. The final section summarizes the magnitudes of uncertainty associated with the various measurements and assumptions discussed in this chapter.

Before the original paper maps are used, they must be evaluated through a consideration of their scales, methods of production and levels of accuracy. The difficulties involved in reducing all of the data sources into an identical referencing system are enumerated and methods used to extract digital information from paper maps are discussed.

The computer package *Surfer* was used to generate DEMs from the digital information generated. The way *Surfer* converts raw data into a surface will be explained. A description of how surface and volumetric change are calculated with *Surfer* will conclude the chapter.

3.1 Data Sources

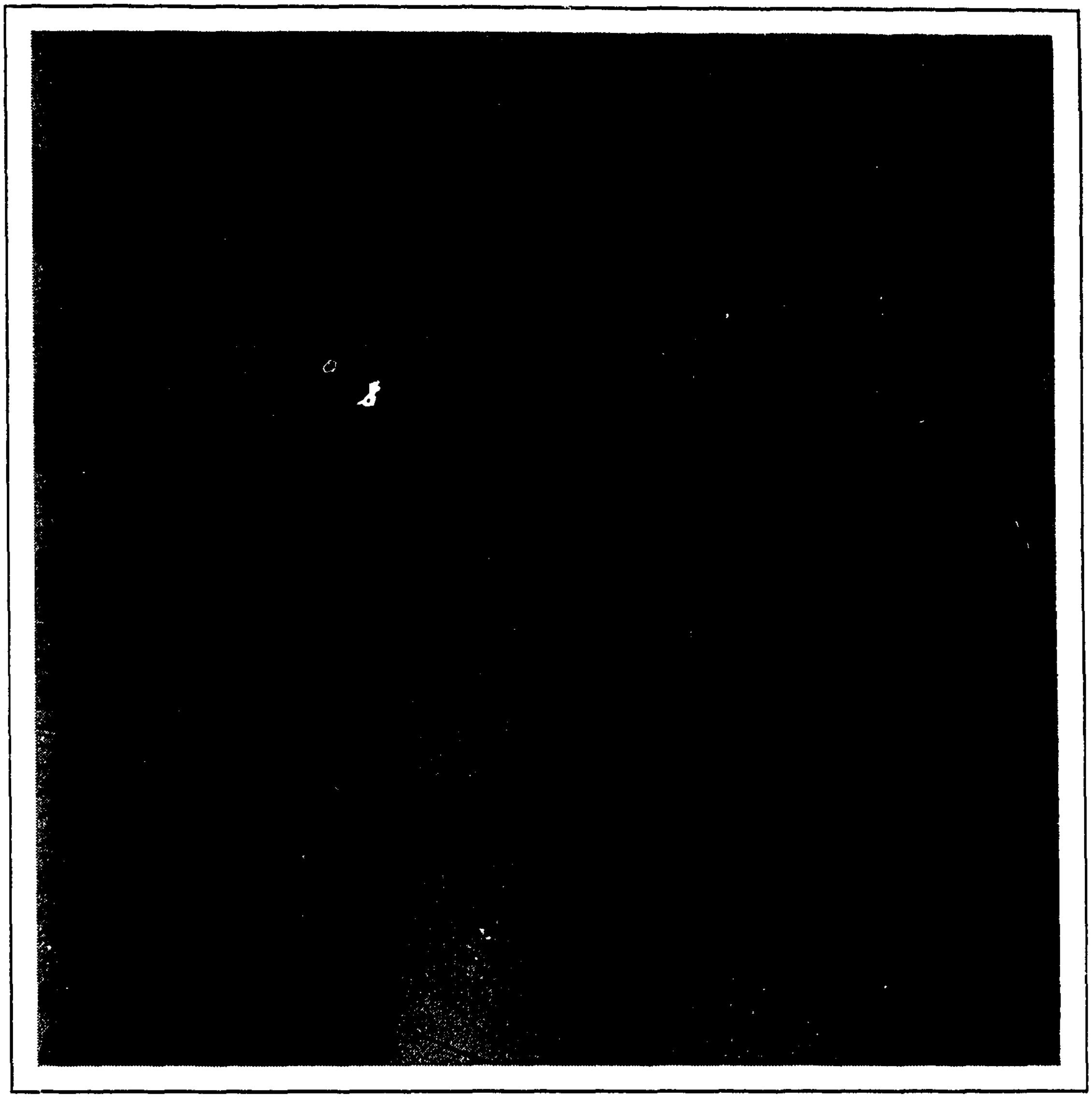
Using DEMs to calculate and show how the glacier has changed over time requires accurate, quantitative information of the glacier at different points in time. For the Athabasca Glacier, this information is stored in the form of contour maps which have been made over the years. These maps were produced using various methods, scales, contour intervals and measurement systems. Table 3.1 summarizes the relevant information concerning the maps covering the glacier which were acquired and used for this study.

Table 3.1: Summary of Data Sources

Year	Producing Body	Method of Production	Scale		Contour Interval
			Photo	Map	
1919	Alberta-British Columbia Boundary Commission	Terrestrial Non-stereoscopic photogrammetry	-	1:62,500	100 feet
1955	National Topographic Service	Aerial photogrammetry	1:10,700	1:50,000	100 feet
1959	Water Resources	Aerial photogrammetry	1:4,100	1:4,800	10 feet
1965-79	Inland Waters	Terrestrial photogrammetry	-	1:10,000	1965, 1967: 25 feet. 1969-1979: 5 metres
1977	Parks Canada	Aerial photogrammetry	1:10,000	1:50,000	20 metres
1979	National Topographic Service	Aerial photogrammetry	1:2,700	1:50,000	100 feet

See figures 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 and appendix 6.

Glaciers are constantly changing. Outside the tropics and monsoon areas, they lose mass in the summer and gain mass in the winter. The amount of change in a year is the mass balance of the glacier. Simply stated, mass balance is the sum of the winter balance and the summer balance, or the sum of total accumulation and total ablation. The summer balance, dominated by melt, is usually negative. The winter balance,



Scale
0 1 2 km

Figure 3.1: Section of Boundary Commission map (1919) showing the Athabasca Glacier. Scale of original 1:62,500.

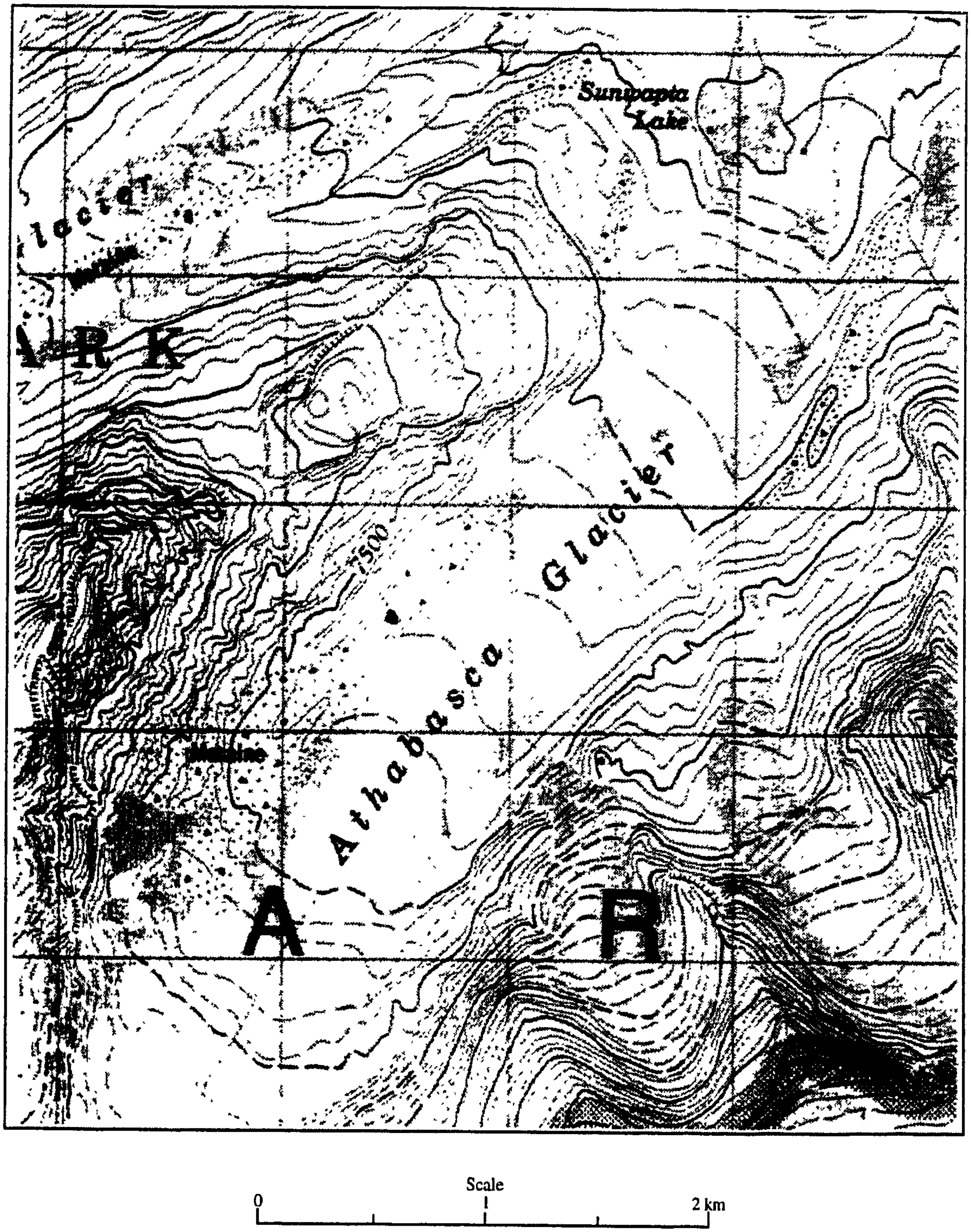


Figure 3.2: National Topographic Map Sheet 83C/3 edition 1 (1955). Contour interval 100 feet. Only visible ice included as part of glacier. Scale of original 1:50,000.

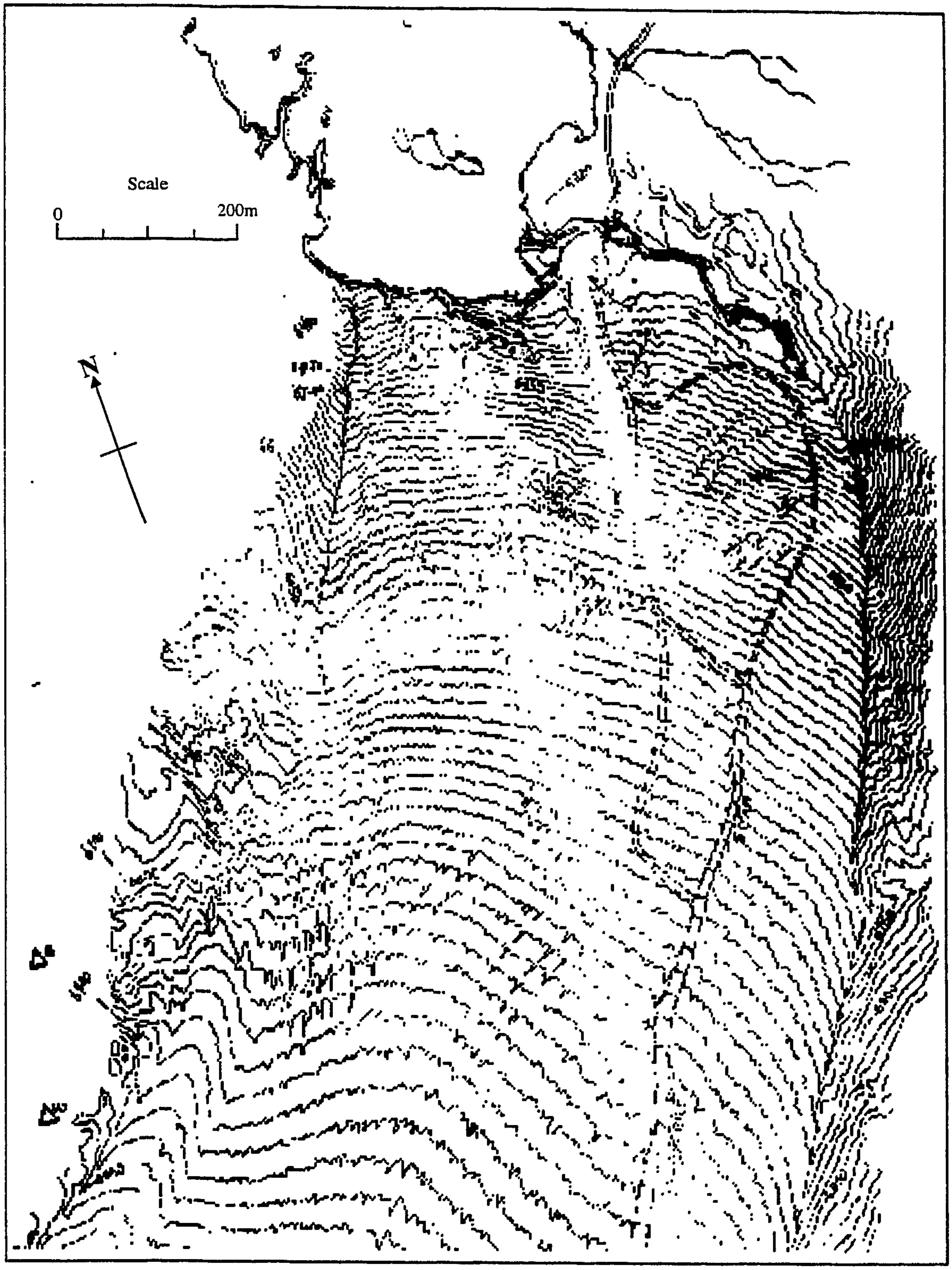
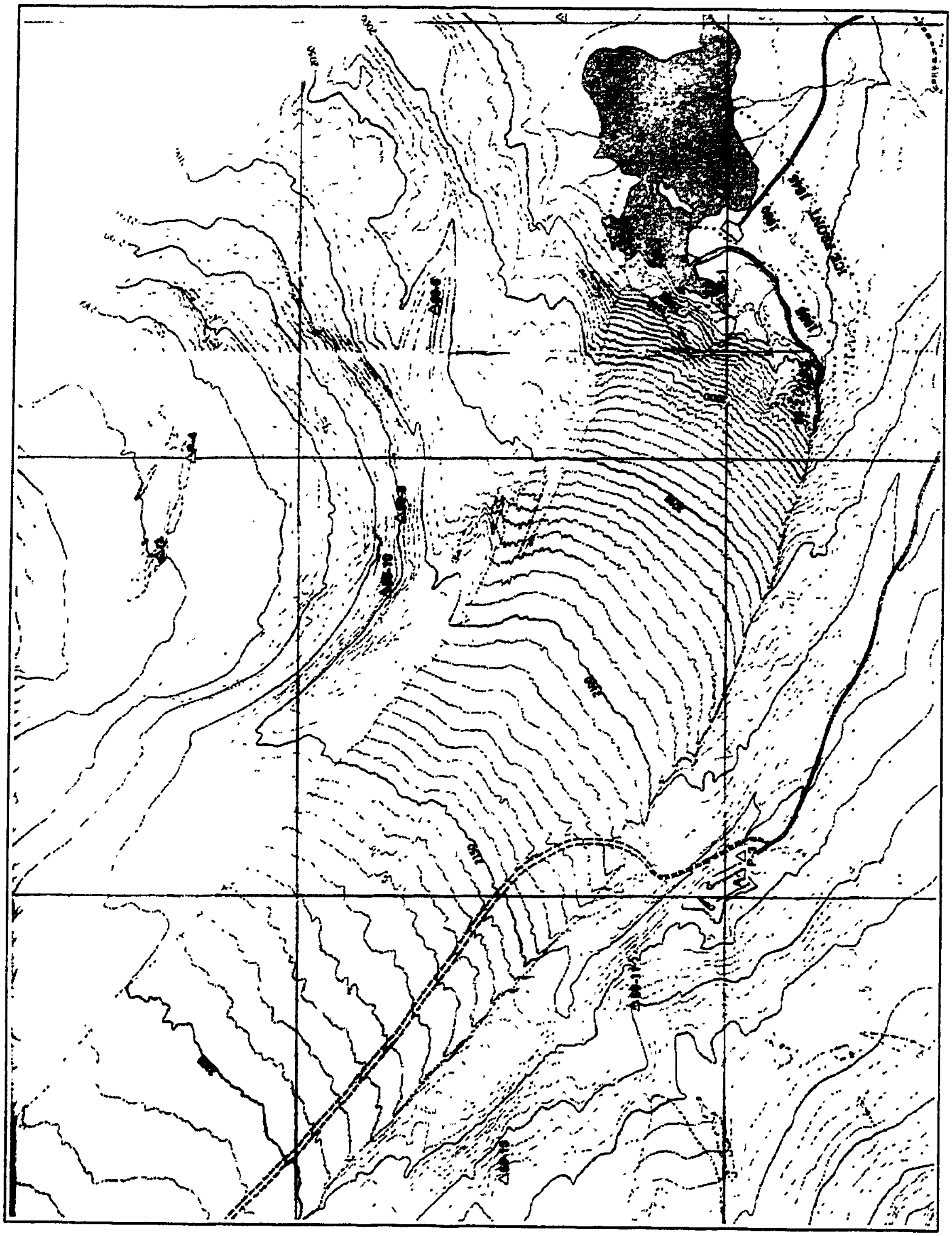


Figure 3.3: Water Resources map (1959). Contour interval 10 feet. Only clear ice included as glacierized; debris-covered ice at terminus not contoured. Scale of original 1:4800.



Scale

0 200 400 600 800 1000m

Figure 3.4: Inland Waters map (1971). Original scale 1:10,000. Scale of this copy 1:14,000. Contour interval 5 metres on ice. Glacier defined as including clear ice only.

Figure 3.5: Parks Canada Map (1977). Contour interval 20 metres on ice. Debris-covered ice and lateral moraines are included as glacierized. Scale of original map 1:50,000.

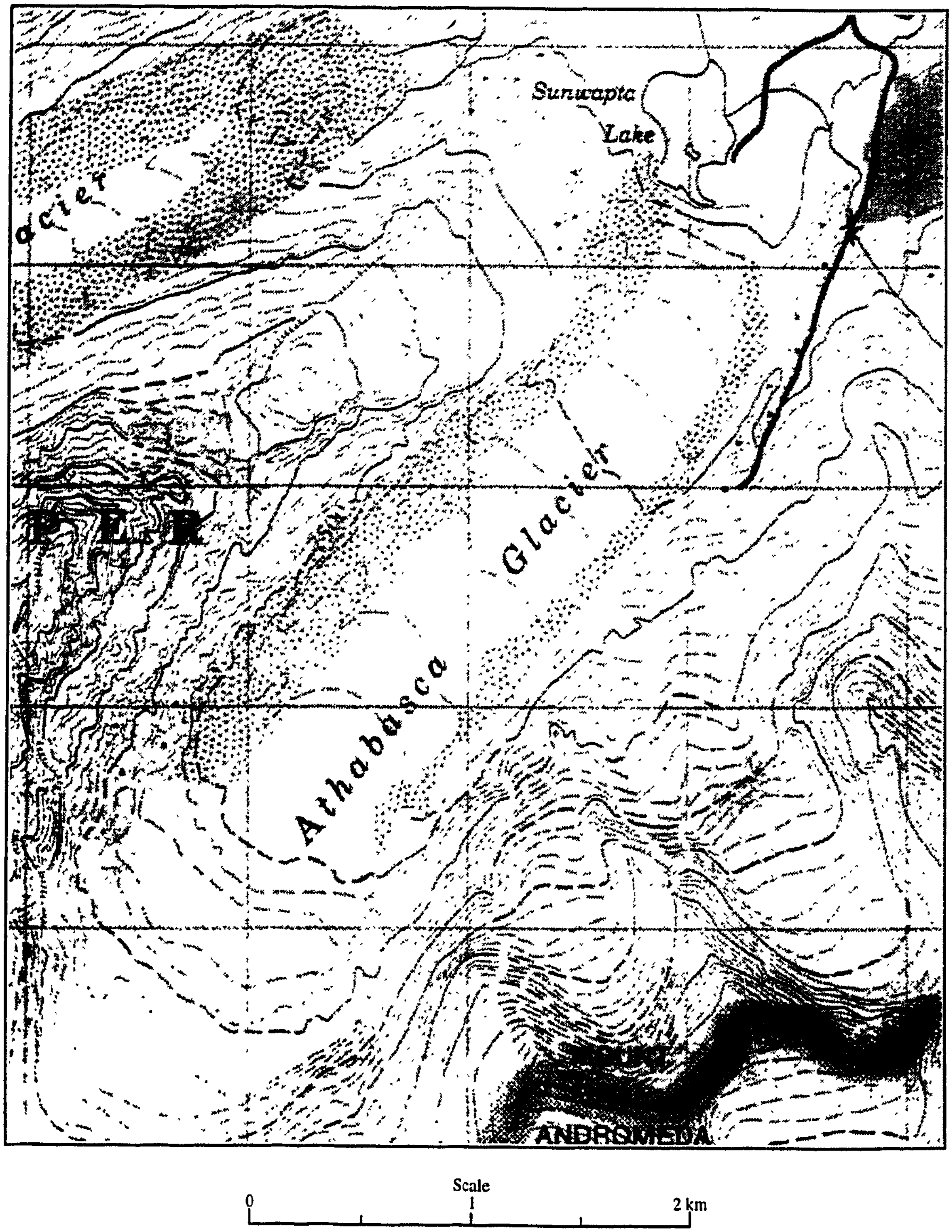


Figure 3.6: National Topographic Map Sheet 83C/3 edition 2 (1979). Contour interval 100 feet. Debris-covered ice and lateral moraines included as glacierized. Scale of original 1:50,000.

dominated by snowfall, is usually positive (Østrem and Brugman, 1991). To quantify the amount of volumetric change in a year would require a data series consisting of information acquired immediately prior to the end of either the summer or the winter season. At the Athabasca Glacier, this usually occurs in September or May, respectively.

This thesis does not measure mass balance on a yearly basis since information was not collected on a yearly basis. In addition, the information that was collected was not acquired at the end of the ablation season, but rather part way through it. The interval between successive measurements made in different years contains both accumulation and ablation seasons. Thus the values of surface and volumetric change derived in this thesis do not reflect changes between one hydrological year and the next. Change is measured between one map and the next. Table 3.2 shows the dates of measurement of each map.

Table 3.2: Date of Photography of Source Maps

Producing Body	Year	Date
Boundary Commission	1919	August 1, 9
National Topographic Service	1955	August 28
Water Resources	1959	August 1
Inland Waters	1965	July 26
Inland Waters	1967	July 25
Inland Waters	1969	July 25
Inland Waters	1971	August 12, 16
Inland Waters	1973	August 3
Inland Waters	1975	August 27
Inland Waters	1977	August 13
Parks Canada	1977	August 16
National Topographic Service	1979	August 7
Inland Waters	1979	August 10

3.1.1 Definition of Borders

In addition to their differences of scale, date, body of creation and method of display, some of these maps have different definitions of the edge of the glacier. Clearly, the area that the glacier encompasses is important in determining the magnitude of change. Different definitions of the border of the glacier will lead to different values of change. It is often difficult to distinguish the edge of a glacier using photography alone. Many subjective judgements must be made between clear-ice portions of glaciers and various forms of glacier-cored debris (Young *et al.*, 1978). It is still more difficult to distinguish the edge of the glacier from maps alone. Being several steps removed from reality, they have only as much information as the cartographer sees fit to place on them and often lack a standard nomenclature and symbolism (Henoch, 1969; Sebert, 1969). When glacier borders are determined from maps, aerial photography from the nearest year to mapping can assist in the verification of the border, but some imprecision of placement is to be expected.

The border of the glacier marked on all of the maps except Parks Canada 1977 and National Topographic Service 1979 is the edge of the clear ice. This definition has the advantage of often being simple to repeat and of producing a clear, obvious border. Unfortunately, this definition of the glacier is not complete. When photographs (both aerial and terrestrial) of the glacier are studied, it becomes evident that the western portion of the glacier is covered with debris, deposited from where part of the glacier flows over an icefall as it leaves the Columbia Icefield.

Aerial photography of the glacier from 1959 was studied and compared with the map of the glacier prepared from those photos and with later maps to determine the extent of buried ice. These photos showed that there was no significant layer of flowing debris-covered ice on the eastern portion of the glacier. For the western portion, photographic interpretation indicated that the edge of the clear ice was located on the line connecting the v-notches of the contours on the map after the curving portion of the contour lines that represents the surface of the clear ice. Immediately to the west of the clear ice surface the surface is debris-covered. Crevasses are visible on this portion of the terrain on the 1959 photography, indicating that there is flowing ice beneath the debris. The border between the flowing debris-covered ice and the stagnant ice in the lateral moraines is located on the next major sequence of v-notches west of that. Figure

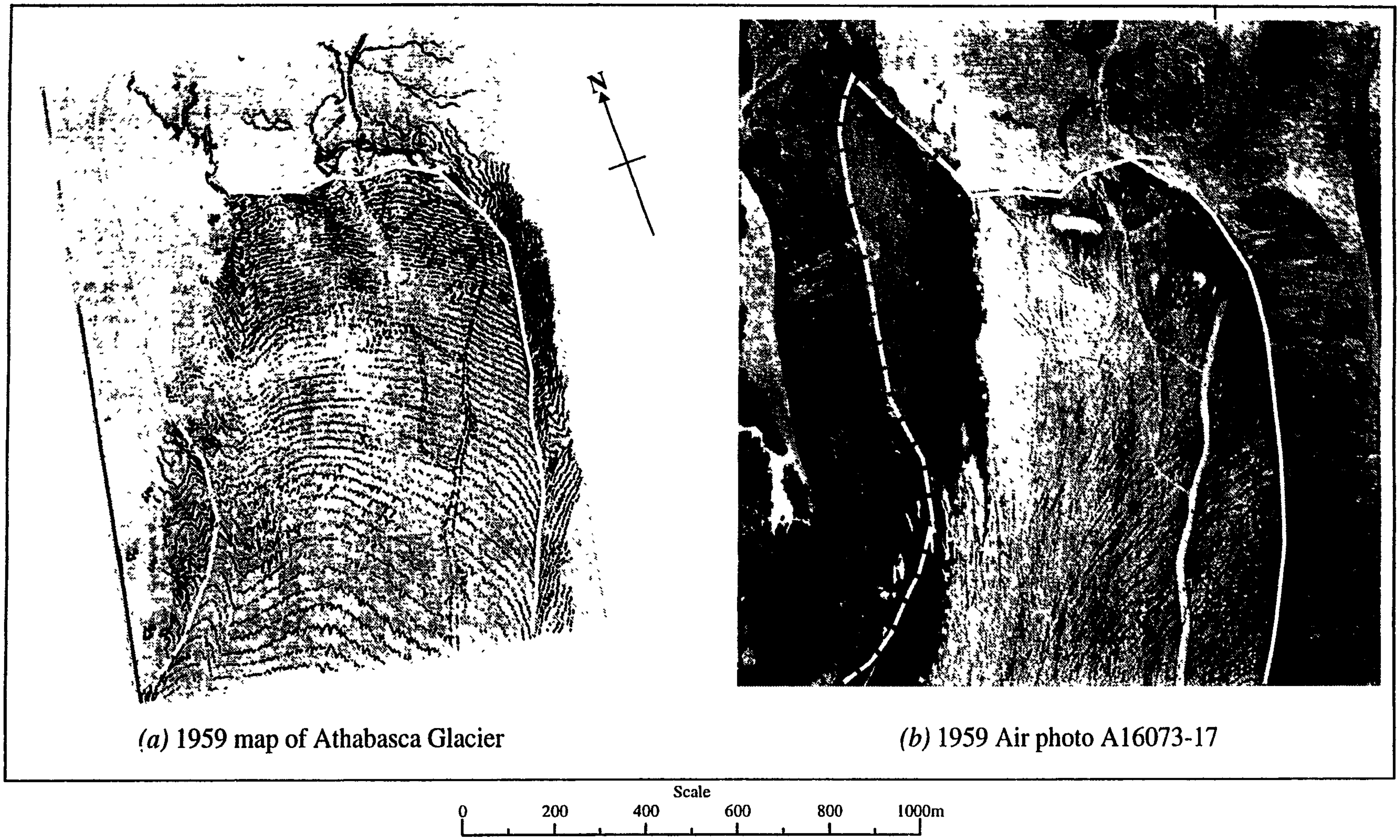


Figure 3.7: Definition of borders. Map and photo at scale c.1:13,300. Solid line indicates border shared by both border definitions. Dotted line indicates border of clear ice. Dashed line shows border of moraine-covered ice. Moraine-covered area only partially contoured.

3.7 partially illustrates these borders. Figure 3.7*a* shows a portion of the 1:4,800 map produced in 1959. Figure 3.7*b* is one of the air photos it was produced from. It can be seen that much of the terminus of the debris-covered ice is not contoured on this map. However, where the contouring begins, the relative placement of the borders and the associated v-notches in contour lines can be seen. On all of the other maps except for the 1919 map, these notches were well-defined but not otherwise marked. On the 1919 map, they were at best indistinct.

Previous volumetric change work was done using the large-scale maps which defined the glacier as consisting of clear ice only. To compare this work with values calculated from DEMs, the same definition of the glacier must be used. However, to produce values which are representative of the true volumetric change of the ablation zone, the sections of debris covered ice must be included.

To allow for both comparison with previous work and calculation of changes which include all of the ablation area of the glacier, two definitions of the glacier border were used. To compare results produced using DEMs with previous volumetric change work, one border was defined as being the edge of the visible ice. The other border was defined as including both clear ice and flowing debris-covered ice but excluding lateral moraines. The glacier-proximal portions of the lateral moraines are ice-cored, but the ice in them is stationary and is not included in this study. Due to glacier fluctuation, the border of the glacier was defined separately for each year and for both definitions.

3.1.2 Accuracy of Vertical Estimation

Contour maps of the Athabasca Glacier were the primary sources of information for this thesis. Several different methods were used to prepare the different maps:

terrestrial photogrammetry, aerial photogrammetry, and terrestrial non-parallel photogrammetry as discussed in chapter two. The precision of these is discussed below. In addition, the various assumptions made about the maps are enumerated as follows:

1. Different map series could be compared meaningfully when problems with georeferencing were resolved.

2. Differences in borders due to varying interpretations of ice-edge positions on different maps did not significantly reduce the ability to compare different intervals.

3. The contours on the 1919 map are of similar accuracy over the entire area of the glacier.

4. Converting imperial to metric measurements did not appreciably reduce precision of measurement.

5. The only significant precision problems stem from difficulties in estimating elevation:

    a. There was no systematic difference between precision of measurement of clear ice, debris-covered ice and bedrock. The increased imprecision associated with snow-covered regions could be ignored due to their severely limited extent.

    b. The accuracy of vertical estimation was very largely attributable to the distance between the photogrammetric station and the object being measured.

    c. Errors in vertical position due to horizontal displacement were very small due to the low slope of the surface of the glacier.

Some of these assumptions are clearly more sound than others. The assumptions concerning imperial to metric conversion, georeferencing and borders are reasonable. Of the vertical precision assumptions, 5(c) is most open to question since this assumption does not hold in the steep terminus and icefall regions. Of the assumptions which were made, the most unreliable is that concerning the 1919 map. Problems with this map will be discussed in more detail in section 3.1.2.1.

Once these assumptions were made, the task of quantifying the magnitude of uncertainty that the precision of vertical estimation produces over the surface of the glacier may proceed. These values must be calculated for each of the methods used to produce the maps.

3.1.2.1 Terrestrial Non-Stereoscopic Photogrammetry

A 1:62,500 map of the Athabasca Glacier made from information collected in 1919 was published in Cautley and Wheeler, 1924*b* (figure 3.1). The map was prepared using photogrammetric methods from photographs taken from the summits of Mt. Wilcox and Nigel Peak.

Cautley *et al.* (1924) did not state the level of accuracy of vertical estimation that their non-stereoscopic photogrammetry entailed. As a result, this figure had to be estimated using certain assumptions. The Boundary Commission report stated that the camera used had a fixed focus and a wide angle lens which covered 52° of arc. The width of the photographic plate used was 6.5 inches (165.1 mm).

Mapping was done using photographic enlargements having a width of 13 inches (330.2 mm). From simple trigonometry, the effective focal length of the enlargement is 338.5 mm. If it is assumed that measurements on the enlarged photos were made to an accuracy of 0.5 mm with the equipment of the day, this means that measurements could be made to an accuracy of 0.0846° of arc.

Another factor influencing the precision of vertical estimation stems from the fact that a large portion of the surface of the ablation area of the glacier is invisible from the Mt. Wilcox photographic station, obscured by a rocky rise between the ablation areas of the Athabasca and Dome Glaciers. The contour lines in the obscured portion of the map must therefore have been placed by estimation. It is impossible to quantify the amount of error that this adds to the measurement. For purposes of calculating the error in volume and elevation change, it was assumed that the trigonometric estimate of precision listed above could be applied over the entire study area. It should be recognized that in the occluded area, this is a very optimistic assumption.

When these assumptions are made, the magnitude of uncertainty of vertical estimation can be quantified. The distance from Nigel Peak (the more distant of the photo stations) to the 1919 terminus was 4400 m. The distance to the upper limit of the study area was approximately 9400 m. Thus, the precision of vertical estimation is $\approx$6.5 metres at the terminus and $\approx$14 metres at the upper icefall. This is a substantial margin of error, larger than that for any other map. It is necessary to accept this high margin of error in order to extend the record of volume and surface change to include the earliest data. A planimetric representation of the variation in the precision of vertical estimation of the 1919 map is shown in figure 3.8*a*.

3.1.2.2 Terrestrial Stereoscopic Photogrammetry

Eight maps at a scale of 1:10,000 were made using terrestrial photogrammetry in odd-numbered years between 1965 and 1979. Making these maps required

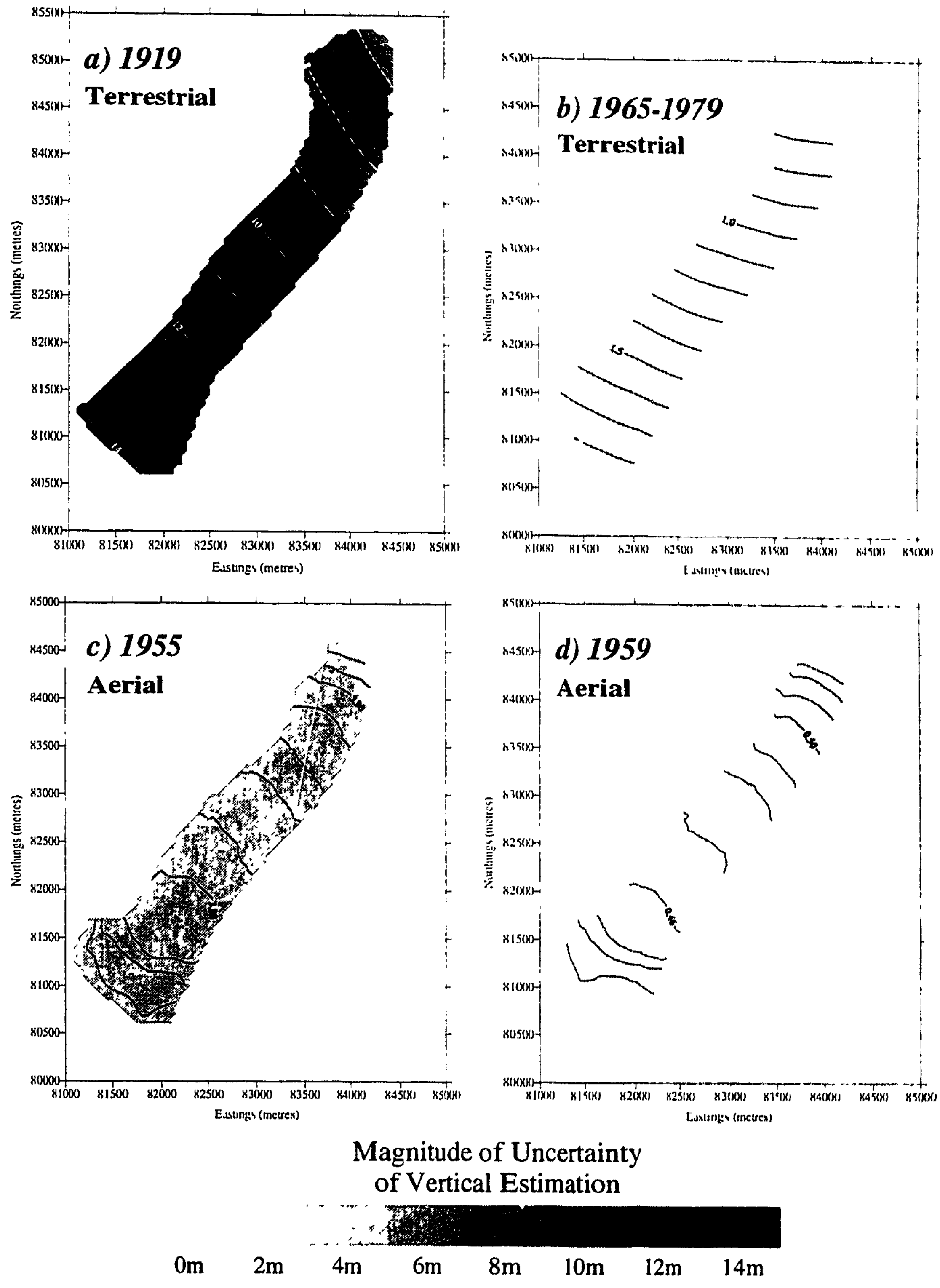


Figure 3.8: Planimetric representation of uncertainty of vertical estimation of base maps. Maps (a) and (b) demonstrate the imprecision associated with terrestrial photogrammetry, while (c) and (d) show the imprecision of aerial photogrammetry.

photogrammetric stations overlooking the glacier. For each of these maps, identical photogrammetric stations were used, located on Wilcox Ridge, some distance from and overlooking the terminus of the glacier.

It was stated in Young *et al.* (1978) that readings in the vertical plane from these photos can be made to a fineness of 1 minute of arc. Given the distance to the point of photograp ıv and the angular precision of measurement, the precision of vertical estimation that could be achieved is defined as the product of the tangent of this angle and the distance to the farther of the photo stations used:

$$Z_n = D \cdot \tan\alpha$$

where Z_n = uncertainty factor in vertical estimation
D = distance to the photogrammetric station
α = precision of angular measurement

As the distance from the photogrammetric stations increases, the uncertainty of vertical estimation also increases. In the Inland Waters surveys at the terminus of the glacier, the uncertainty factor is ≈ 0.5 metres. At the 2400 metre contour mark, more than 7 kilometres from the stations, the uncertainty factor is ≈ 2.1 metres. This is shown in figure 3.8*b*.

3.1.2.3 Aerial Photogrammetry

Four maps have been made of the Athabasca Glacier using aerial photogrammetry. Three of these maps were at a scale of 1:50,000. The remaining map was at a substantially larger scale and was produced from low-elevation photography.

Estimates of height error must be produced for these maps as well. In the case of aerial photogrammetry, standard height error is a function of the height above ground that the photo was taken, the focal length of the camera, the parallax angle of the photography, the slope of the terrain and other features (Ghosh, 1988). As stated before, it has been assumed that the only significant errors are errors of vertical estimation. Several studies have been undertaken to determine the standard height error of aerial photogrammetry, both by empirical and theoretical methods (Blachut, 1961; Paterson, 1966; Young and Arnold, 1977; Ghosh, 1988).

The uncertainty of vertical estimation for the 1959 map was substantially lower than for the other aerial based maps. This is due to the fact that the 1959 map was the subject of an empirical study (Paterson, 1966). Reid, in 1959, was the first to place ground control points (points of surveyed location and elevation) around the glacier. A total of twenty-one survey plugs were placed on either side of the valley of the glacier and in the glacial forefield. No markers were placed on the glacier itself. These survey plugs were marked and air photography was flown. Photogrammetric work was done on the pictures and a map having a scale of 1:4,800 was produced (Reid, 1961). Appendix 3 lists the major components of that flight of air photography, as well as other air photos which have been taken of the glacier.

The accuracy of contouring on this map was checked independently by Paterson (1966). Paterson placed fifty-nine stakes on the glacier surface and surveyed their elevations with theodolite triangulation from the stations established by Reid. The position of the stakes was surveyed immediately before and directly after the air photography was flown. The position of the stakes at the time of photography was determined by linear interpolation between the two surveyed positional values.

Paterson determined that the root mean square difference between the elevation of each marker and the corresponding location on the maps was 49 cm, or 2.0×10^{-4}

of the height above ground. This is approximately two times the error as determined by theoretical studies (Blachut, 1961; Konecny, 1963; Paterson, 1966). The effect of the accuracy of measurement on the measurement of elevation and volumetric change is discussed in section 3.3.3.6.

The function used to quantify the error of vertical estimation in the other aerial photogrammetry was based on the mean of the range of empirical values reported in Young and Arnold (1977). This was 4.5×10^{-4} of the flying height of the airplane. This value was calculated from studies comparing other glacial maps with surveyed spot heights. It was thought that this value would be more appropriate to use with the untested aerial photogrammetry of the Athabasca Glacier, since no spot heights were surveyed for comparison when those maps were made.

The survey plugs that Reid inserted in 1959 were used as the basis of subsequent surveys of the glacier. These have included the bi-yearly terrestrial photogrammetric surveys of 1965-1979, as well as the surveys of Energy, Mines and Resources in 1980, Trombley in 1986 and Brugman in 1993 (Brugman and Demuth, in preparation).

Figure 3.8*b,c,d* shows the magnitude of uncertainty of vertical estimation for the glacier surface for the stereoscopic photogrammetry. In addition, table 3.3 lists the uncertainty at 1950 metres (close to the terminus of the glacier) and at 2400 metres, the maximum elevation studied, along with functions that describe the magnitude of uncertainty of vertical estimation at each point on the glacier surface. The only exception is the 1979 photography, which recorded only the terminus of the glacier.

Figure 3.8 and table 3.3 show that the calculated uncertainty for the high altitude aerial photogrammetry was large in comparison with the low altitude aerial photogrammetry and the terrestrial photogrammetry of 1965-1979. The uncertainty values for the 1959 photogrammetry, on the other hand, were low due to the low flying height of the airplane. Terrestrial stereophotogrammetry has an uncertainty of vertical estimation that falls between high altitude and low altitude aerial photogrammetry. As noted previously, the uncertainty of vertical estimation on the 1919 map is substantial.

These values of standard error should be recognized as minimum estimates under ideal conditions for precision measurements based on distance. No other source of error is considered in table 3.3. In snow-covered areas or other zones with minimal contrast, the uncertainty factor in the estimation of vertical accuracy will be higher.

Table 3.3: The accuracy of vertical estimation
for different maps or map series

		Standard Height Error (metres)	
Interval	Function	1950 m	2400 m
1919	$Z_n = D \cdot \tan\alpha$	≈6.50	≈14.00
1955	$Z_n = h \cdot 4.5 \times 10^{-4}$	3.94	3.74
1959	$Z_n = h \cdot 2.0 \times 10^{-4}$	0.53	0.43
1965-79	$Z_n = D \cdot \tan\beta$	≈0.50	≈1.80
1977	$Z_n = h \cdot 4.5 \times 10^{-4}$	3.62	3.42
1979	$Z_n = h \cdot 4.5 \times 10^{-4}$	0.34	-

where Z_n = the uncertainty of vertical estimation (metres)
D = distance from terrestrial photogrammetric station
h = height above ground of aerial photogrammetry flight
α = precision of angular measurement of 1919 photogrammetry
β = precision of angular measurement of terrestrial photogrammetry, 1965-1979

3.1.2.4 Imperial/Metric Conversion

Many of the maps of the Athabasca Glacier were produced with contours measured in imperial units. All of the maps of the glacier produced in or before 1967 were contoured in feet.

Clearly, it was necessary to convert the imperial measures to metric to produce results in a common scale. The conversion factor between feet and metres is 0.3048 ft/m. Since there were four significant figures in both the elevation values delimited in feet and the conversion value, four significant figures were retained in each metre elevation value.

3.1.3 Problems of Georeferencing

Another impediment to the direct comparison of the maps stems from problems associated with their georeferencing. Georeferencing refers to the coordinate system used to locate features on the surface of the earth. Maps can be registered using latitude and longitude or Universal Transverse Mercator (UTM) coordinates. On each map there are printed points which state explicitly their location in UTMs or latitude and longitude. For meaningful measurement of both elevation and volumetric change, points of identical location must be compared between one year and the next. If georeferencing is the same on all maps, then points in the same location can be compared without difficulty. Unfortunately, each of the different series of maps is located differently with respect to the local UTM network. These differences must be accounted for and corrected.

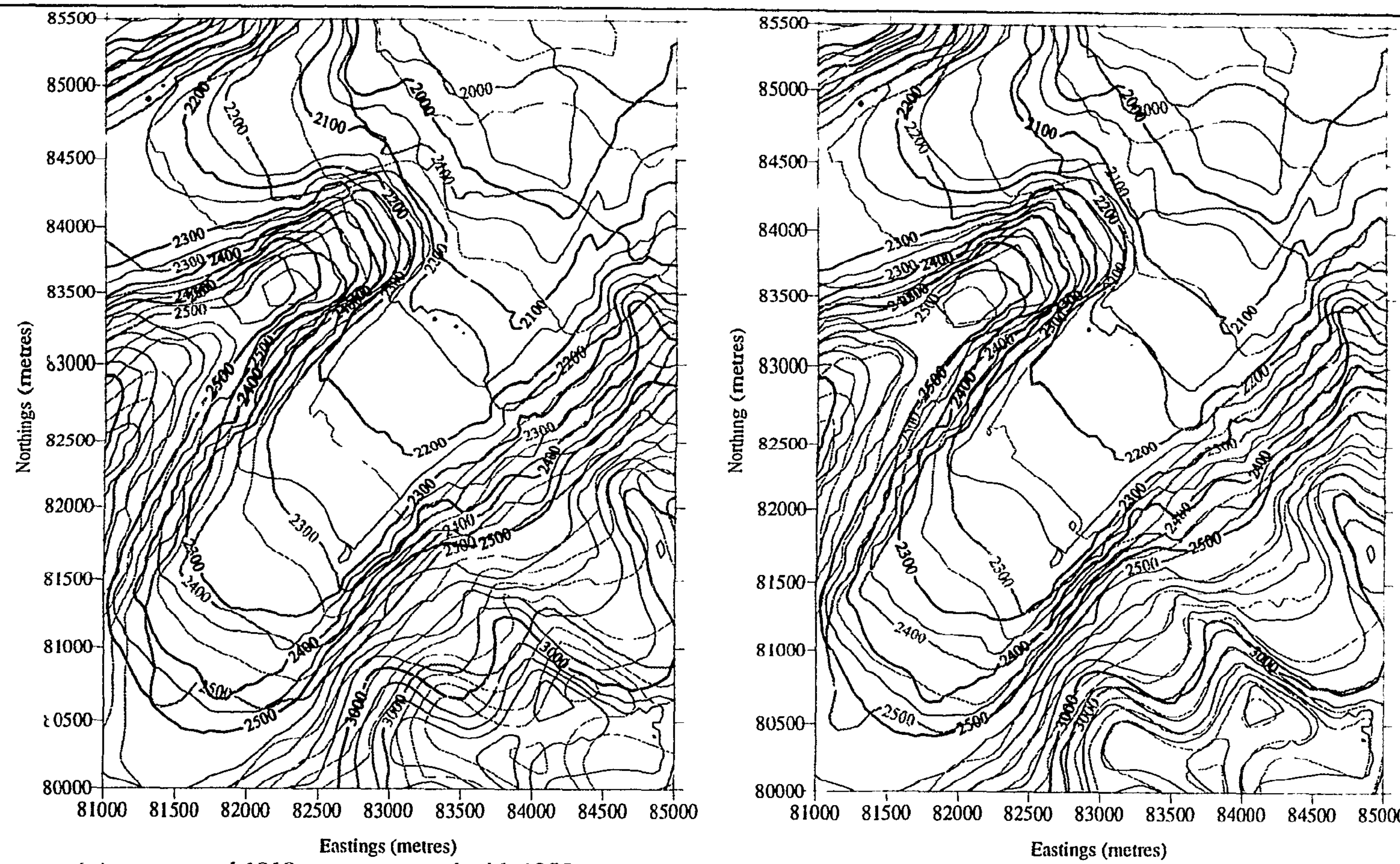
In each case, due to its superior labelling of georeferencing points and since it is referenced in relation to the national georeferencing network, the 1955 map is assumed to be the most positionally accurate of the maps. Consequently, the georeferencing of each of the other maps or map series was compared to it. The positioning of the small-scale maps will be discussed first, followed by a discussion of the georeferencing of the large-scale maps.

3.1.3.1 1919: British Columbia-Alberta Boundary Commission Map

The map produced by the Boundary Commission from 1919 photographs was prepared using terrestrial non-stereoscopic photogrammetry, based on occupied stations whose positions were determined using ground-based surveying techniques (Cautley and Wheeler, 1924*a*). The ground-based surveys were very accurate, with a measured error of one foot over fifty-three miles (Cautley *et al.*, 1924).

The positions of objects measured using Cautley and Wheeler's photogrammetry were not as accurate however. When the 1919 map was compared to the 1955 map, various problems emerged. Close to the photogrammetric stations, the terminus of the glacier and the slopes around it appear to be in the correct location. However, the peaks by the icefalls, well away from the photogrammetric stations, show definite displacement when compared with the same mountains represented on the 1955 map. Figure 3.9*a* illustrates this. Studying this figure, it can be seen that the difficulty appears to be a compression of the 1919 map in the north-south direction.

To resolve the distortion in the 1919 map, a comparison was performed between the 1919 and 1955 maps. Identical points surrounding the glacier which appeared on both maps were noted and their precise positions were obtained using the digitizing program *Tosca* in conjunction with a Summagraphics SummaSketch II digitizing tablet. Table 3.4 lists the comparison points used; figure 3.10 shows the location of these points surrounding the glacier. It can be seen that all of these positions are mountain peaks located at high elevations. It was necessary to estimate the exact position of the summits on 1919 and 1955 maps since not all of the summits were located precisely on those



(a) uncorrected 1919 map compared with 1955 map.

(b) corrected 1919 map compared with 1955 map.

Contour Interval:
50m below 2500m
100m above 2500m

1919 ————
1955 ————

Figure 3.9: 1919-1955 georeferencing comparison.

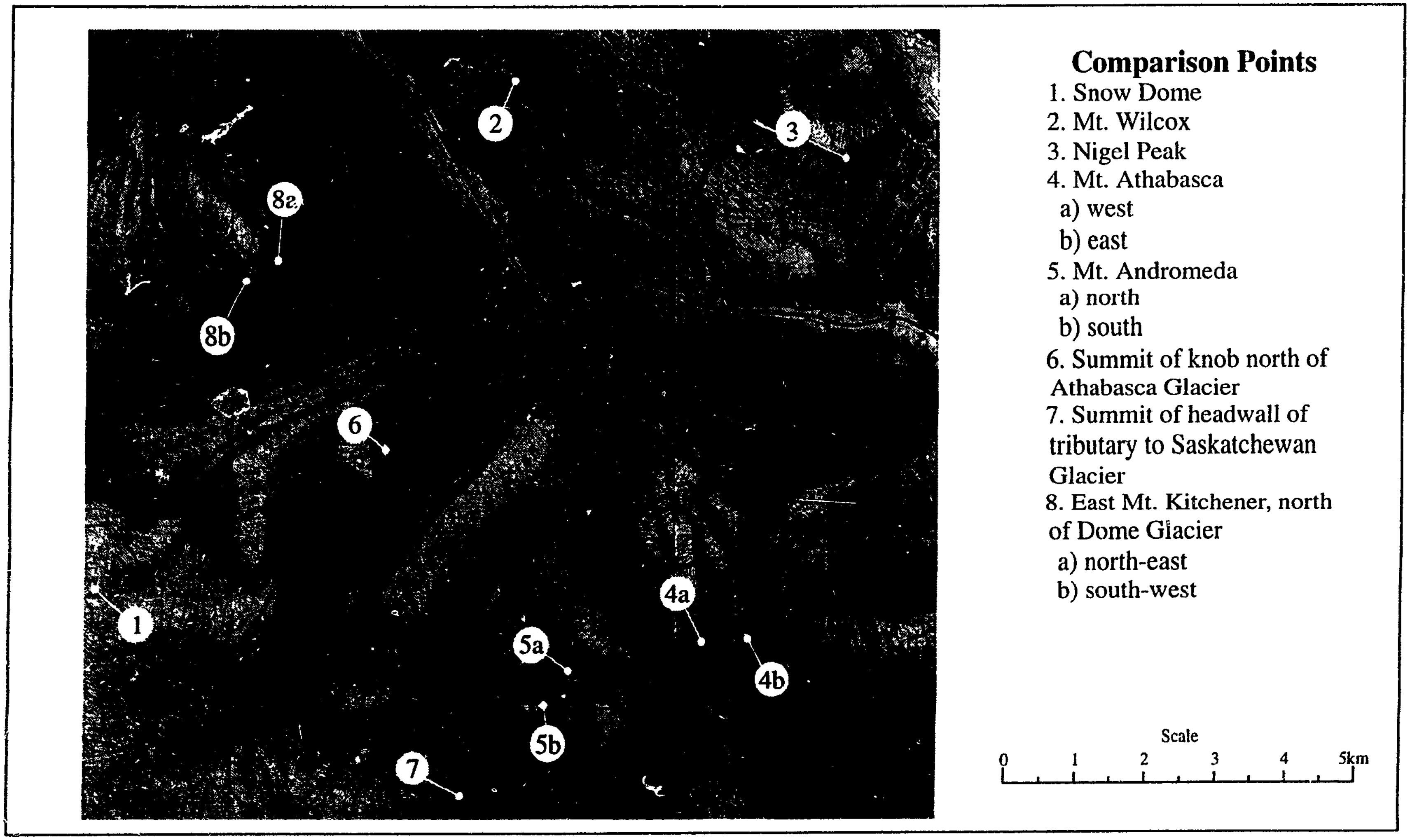


Figure 3.10: Parks Canada 1977 map showing location of comparison points used to bring the georeferencing of the 1919 and 1977 small- scale maps into agreement with the 1955 map. Scale of original 1:50,000. Scale of copy 1:86,500

maps. It was not possible to obtain any points for comparison at valley floors due to the lack of prominent invariant features there that appeared on all maps.

The georeferencing points were first compared for displacement. The position of each point on one map was compared with the corresponding points on the other map. Their differences in position in the x and y directions (east-west and north-south), derived from UTM coordinates, were recorded and the mean displacement was taken. This was found to be (x,y) $(-83.7,\ -88.7)$ metres.

Table 3.4: Comparison points used to georeference the 1919 and 1977 maps

			1919	1955	1977
1.	Snow Dome		x	x	x
2.	Mt. Wilcox		x	x	x
3.	Nigel Peak		x	x	
4a)	Mt. Athabasca	west	x	x	x
b)		east	x	x	x
5a)	Mt. Andromeda	north	x	x	x
b)		south	x	x	x
6.	Summit of knob north of Athabasca Glacier		x	x	x
7.	Summit of headwall tributary to Saskatchewan Glacier		x	x	x
8a)	East Mt. Kitchener, north of Dome Glacier	northeast		x	x
b)		southwest		x	x

When attempting to discern whether compression exists while having a limited number of comparison points, linear regression is a suitable method to use (P. Treitz, personal communication, 1994). The difference in y-values of the matching points recorded in 1919 and 1955 was taken and a regression was done between the resulting differences and the corresponding original 1919 y-values. If there is a systematic compression in the north-south direction, comparing the difference in corresponding y-values with the original 1919 y-values should produce a significant trend.

The points were compared, and a trend emerged. The correlation coefficient (r) between the points and the differences was 0.6976. For nine points, there were seven degrees of freedom. A statistical t-test was done which produced a value of 2.576. This is significant at the 95% confidence level: a definite trend exists. This accounts for approximately half of the variance. Of that portion that was not accounted for, part of it is certainly due to the need to estimate the precise location of the various peaks on the different maps. This t-test was repeated in the east-west direction, and as expected, there was no significant correlation at any level between the 1919 x-values and the difference between corresponding 1919 and 1955 x-values. Visual inspection indicated that it would not be necessary to rotate the 1919 data to match the 1955 data.

Thus, after the 1919 map was digitized, the data file so produced was modified using the values suggested here. Each point was displaced (x,y) $(-83.7, -88.7)$. Following this, y-values were modified using the regression equation produced:

$$y_{new} = 1.01194 y_{old} - 1074.4$$

These modifications were made using a simple QBASIC program.

Figure 3.9*b* illustrates the comparison of contour lines of the modified 1919 data file with the 1955 data file. This is an improvement over the unmodified 1919 data set, shown in figure 3.9*a*. All subsequent DEM generation as well as elevation and volumetric change calculations using 1919 data were made using the modified data set.

3.1.3.2 1977: Parks Canada Map

The Parks Canada map (1977A) was produced from air photos taken only three days after the terrestrial photography for the 1977 1:10,000 Inland Waters map (1977T) was done. Thus, the map prepared for Parks Canada allows the comparison of measurements between a 1:50,000 and a 1:10,000 map.

Although the Parks Canada map is a very handsome one (figures 3.5, 3.10), it proved difficult to georeference. Unlike the other 1:50,000 or 1:10,000 maps, the UTM referencing grid was not printed to overlay the other features on the map. Instead, the UTM coordinates were printed on bars surrounding the map. It was necessary to base georeferencing for digitizing upon the measurement of the positions of the bars with respect to the map. Prior to digitizing, the distance between the bars and the edge of the map was physically measured with a ruler. The obvious drawback of this method is that measurements more precise than the finest gradation on the ruler (1 millimetre) become problematic.

Due to these necessary measurements, the georeferencing system used to digitize this map matches neither Reid's estimate nor the NTS referencing. To bring the georeferencing for this map into agreement with the 1955 map the method used to bring the 1919 and 1955 maps into accord was used: the peaks of mountains surrounding the glacier, which are prominent and assumed to be invariant, were located on both maps and compared. Table 3.4 lists the points used to perform this task.

When this comparison was made, it was seen that there was a slight displacement between the 1955 and 1977 maps, amounting to (x,y) $(-1.4, 22.5)$ metres. The precision of measurement of georeferencing on the 1977 map was one millimetre. At a scale of 1:50,000, this is the equivalent of 50 metres on the ground. The magnitude of the correcting displacement is less than this in the x- and y-directions, suggesting that the measurements made with the ruler were as precise as can be expected.

Tests for linear distortion with respect to the 1955 map were also performed using these comparison points. As expected, no systematic compressions or rarefactions were noted. Thus, after the data from the map was converted to digital format, it was translated (x,y) $(-1.4, 22.5)$ metres to match the 1955 map.

3.1.3.3 1979: NTS 83C/3 edition 2

The second edition NTS map covering the Athabasca Glacier is unsuitable for use in this study. Only the first two contours within the terminus of the glacier (6400 and 6500 feet) were different from the contours on the 1955 map. The remainder are identical in both years. It is likely that only photos taken of the terminus of the glacier were used to update this edition of the map. The terminus has been replotted and the borders of the glacier have been re-interpreted; however, all the contours above 6500 feet are identical with the 1955 contours. This is incorrect and renders the map unusable in this study.

3.1.3.4 Large-scale maps: Survey of I.A. Reid

In 1959, Reid did the first precise survey of the Athabasca Glacier that included the valley walls surrounding the glacier (Reid, 1961). His survey established ground control points for the aerial photogrammetry which was flown on August 1 of that year. Previous work by Water Resources Branch did not involve detailed surveys.

Reid's survey was very precise and self-consistent (Brugman and Demuth, in preparation). Later photogrammetric work, including terrestrial photogrammetric surveys made for the 1:10,000 map series either extended this survey with additional survey points or simply used his ground control points (Reid and Charbonneau, 1981). When the 1:10,000 maps were compared with each other, it could be seen that their georeferencing was self-consistent. However, Reid's survey contains a basic flaw. He did not extend his survey to the nearest benchmark. Instead, he estimated the location of his principal survey point from a large-scale map (Reid, 1961).

The result of this is that when the small-scale 1955 map, tied to the national benchmark system, is compared with the larger scale maps based on Reid's survey, a definite displacement can be observed. The maps based on Reid's survey are displaced to the east and south of the 1955 map.

To resolve the differences noted between the large-scale maps based on Reid's estimated position and the small-scale maps which are related to the national series of benchmarks, the method of comparing the positions of nearby mountain peaks in the different map series could not be used. Mountain peaks were marked on no large-scale map. It was necessary to identify prominent features that appeared on both the large- and small-scale maps. After observing the maps, the crests of the lateral moraines surrounding the terminus of the glacier were selected. It was assumed that these crests had been mapped precisely in both years and would be in the same position in 1955 as in 1967, the first of the large-scale maps to include the contour crests. Accordingly, the positions of the contour lines which crossed the crests of the moraines were determined for the large-scale and the small-scale maps. The agreement between the two sets of values was good, so the average (x,y) displacement between corresponding moraine crests at corresponding elevations was the amount by which Reid's estimate of the location of his primary benchmark was in error. This displacement was (x,y) (148.3, -63.8) metres.

The second assumption, that the crests of the moraines did not appreciably change position between 1955 and 1967, is more questionable than the first. The lateral moraines are subject to a great deal of mass-wasting, visible from air photo interpretation. Figure 3.7*a* shows very recent mass wasting on the lateral moraine to the west of the terminus. Other, not as recent mass-wasting flows are also clearly visible on that photograph. Thus, to state that the crests of the moraines stayed in precisely the same place while twelve years elapsed is a large assumption. However, it is unlikely that the magnitude of the change on the moraines would have been great enough to cause this estimate to be seriously incorrect.

Although the position of the glacier on the small-scale 1955 map is tied to the national benchmark system and is therefore more accurate, for this study the exact position of the glacier in space is irrelevant so long as all the various maps are self-consistent. Since there were far more large-scale maps which used the erroneous referencing than there were small scale maps that were correct, it was more convenient to convert the small-scale maps to use Reid's coordinate system. Thus, the 1955 map and the corrected 1919 and Parks Canada 1977 maps were all displaced (x,y) $(-148.3, 63.8)$ metres to bring them into agreement with Reid's survey.

3.2 Production of Results: *Surfer*

Surfer 5.01 for Windows, a raster-based interpolation package, was the main tool of data generation and analysis for this thesis. This program was selected for a variety of reasons. Since it is a raster-based program, its data structure is simple to understand and utilize. This thesis consists of extensive comparison of many DEMs. Using a raster-based data structure means that when DEMs are compared, grid nodes of identical position and means of calculation in different DEMs are directly contrasted. A triangular irregular network model approach, such as is used in *Arc-Info*, would compare nodes that were necessarily in different positions when calculating elevation change.

In addition, *Surfer* is a user-friendly package that is extremely easy to learn. Little time is required to become expert in the program. Usable output can be prepared within a day after first sitting down to the package.

3.2.1 Explanation of griding

Surfer is a raster-based interpolating package. It represents surfaces by overlaying them with a regular rectangular grid in the x-y plane. The fineness of the grid (the distance between nodes) is set by the user. Nodes in the x-y grid are given z-values by interpolating from surrounding irregularly spaced data points.

The method of interpolation used was universal Kriging with a linear semivariogram, which was one of the interpolation options offered in *Surfer*. Kriging assumes that the spatial variation of any variable can be expressed as the sum of three major components: a structural component, associated with a constant mean value or a constant trend; a random, spatially correlated component; and a random noise term (Burrough, 1986).

Kriging proceeds by calculating and isolating the structural component at a particular location. Once structural effects have been removed, the remaining variation (the semivariance ($\gamma(h)$)) can be assumed to be a function of distance. When semivariance is plotted against sample spacing (the lag(h)), it can be seen that often two populations are the result. Close to the point being calculated, the sample data shows definite correlation. In this zone, the closer together sites are, the more similar they are likely to be. This condition persists until the range of the function is reached. Beyond the range is the sill. Sample points from the sill have no effect on the calculation of interpolation: they are spatially independent (Burrough, 1986). These conditions are illustrated in the sample semivariogram in figure 3.11.

Interpolation requires data points. In the *Surfer* calculation of Kriging, the area around each grid node is divided into eight 45° regions, or octants. From each octant, the three irregularly spaced data points which are closest to the grid node whose value is being interpolated are selected. These twenty-four points are used to interpolate the grid node value.

A linear model is used when the magnitude of the range exceeds the distances over which one wishes to interpolate. This is the variety of semivariogram used in the interpolations done in this thesis. Mapping the distribution of digitized points indicated that on the surface of the glacier, where the data points were most densely packed, it was highly likely that none of the twenty-four points chosen would be from the sill of the semivariogram. Hence, a linear variogram would be an acceptable way of modelling the data.

Every node in the grid was interpolated using this process. When the process was complete, the result was a grid file which represented the irregular data points by a regular rectangular series of (x,y,z) points.

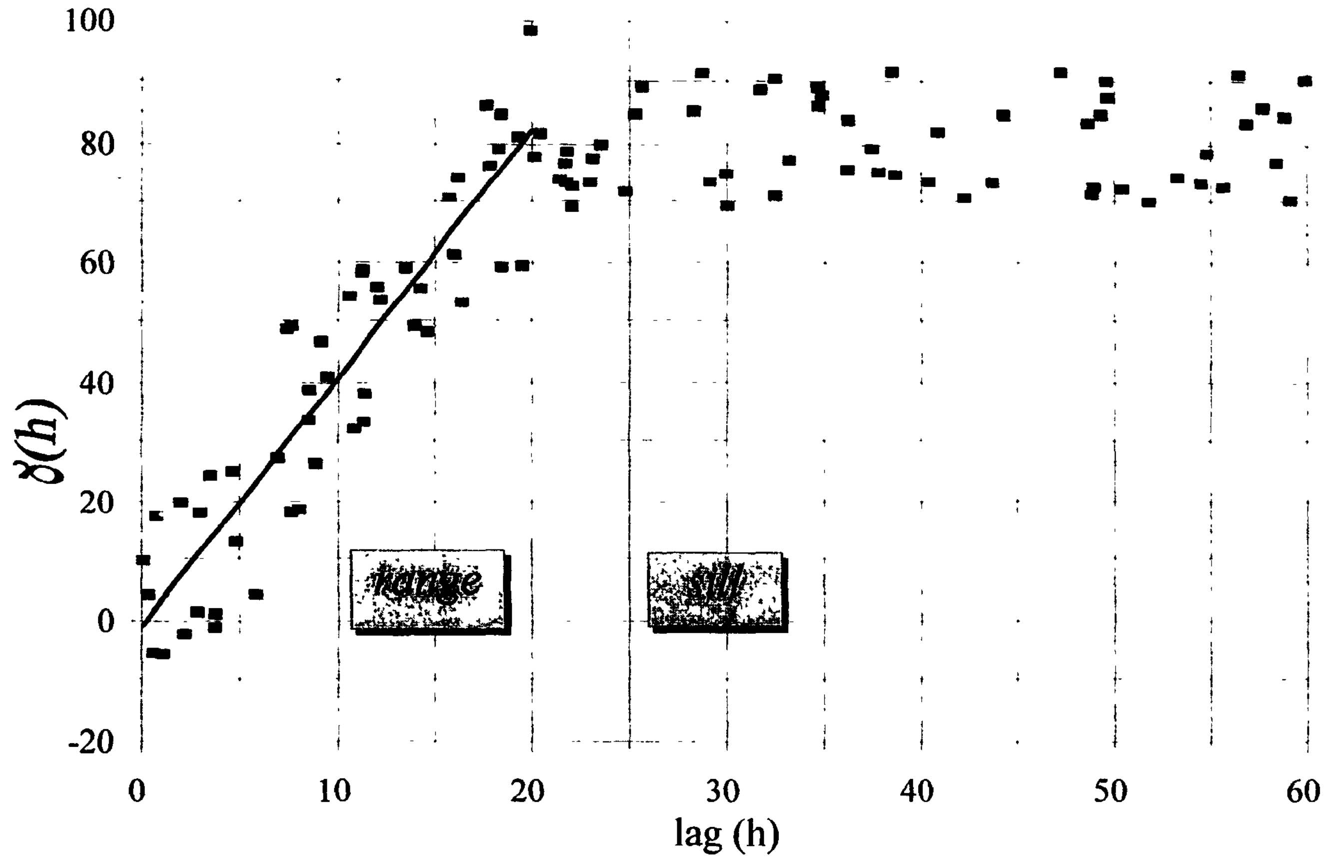


Figure 3.11: Sample semivariogram. After Burrough (1986) p.156.

3.3 Analysis

To produce usable DEMs from the original paper maps, several steps are necessary. These are: digitizing; converting digitized files to *Surfer* format; and correcting problems arising from the previous steps. Once these steps are completed, surface and volumetric change results can be calculated. The flowchart shown in figure 3.12 summarizes the steps required to convert paper maps into DEMs and how these are used to produce change results.

3.3.1 Digitizing

In this context, digitizing is a process of converting the contour lines and borders on a paper map into a series of location points in a computer file. Digitizing was done using the program *Tosca* in conjunction with a Summagraphics SummaSketch II Professional digitizing tablet. A discussion of the method used follows.

Digitizing proceeds in three steps: georeferencing, tracing lines, and saving results. Georeferencing consists of locating the map on the digitizing tablet. On each map there are printed points which state explicitly their location on the surface of the earth. These may be in either latitude and longitude or in the UTM projection. The DEMs produced in this thesis were referenced exclusively in UTM coordinates. The 1919 map was produced using an equirectangular projection and was referenced in latitude and longitude. Its georeferencing points were converted into the UTM system using *Gsrug*, a computer program located at the University Map and Design Library at the University of Waterloo.

To reduce the size of the files produced, the UTM easting and northing values for each of the data files were truncated. Only the five digits before the decimal place were

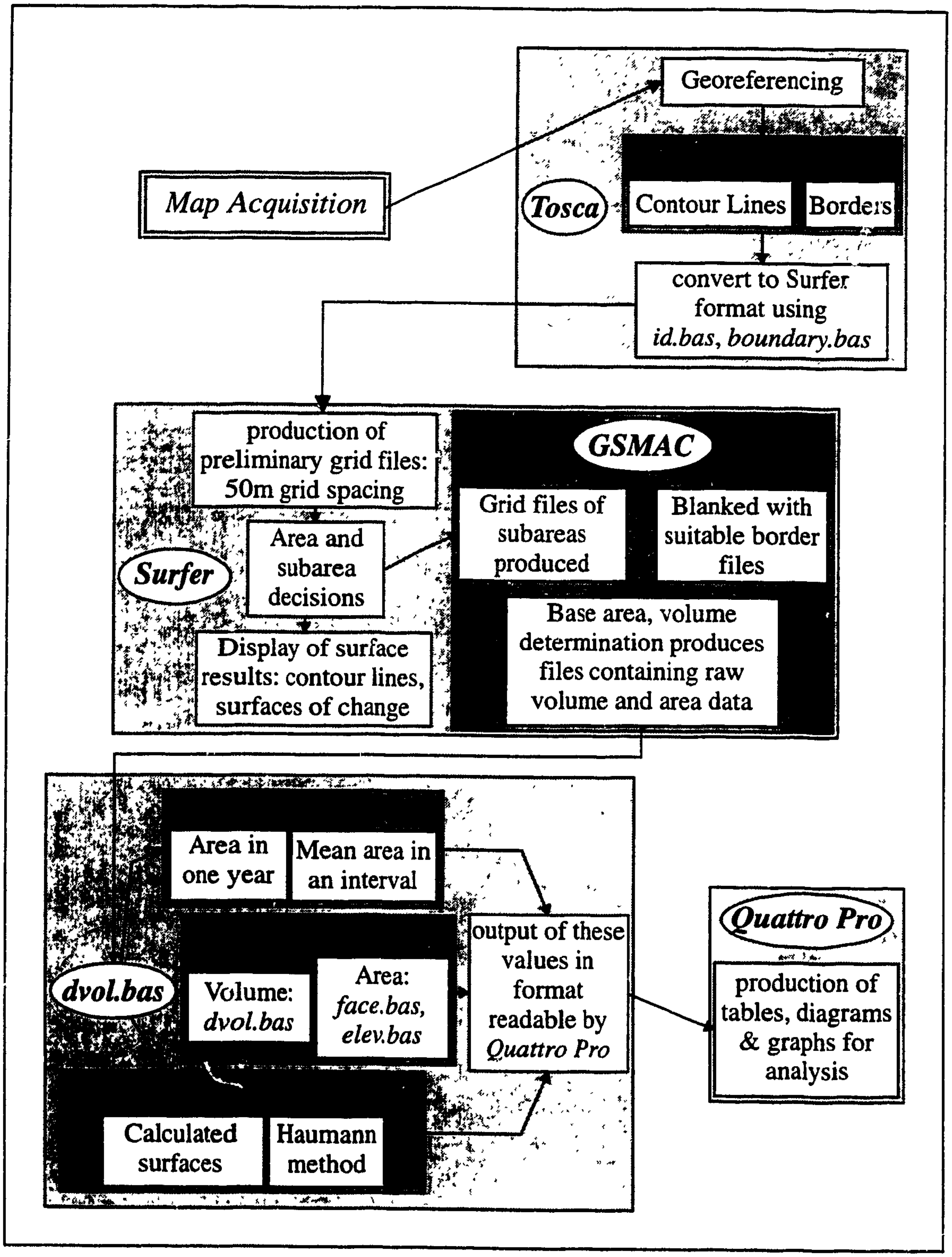


Figure 3.12: Steps followed to convert paper maps to DEMs and extraction of change data. Packages or programs used in the process are denoted by having an ellipse drawn around them. Each ellipse is imbedded in a shaded box. Steps taken in a particular package or program occupy boxes within the appropriate shaded box.

used. Thus, the value 5784000 easting, 481000 northing becomes 84000 easting, 81000 northing.

The map is taped to the digitizing tablet and the georeferencing points are digitized: the program is told their location on the digitizing tablet. With this done, the digitizing tablet is able to adjust all subsequent points for location and scale (Burrough, 1986).

The precision of location of the registration points is measured by the root mean square (RMS) of the difference between the location of the registration points as digitized and their ideal positions if they could be located precisely on the digitizing tablet. For terrestrial maps having a scale of 1:10,000, the RMS values tended to be in the range 13-17 m. The 1919 map had precise registration points but a small scale, allowing an RMS of registration of 85.8 m. The RMS value for the 1955 map was 15.5 m. The 1959 map had an RMS of 2.30 m. Finally, although the 1977A map lacked registration points overlaying the map surface, its RMS was 16.6 m.

Of these maps, only the 1955, 1959 and 1977A meet the United States National Map Accuracy Standards requirements. It is likely that the limited number of registration points that could be digitized on the 1919 and the 1:10,000 maps due to the size of the digitizing tablet used was responsible for the registration difficulties.

3.3.1.1 Digitizing Contour Lines

Once a map is georeferenced, its contour lines may be digitized. Digitizing consists of tracing each contour line with the cross-hairs of the digitizing puck. While tracing the lines, location values are inputted by pressing the appropriate button on the puck. For each centimetre of contour line on the glacier, an average of 7-8 points were digitized. This produced output of exact UTM positions along a contour line of known elevation. This process was repeated for every contour line being digitized and resulted in a file of (x,y,z) points for the area of the glacier. Table 3.5 presents the number of points that were digitized from each source map.

Table 3.5: Number of elevation points digitized from each source map

Year	Scale	Number of Points
1919	1:62,500	4,731
1955	1:50,000	6,896
1959	1:4,800	33,378
1965	1:10,000	16,725
1967	1:10,000	18,178
1969	1:10,000	25,237

Year	Scale	Number of Points
1971	1:10,000	20,092
1973	1:10,000	20,673
1975	1:10,000	19,348
1977T	1:10,000	11,964
1977A	1:50,000	6,343
1979	1:10,000	18,488

On the small-scale maps, every contour line on the glacier was digitized. In addition, some of the contours of the surrounding non-glaciated area were also digitized, but with a much lower frequency of points per unit length of contour line than were recorded on the glacier. Digitizing the extraglacial regions produces the effect of continuous variation in elevation over the entire area of the DEM, despite the extraglacial regions having substantially fewer data points to build the surface.

On the large-scale maps, the entire area surrounding the glacier was not digitized. On no large-scale map was the surrounding area completely detailed; in addition, distant areas were irrelevant or peripheral to this study. As with the small-scale maps, the contour lines that were digitized with the greatest density of points were the contours on the glacier, the cliff walls between which the glacier flows and the area of the terminus.

Some extraglacial contours that appeared were also digitized; however, once again they had a much lower density of points per unit length of line. The uncontoured extraglacial regions displayed peculiar contour effects. This was of limited interest however, since these took place away from the area of study.

The large-scale maps had substantially finer contour intervals than the small-scale maps. A finer contour interval and a larger scale provide a more data-rich representation of the surface of the glacier. Digitizing the additional contour lines allows fine variations on the surface to be portrayed more accurately but cause the total length of the contour lines on a map to increase immensely, which increases the number of points to be digitized. To reduce the task of digitizing to a more manageable level, on each large-scale map only a sample of the contour lines were digitized. Table 3.6 lists the contour lines on the glacier that were digitized on each series of maps of the glacier.

Table 3.6: Digitization of Contour Lines

Interval	Producing Body	Contour Interval of Map	Contour Interval Digitized
1919	Boundary Commission	100 feet	100 feet
1955	NTS	100 feet	100 feet
1959	Water Resources	10 feet	20 feet
1965-67	Inland Waters	25 feet	25 feet
1969-79	Inland Waters	5 metres	10 metres
1977	Parks Canada	20 metres	20 metres

3.3.1.2 Digitizing Borders

After the contour lines on a map were digitized, the glacial borders (discussed in section 3.1.1) were digitized. The border of the clear ice is well defined on each of the original photographs used for mapping, was accurately delimited on each map, and could be digitized directly. The more inclusive definition of the glacier, which included clear ice and debris-covered ice, was identical to the clear ice border along the eastern portion of the glacier and the eastern portion of the terminus and was digitized identically. Working with aerial photography from 1959, 1968 and 1979, estimates of the terminus of the debris-covered portion of the glacier were made for each map. The western flowing ice/valley wall border was defined as described in section 3.1.1 and shown in figure 3.7: the border was taken to be the series of v-notches in the contours west of the clear ice boundary. After digitizing, each map had two files that defined different borders.

Three of the maps had definitions of the glacier border which were substantially different from other border definitions. These were the 1959, 1965 and 1977A maps. Figure 3.7 shows that the 1959 map did not contour the entire area of the glacier: part of the debris-covered terminus was uncontoured. On the 1965 map, the debris-covered ice of a higher series of elevations were not contoured. In both cases, it was necessary to truncate the border of the total ice surface to match the edge of the contoured zone. The volumetric change results presented in chapter four (tables 4.1, 4.2) are affected by this curtailment. Estimates of the additional volumetric change over the uncontoured portions of those maps are also presented in that chapter (table 4.3).

Once digitizing was complete and comparisons could be made, it was seen that the 1977A border was substantially different in lateral cross-section and had a different interpretation of the position of the terminus of the glacier than the same region of the

1977T map. This is shown in figure 3.13. The border as defined on the 1977A map is more inclusive at the terminus, and especially so at the debris-covered portion of the terminus. The lateral definition of borders is fairly similar from the terminus to above the 2200 m contour mark. Between 2200 and 2250 m, the interpretation of the border varies. The southeast border of the 1977A map contains substantially less area than the 1977T border.

These differences in border definition are one of the sources of the marked disparity in elevation zone areas that are noted in table 4.4 and figure 4.3 in chapter four.

3.3.1.3 Horizontal Digitizing Accuracy

It cannot be assumed that digitizing procedures are infallible. Errors are associated with both the source map and with the digitizing process. On the source maps, the contour lines are not infinitely thin. A line that is half a millimetre thick on a 1:10,000 map represents five metres on the ground. A line of the same thickness on a 1:50,000 map represents a region 25 metres wide. Although the operator would reasonably strive to digitize only the middle of the line, in practice, a contour line represents a zone of uncertainty (Burrough, 1986)

Regarding the digitizing process, all of the digitization for this thesis was done manually. The level of error is directly proportional to the proficiency of the operator. To determine the accuracy of operator digitization, an empirical test was performed. A sheet of graph paper was registered on the digitizing tablet. Five straight 10-cm horizontal lines were digitized on the map, with an average spacing between digitized points being comparable to that used for digitizing the contour maps: about 7 points per

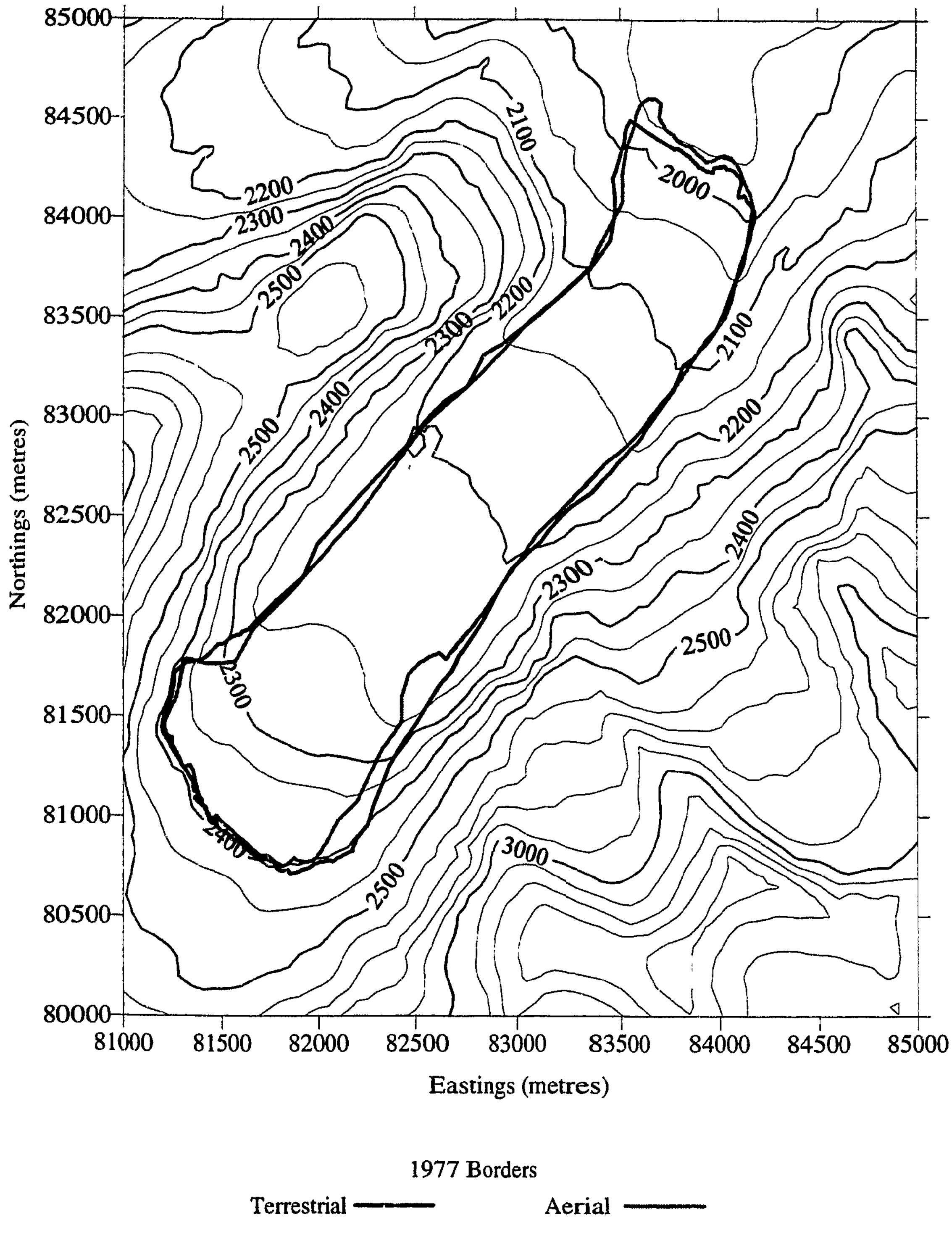


Figure 3.13: Comparison of location of borders of 1977 maps: aerial and terrestrial. Contours from 1977 Parks Canada map.

centimetre. On these five horizontal lines, a total of 275 points were digitized. The displacement of the points perpendicular to the line was normally distributed. The average displacement was 0.031 mm below the line; the standard error of this distance was 0.0962 mm. This process was repeated on five 10-cm vertical lines with very similar results. Performing this test on orthogonal lines allows distance to the original line to be readily calculated and tested to determine whether any systematic difference existed between digitizing lines in different orthogonal directions. Table 3.7 summarizes this information.

Table 3.7: Magnitude of Displacement from the Mean of Digitized Points for straight lines

	Number of Points	Average (mm)	Standard Error (mm)	Magnitude of Maximum Displacement (mm)
Horizontal	275	-0.0310	0.0963	0.3114
Vertical	303	-0.0075	0.0900	0.2376
Total	578	-0.0187	0.0937	0.3114

As can be seen, the magnitude of digitizing error is slight. The overwhelming majority of points digitized lie very close to the line they represent. Digitizing error is also normally distributed, suggesting that what errors are made would not have a great effect on the surface as a whole. This is in accord with the assumption of no systematic horizontal error, made in section 3.1.2. Table 3.8 shows the magnitude of uncertainty this digitizing error represents on the ground at each of the four scales of map used. As the scale shrinks, the magnitude of uncertainty grows. Even at the three standard error level however, the large-scale maps which were used for the majority of the results have a comparatively slight displacement. The small-scale maps display a substantial horizontal precision problem, however. This adds further uncertainty to measurements made from corresponding DEMs.

Table 3.8: Horizontal Uncertainty Caused by Digitizing Distribution

	standard error (metres)		
	1	2	3
1:4,800	0.90	1.80	2.70
1:10,000	1.87	3.75	5.62
1:50,000	9.37	18.74	28.11
1:62,500	11.71	23.43	35.14

3.3.2 Data Conversion: *Tosca* to *Surfer* Format

The files produced from digitizing in *Tosca* were saved in binary format. This was readily convertible into ASCII by the program *Idrisi*, of which *Tosca* is a subprogram. In ASCII, the files were of the format

$$z_1,n_1$$
$$x_{11},y_{11}$$
$$x_{12},y_{12}$$
$$:,:$$
$$x_{1n1},y_{1n1}$$
$$x_2,n_2$$
$$x_{21},y_{21}$$
$$:,:$$
$$x_{2n2},y_{2n2}$$
$$:,:$$

where z is the elevation of the data point, n is the number of data points of that elevation, and (x,y) are the positions in Cartesian space of the given elevation. With an ASCII file in this format, contour lines which were inadvertently mislabelled while digitizing could be easily corrected.

To produce a surface from data points, *Surfer* requires a file with the data points in the format (*x,y,z*). The ASCII file of the points digitized in *Tosca* was converted into a *Surfer*-readable format using the program ID.BAS, reproduced in Appendix 7. ID.BAS was also used to apply the displacement corrections which were noted in section 3.1.3 and the imperial-metric conversions noted in section 3.1.2.5. Preliminary griding work using *Surfer* was done with the files to ensure that any mislabelled contour lines which were neither identified while digitizing nor corrected before converting from *Tosca* to *Surfer* format were discovered and corrected.

The digitized borders which were produced were also converted from binary format to ASCII. To enclose an area with a border, *Surfer* requires that the border file be in the format

$$\begin{array}{c} n,a \\ x_1,y_1 \\ \vdots,\vdots \\ x_n,y_n \end{array}$$

where n = the number of points in the blanking file
a = which area should be blanked (inside or outside the line)
x,y = the Cartesian points which define the location of the border.

Converting from the ASCII *Tosca* file to the ASCII *Surfer* border file was a task that required only that the first two digits of the ASCII *Tosca* file be rearranged.

3.3.3 Calculating Change

From the DEMs of glacier surfaces, values of volumetric and elevation change were calculated. Areas contained in elevation zones were also determined: both mean areas in successive years and areas in single years. In addition, the values calculated with *Surfer* were compared with results produced without DEMs that were published in the Inland Waters series Glacier Surveys in Alberta.

The calculation of volumetric and area measures was a two-step process. It was necessary to calculate raw values using a program written in GSMAC, the *Surfer* programming language. These values were then read, processed and prepared for presentation in the QBASIC program *DVOL.BAS*.

Figures showing elevation change over an area are very easy to prepare using *Surfer*, but average values of elevation change are not readily computable in that package. Consequently the programs *FACE.BAS* and *ELCH.BAS* were written to produce average values of elevation change over elevation zones. The summary figures produced in this program were read into the results of *DVOL.BAS* and consequently appear in tables in chapter four.

The following subsections will discuss how values were calculated from the DEMs produced. The meaning of the raw values that the program *DVOL.BAS* uses to calculate final values will be explained. This will be followed by subsections discussing how these raw values were converted into meaningful measurements of volumetric change, area, and mean area. Following this is a description of the calculation of elevation change figures and values. Finally, the effects of the uncertainty of vertical estimation on the original maps will be quantified.

3.3.3.1 Production of Grid Files

Quantifying the values of surface and volumetric change requires the production of many DEMs from a single data set. Some DEMs are used for the production of diagrams showing the entire study area of the glacier surface. Other DEMs are prepared from subsections of the areas, and are used to perform detailed calculations of surface and volumetric change.

Surfer is a raster-based surfacing package. Data points representing the surface are separated by a user-specified distance. When showing changes over the entire ablation area of the glacier, a coarse node separation is satisfactory. Note that as discussed in appendix 8, the finer the interval between grid nodes, the more precise are the resulting area calculations. The finer the spacing between grid nodes however, the longer the computer must take to produce the DEM, and the larger the file is in the computer. It was necessary to produce a compromise between file size, grid spacing, speed of DEM calculation, and accuracy of results.

It was decided to produce different classes of DEMs having different spacings between grid nodes, and use the different classes of DEMs for different purposes. Two different node spacings were used: 50 m and 5 m. The DEMs that had a 50 m grid spacing were used to produce diagrams that show contour lines, elevation differences and borders over the complete surface of the glacier below 2400 m.

It was necessary to use DEMs that had different extents to show change over the entire study area of the glacier properly. In 1955 and later, the terminus of the glacier was south of the 85000 north UTM line, so for the DEMs produced from maps made after that date, the 85000 north UTM was the northern extent of the gridded area. However, the terminus of the glacier in 1919 was substantially farther north. As a result, the 1919 50 m spacing DEM and the 1955 DEM that it was compared with have northern borders at the 85500 UTM line. The vertices of the borders of the gridded areas appear in table 3.9.

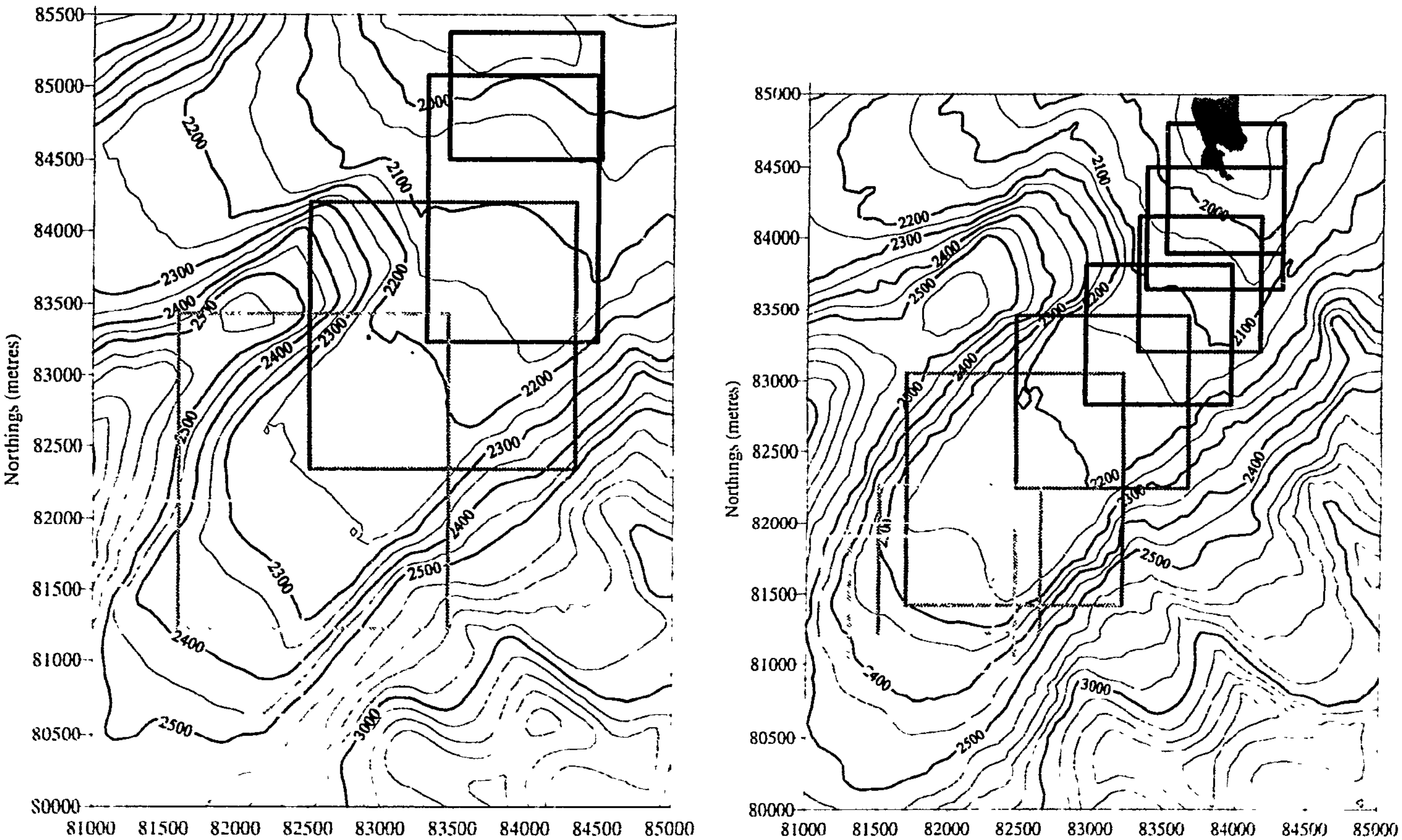
Table 3.9: Vertices of the borders of the gridded areas for DEMs with 50 m grid spacings

a. 1919, 1955 DEMs	
Truncated UTM Coordinates	
Eastings	Northings
81000	80000
81000	85500
85000	85500
85000	80000

b. 1955-1979 DEMs	
Truncated UTM Coordinates	
Eastings	Northings
81000	80000
81000	85000
85000	85000
85000	80000

DEMs having a 5 m grid node spacing were used to produce values of surface and volumetric change. Except for the terminus region of the glacier, for most of the intervals between DEMs, surface and volumetric change was calculated over 50 m elevation zones. Thus, each series of DEMs dealing with a given elevation zone need only enclose that particular elevation zone covered by the glacier on each DEM. The extent of an individual elevation zone covers a much smaller area than is enclosed by the entire study area. This greatly reduces the length of time required to produce the DEMs, the size of the DEM files that are generated, and the time required to produce measures of elevation and volumetric change from the DEMs. The vertices of these elevation zones are listed in table 3.10*a* and shown in figure 3.14*b*.

Once again, the 1919 DEM proved to be the exception. Since the glacier downwasted greatly between 1919 and 1955, the corresponding elevation zones for the 1919 DEM are substantially northeast of those from the later DEMs. If the edges of the 5 m grid node spacing DEMs were expanded to include the corresponding 1919 elevation zones, the files containing the DEMs covering the subareas would be unmanageably



(a) subareas used for 1919, 1955 maps

1919 Boundary Commission map

Borders

terminus-2000 —
2000-2100 —
2100-2200 —
2200-2300 ·····
2300-2400

Contour Interval:
50m below 2500 m
100m above 2500 m

(b) subareas used for 1955-1979 maps

1977 Parks Canada map

Borders

terminus-2000 —
2000-2050 —
2050-2100 —
2100-2150 —
2150-2200 —
2200-2250 —
2250-2300 · ·
2300-2350 – –
2350-2400

Figure 3.14: Locations of subarea borders

large. It was decided to compare the 1919 data to the 1955 data separately. For that interval between DEMs, a series of 100 m elevation zones were compared. The vertices of those elevation zones are shown in table 3.10*b* and figure 3.14*a*.

Table 3.10: Subarea Vertices
a. 1955-1979

Elevation Zone (metres)	Eastings (UTM)		Northings (UTM)		Area (km^2)
	min.	max.	min.	max.	
terminus-2000	83490	84300	83900	84800	0.73
2000-2050	83350	84300	83650	84500	0.81
2050-2100	83300	84150	83200	84150	0.81
2100-2150	82930	83950	82825	83825	1.02
2150-2200	82450	83650	82250	83450	1.44
2200-2250	81690	83200	81425	83050	2.45
2250-2300	81500	82625	81225	82275	1.18
2300-2350	81300	82450	80950	82000	1.21
2350-2400	81100	82300	80625	81900	1.53

b. 1919-1955

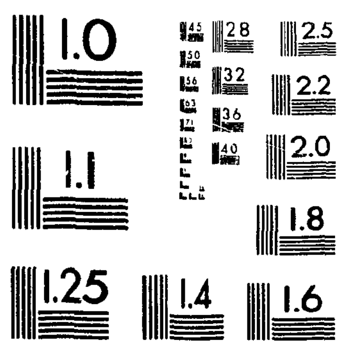
Elevation Zone (metres)	Eastings (UTM)		Northings (UTM)		Area (km^2)
	min.	max.	min.	max.	
terminus-2000	83450	84500	84050	85375	1.39
2000-2100	83300	84480	83225	85075	2.18
2100-2200	82490	84325	82350	84200	3.39
2200-2300	81590	83450	81230	83420	4.07
2300-2400	81200	82450	80630	82125	1.87

Once the extent of terrain that each DEM will cover is decided, the DEMs must be generated. In *Surfer*, this process is known as griding. The (x,y,z) raw data that was produced from digitizing contour lines is used to interpolate z-values for each of the grid nodes on the regular rectangular grid that *Surfer* uses to represent the surface. In this thesis, the method of interpolation used was universal Kriging with a linear semivariogram, discussed in section 3.2.1 above. Griding was done with the GSMAC program *SURELEV.BAS*.

The precision of the interpolation routine was measured by comparing the elevations of the data points digitized with the elevations of the surface produced from those data points at each of the data point locations. The differences between the elevations of the data points and the generated surface are the residuals. The total RMS of the residuals for each year of mapping appears in table 3.11. This table shows that there was a good match between the data points and the generated surfaces. For each of the surfaces, the RMS of flat areas was very low, often averaging less than 9 cm. For steeper areas of the glacier, such as the terminus and especially the icefalls, the RMS was higher, but in no 50 m elevation zone did the RMS exceed 30 cm. This demonstrates that the interpolation algorithm was quite precise in fitting a surface to the data points.

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT



PRECISION℠ RESOLUTION TARGETS

Table 3.11: Residual RMS measurements comparing generated DEM surfaces (5 m grid spacing) with digitized data points

Year	number of points	RMS (m)
1919	337	0.119
1955	616	0.123
1959	24499	0.140
1965	8137	0.129
1967	8679	0.137
1969	9711	0.120

Year	number of points	RMS (m)
1971	8720	0.138
1973	9498	0.139
1975	7402	0.126
1977T	6261	0.140
1979	8917	0.135
1977A	637	0.142

3.3.3.2 Production of Raw Values

After they are produced, each of the subarea files has elevations attached to each grid node. To produce the required data to measure volumetric change and area, several steps must be taken. First, the file must be blanked with one of the relevant blanking files.

Blanking files are representations of the borders of the glacier. They were produced from every glacial map, for both the clear ice and the complete ice surface as discussed in section 3.3.1.2. A blanking file can be structured in one of two ways: it can be set to remove from consideration all grid nodes that fall outside its border or it can remove all grid nodes within its border. For every border digitized, both varieties of blanking file were produced.

Many additional DEMs were prepared by successively blanking the original unblanked subarea DEM file with the relevant blanking files. The original subarea file was blanked with the complete ice border files from the year of mapping and by the border files from the previous and the succeeding mappings to allow different DEMs to be compared using identical borders, as will be discussed in section 3.3.3.4. In addition, the DEMs produced from terrestrial photogrammetry from 1969 to 1979 were also blanked with blanking files representing the clear ice surface from the year of mapping and from the subsequent mapping to allow for calculation of clear ice values for comparison with previous volumetric change calculations. In each case, the blanking files applied were of both varieties: those that blanked outside the border and those that blanked within. Thus, any subarea DEM from 1971, for example, would be used to produce ten DEMs, each blanked using a different blanking file.

Once a *Surfer* subarea file has been blanked, raw data can be produced from it. The subarea file contains a given elevation zone of the glacier. For each elevation in the subarea file that is divisible by 10, a report was produced with *Surfer*. This procedure was repeated with each of the blanked files produced from the unblanked file. The report shows the results of area and volumetric calculations made by the package for the unblanked portion of the surface with respect to the given 10 m elevation. An example of the file produced by this report appears in figure 3.15. Of the information appearing in files like this, only the values of positive volume and positive planar area, shown in bold, were used. These values were used to calculate volumetric change and area values. One of the functions of the program *SURELEV.BAS* was to produce these report files from the various DEM subarea files.

3.3.3.3 Area Calculation

The amount of area within a given elevation zone is the difference of the total planar area having elevation greater than the lower boundary of the elevation zone and the total planar area having elevation greater than the upper boundary of the elevation

```
VOLUME COMPUTATIONS

UPPER SURFACE
     Grid File:      outifn.grd
     Rows:    0 to 32766
     Cols:    0 to 32766
     Grid size as read:      171 cols by 191 rows
     Delta X:        5
     Delta Y:        5
     X-Range:        83300 to 84150
     Y-Range:        83200 to 84150
     Z-Range:        1964.21 to 2149.13

LOWER SURFACE
     Level Surface defined by Z = 2080

VOLUMES
     Approximated Volume by
     Trapezoidal Rule:        -586795
     Simpson's Rule: -584112
     Simpson's 3/8 Rule:      -585132

CUT & FILL VOLUMES
     Positive Volume [Cuts]: 2.38858E+006
     Negative Volume [Fills]:        2.97502E+006
     Cuts minus Fills:        -586436

AREAS
     Positive Planar Area
     (Upper above Lower):     103765
     Negative Planar Area
     (Lower above Upper):     122810
     Blanked Planar Area:     580925
     Total Planar Area:       807500

     Positive Surface Area
     (Upper above Lower):     108120
     Negative Surface Area
     (Lower above Upper):     125640
```

Figure 3.15: GRID VOLUME result example: raw data

zone. The mean area of an elevation zone in a given interval between mapping is the mean area of the corresponding elevation zones in the DEMs compared. Part of the report file (figure 3.15) produced by the program *SURELEV.BAS* lists the positive planar area of the surface. This is the total area of the unblanked surface of the DEM that lies above a plane of height given in the 'level surface' section of the report. The raw values from these files were read into the QBASIC program *DVOL.BAS* and modified according to the method described in appendix 8 to remove the effects of a systematic underestimation of area. From these modified values, area and mean area values were calculated as described above and prepared for output in the form of tables. Tables showing mean area between DEMs and area of given DEMs are presented in chapter four. An example of the calculation of the area and mean area within a given elevation zone appears in appendix 9.

3.3.3.4 Volumetric Change

The volumetric change of the Athabasca Glacier was measured by two methods: directly using the surfaces generated by *Surfer*; and indirectly by preparing a Haumann estimate of the volume based on area figures produced by *Surfer*. The method using the *Surfer* surfaces measures volume directly from the calculated surfaces. The Haumann estimate is not as precise, but since it was the method used in previous volumetric change calculations, it was necessary to calculate it to allow a meaningful comparison between previous work and the values calculated using the DEMs generated.

Two assumptions must be made before volumetric change is calculated. These are that the position of the bedrock proximal to the glacier surface reported in the DEMs is invariant; and that the melting stagnant ice on the margins of the flowing ice has a volumetric change which is negligible in comparison to the volumetric change of the flowing ice.

It is probable that the assumption concerning the melting stagnant ice is reasonable, since flowing ice volumetric changes reported can be quite substantial. The bedrock assumption is also likely to be valid, since the contour lines representing those parts of the valley walls immediately surrounding the glacier were digitized with equal density to the on-ice surface contour lines and should be as accurate.

3.3.3.4.1 ***Surfer***

For volumetric change calculations to be meaningful, change must be measured within identical areas. Given glacier shrinkage, if change is being measured between years *A* and *B*, with *B* later than *A*, then it is the volumetric change within the border of the glacier as defined in time *A* that is measured. This border was applied to the DEMs as described in section 3.3.3.2, and volumetric calculations were made using it in DEMs from different years.

The positive volume value highlighted in figure 3.15 represents the total volume between the surface in the subarea and a plane of height given as the 'lower surface' value in figure 3.15. *Surfer* cannot directly measure the volume contained within a surface between two planes. Thus, the volume within an elevation zone is obtained by taking the difference of volumes recorded for successive elevation zones:

$$V_{lh} = V_l - V_h$$

where V_{lh} = the total volume of a surface between elevations l and h
V_l = the volume between the surface of the DEM and elevation l
V_h = the volume between the surface of the DEM and elevation h,
and l and h are elevations, l lower than h.

The volume so calculated includes both ice and rock to the limit of the DEM subsection. The volumetric change in an elevation zone is calculated as being

$$\Delta V_{lhY1Y2} = V_{lhY2} - V_{lhY1}$$

where $\Delta V_{lhY_1Y_2}$ = volumetric change between elevations l and h in the years Y_1 to Y_2
V_{lhY_1} = the volume between elevations l and h in year Y_1
V_{lhY_2} = the volume between elevations l and h in year Y_2
and Y_1 and Y_2 are years of measurement, Y_1 earlier than Y_2.

If the assumptions stated above hold, then the non-glacier volume recorded in a given elevation zone in both DEMs would be identical. When the two volume values are subtracted, the difference of the two figures would be the volumetric change of the glacier ice and the slowly melting stagnant ice at the glacier margin. Figure 3.16 illustrates how volumetric change is measured between elevation zones.

Volumetric change between successive DEMs was calculated in 10-metre elevation zones from the terminus of the glacier to the 2400 metre contour level in the program *DVOL.BAS* from the files produced in *SURELEV.BAS*. *DVOL.BAS* outputted this information in a form readable by the package *QuattroPro for Windows*, which displayed the data so generated, appearing in tables in chapter four. Appendix 7 contains complete listings of the program code of *DVOL.BAS* and *SURELEV.BAS*. Appendix 9 presents an example of how volumetric change is calculated for a given elevation zone in a particular interval.

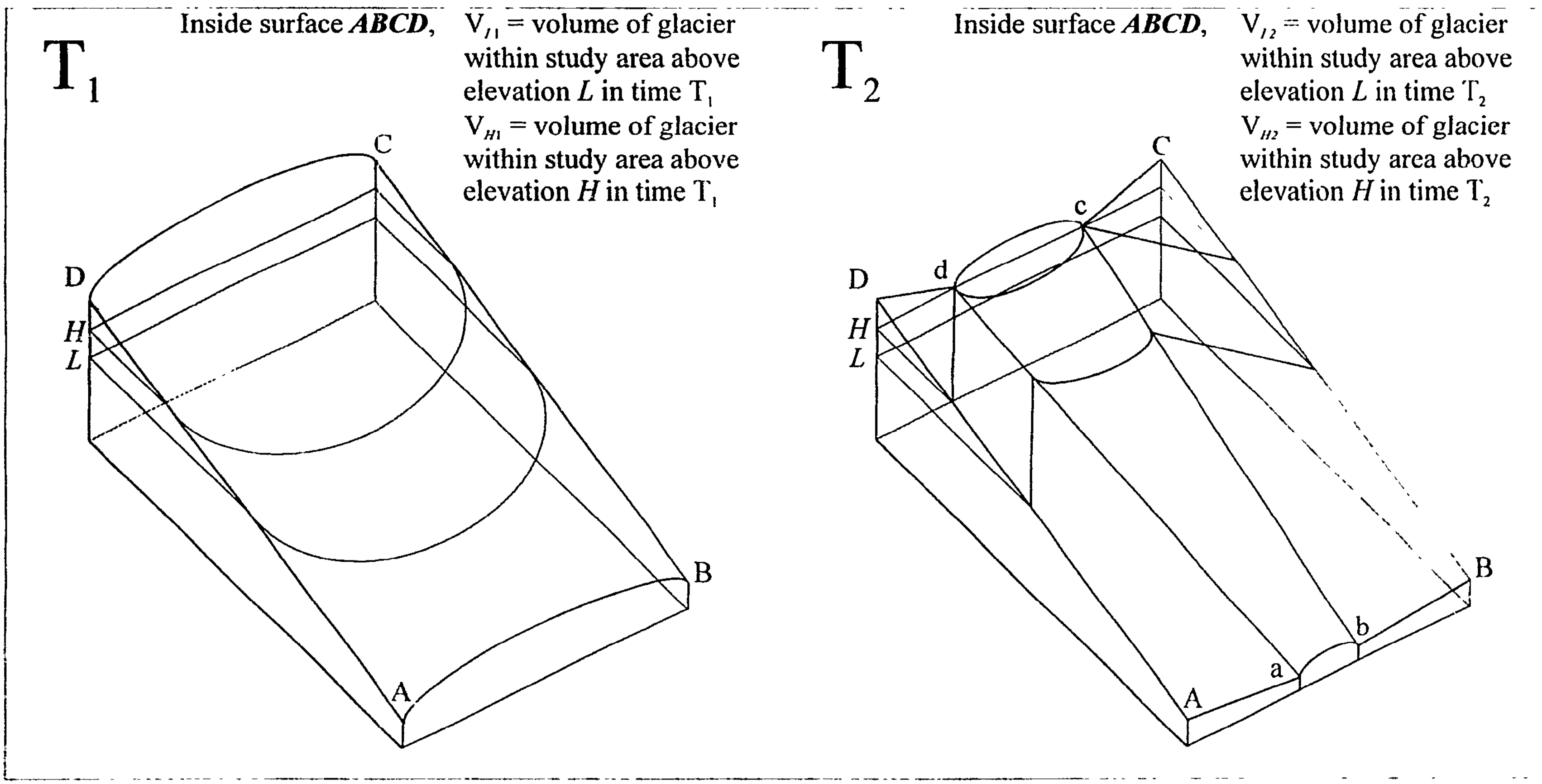


Figure 3.16: Measurement of volumetric change. For volumetric change measurements to be meaningful, measurement must take place over the same area. At time T_1, the glacier surface is ***ABCD***. At time T_2, the glacier has downwasted substantially and is enclosed in area ***abcd***. However, volumetric change is measured over the entire area ***ABCD*** in both years. Volumetric change in elevation zone *LH* is

$$\Delta V = (V_{L1} - V_{H1}) - (V_{L2} - V_{H2}).$$

3.3.3.4.2 The Haumann Method

Previous volumetric change work on the Athabasca Glacier was done in the era before fast and convenient computing power. The amount of volumetric change in an elevation zone could not be measured directly. Instead, it was estimated using the Haumann method, which was a means of estimating volumetric change between elevation zones by comparing areas and differences in areas between the contour lines in question. This is described in more detail in section 2.4.1, figure 2.3, and appendix 4.

Area values were contained in the files produced with *SURELEV.BAS*. After the values were read into the program *DVOL.BAS*, the necessary calculations were made within that program. Table 4.8 contains the values of volume so calculated. These tables compare volumetric change values calculated with the Haumann method using *Surfer*, those calculated using *Surfer* surfaces, and values published by Glacier Surveys in Alberta. Appendix 9 contains an example of how volumetric change is calculated by the Haumann method with raw data produced using *Surfer*.

3.3.3.5 Elevation Change

To calculate elevation change, *Surfer* compares nodes of identical (x,y) location in different DEMs. The difference

$$Z_{diff} = Z_{late} - Z_{early}$$

where Z_{diff} = the difference in z-value of the identical location of two DEMs
Z_{late} = the z-value of the later DEM
Z_{early} = the z-value of the earlier DEM

is the amount of vertical change between one DEM and another at a particular node. Figure 3.17 illustrates how this value is calculated for an individual node. *Surfer* can

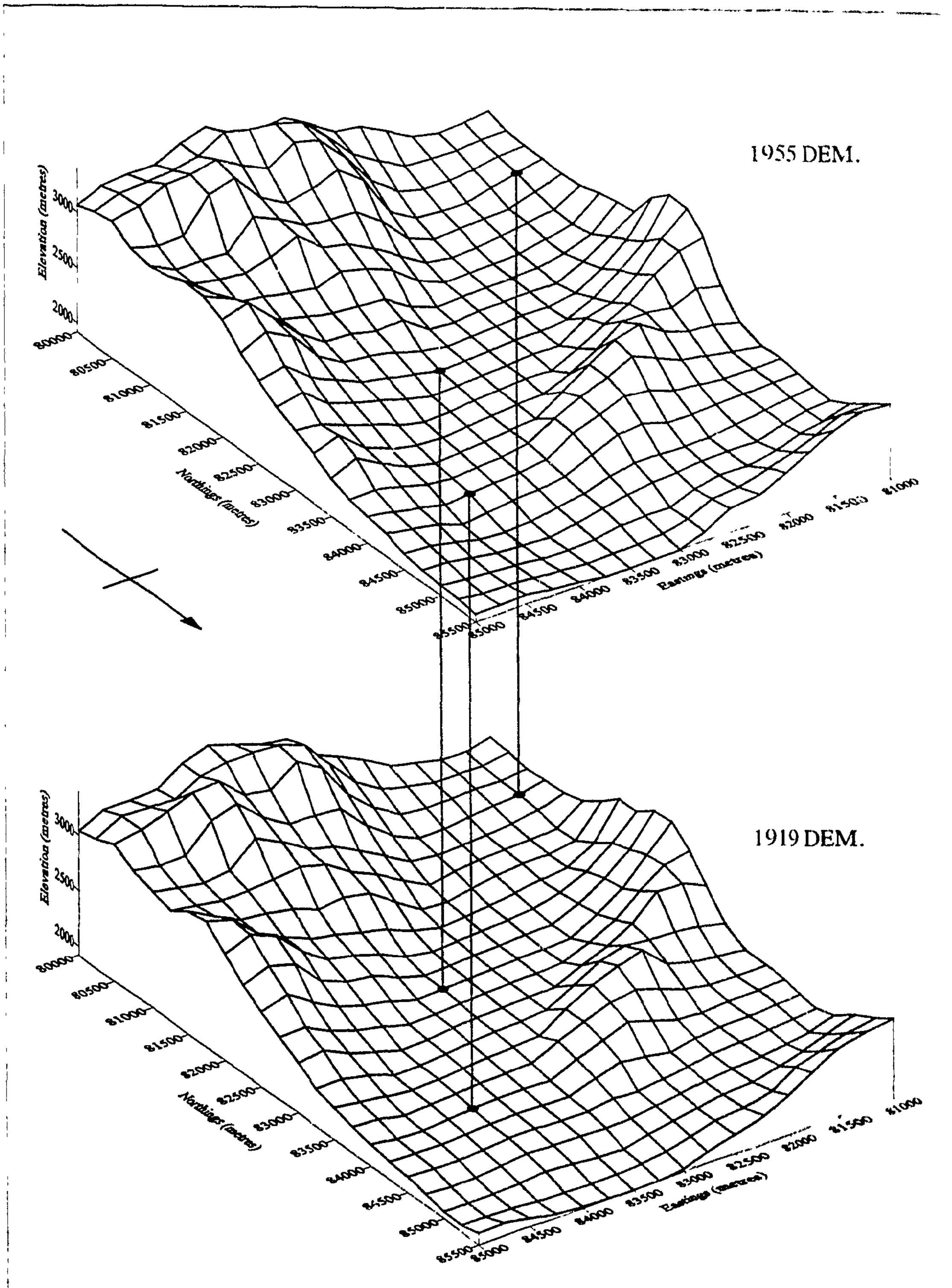


Figure 3.17: Calculation of surface change. Identical nodes are selected on two DEMs and the vertical difference between the values is calculated. The difference in elevation is the elevation change between one DEM and the next.

produce figures which show the distribution of elevation change over all of the nodes of a glacier surface, or over portions of it. Figures showing this appear in chapter four.

In addition, the average value of elevation change in 10 m elevation zones between consecutive DEMs was also calculated and appears in tables 4.1 and 4.2. To calculate this, it was necessary to compare files that showed change between successive DEMs with files containing elevation information for the earlier of the years compared.

For every node in the *Surfer* file of elevation difference between consecutive DEMs, the corresponding point on the elevation DEM from the earlier of the years was compared. This established the original elevation of the location on the surface and how much it changed between one surface and the next. When this was done for every node in the file, the total amount of change in each elevation zone was divided by the number of grid nodes in that zone. This produced values of average elevation change for a given elevation zone.

Two programs were written to accomplish this: *FACE.BAS* and *ELCH.BAS*. *FACE.BAS* was a GSMAC program that converted the subarea DEM files of elevation and elevation change into ASCII format. This allowed the files to be read and their average elevation change values to be quantified in the QBASIC program *ELCH.BAS*. The codes for these programs appear in appendix 7.

3.3.3.6 Quantification of Uncertainty Measures

To quantify measures of uncertainty, the precision of vertical estimation of the individual maps must be expressed for the corresponding DEMs.

In sections 3.1.2.1 to 3.1.2.3, the accuracy of vertical estimation of terrestrial and aerial photogrammetry was discussed. For each map or series of maps, equations were prepared which quantify the amount of photogrammetric uncertainty at any point on the glacier surface. These equations were listed in table 3.3 and are functions of either distance from the photogrammetric station, as is the case with terrestrial photogrammetry, or height of the camera and the elevation of the surface, for aerial photogrammetry. Figure 3.8 shows levels of uncertainty associated with areas of the glacier from four methods of quantifying uncertainty: for the 1919, 1955 and 1959 maps, and for the terrestrial photogrammetric map series 1965-1979. The 1977A uncertainty map is not included due to its great similarity to the 1955 uncertainty map.

To quantify the magnitude of elevation and volumetric uncertainty that arises from this imprecision of vertical estimation, two additional surfaces are generated using the surfaces of elevation and uncertainty: one surface in which the surface of uncertainty is subtracted from the surface of elevation, and one in which the surface of uncertainty is added to the surface of elevation. Figure 3.18 shows the surface of the glacier in cross-section in different years. Although the change in the difference of precision of vertical estimation is difficult to perceive in this diagram, this figure shows that aerial and terrestrial photogrammetry have different responses to increasing elevation of the glacier. With aerial photogrammetry, increasing elevation results in less error due to the surface measured being closer to the measurement platform. With terrestrial photogrammetry, higher glacier elevations occur farther from the measurement platform, resulting in increased uncertainty of vertical measurement.

Each year of mapping is thus represented by three DEMs: one which portrays the surface as mapped and two which represent the elevation as it could be when the maximum possible uncertainty in positive and negative directions is taken into account.

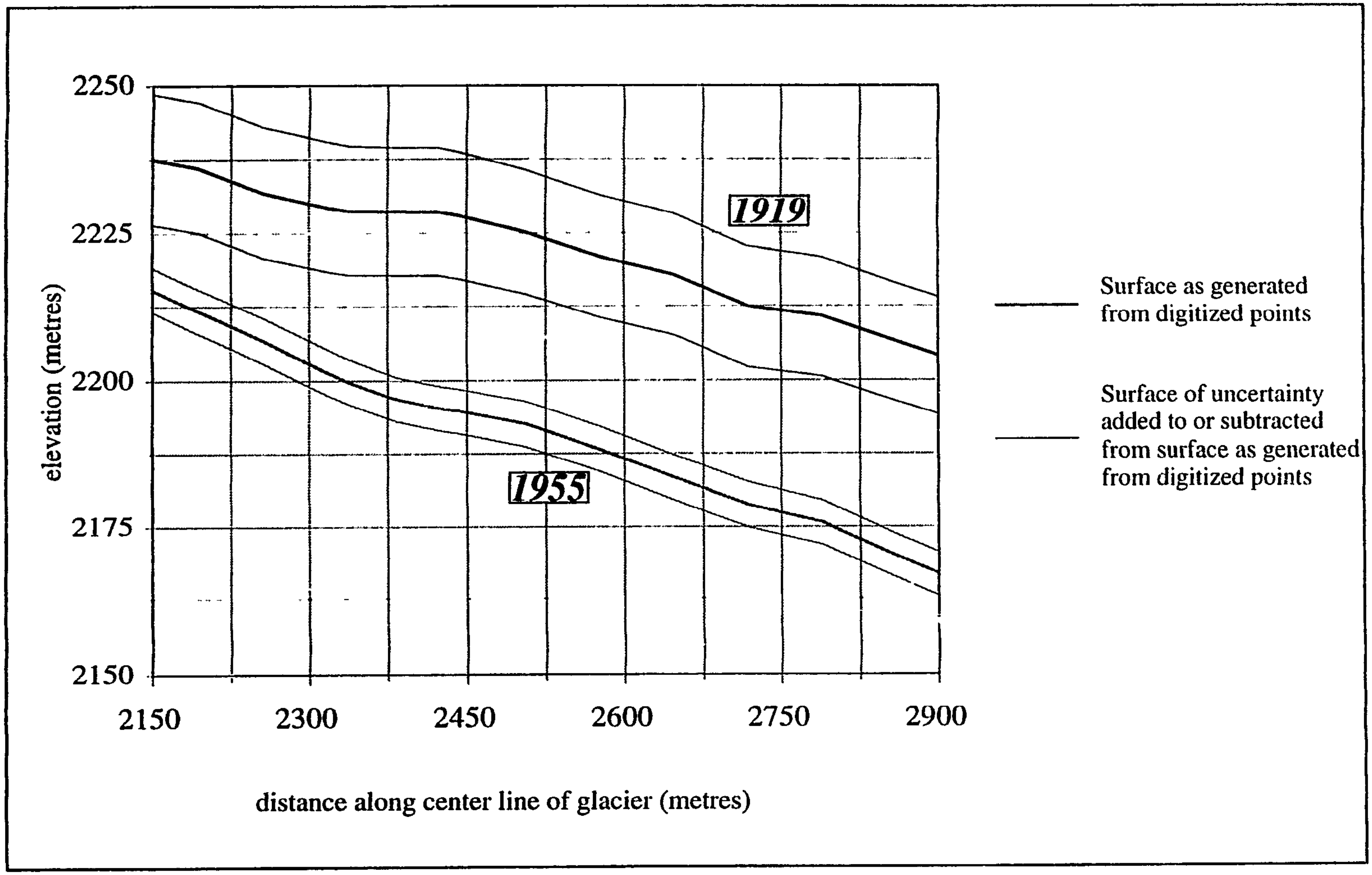


Figure 3.18: Effects of uncertainty of vertical estimation on calculated surfaces. The diagram presents the elevations of cross-sections of the surface down the center line of the glacier (figure 4.8). The magnitude of the uncertainty varies differently for the two years: in 1919, the uncertainty factor increases up the glacier, while the 1955 error decreases up glacier.

Thus, when change over an interval is calculated, a total of six DEMs are compared. There are nine combinations of these six surfaces. Elevation and volumetric change values are calculated for each of these combinations. With a range of measurements, the magnitude of uncertainty of measurement for both elevation and volumetric change was described using standard deviation (σ), a measure of dispersal. If there is little dispersal, then the magnitude of uncertainty is small. If the dispersal is great, then the magnitude of uncertainty is large. This is illustrated for both elevation and volumetric change in figure 3.19.

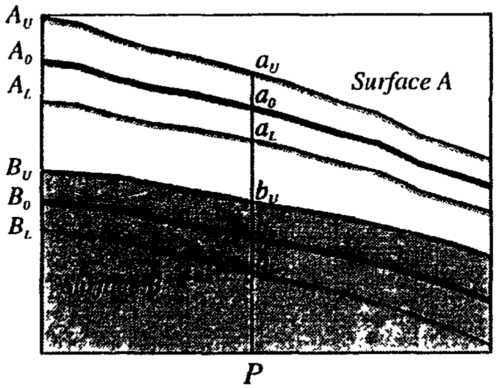
The standard deviation of elevation change (σ_e) was relatively simple to calculate. With uncertainties of vertical estimation known at each node in the surface, the standard deviation of elevation change at that point is

$$\sigma_e = \sqrt{\frac{2}{3}(u_1^2 + u_2^2)}$$

where σ_e = standard deviation of elevation change
u_1 = uncertainty of vertical estimation of DEM_1
u_2 = uncertainty of vertical estimation of DEM_2.

Values produced by this function were compared with elevation change values between DEMs. When σ_e was greater than the elevation change between DEMs, then the variation in measurement is greater than the change measured. These values are marked accordingly in figures 4.1, 4.6 and 4.7 in chapter four. When σ_e is less than the elevation change measured, the variation in measurement is less than the change. It is assumed that any measurement in that case is significant.

It was not as simple to calculate the standard deviation of volumetric change (σ_v). In this case, it was necessary to calculate volumetric change figures for each of the nine



The uncertainty of elevation change between surface *A* and surface *B* at a given point *P* is the standard deviation (σ_e) of the differences:

$$b_U - a_U \quad b_U - a_0 \quad b_U - a_L$$
$$b_0 - a_U \quad b_0 - a_0 \quad b_0 - a_L$$
$$b_L - a_U \quad b_L - a_0 \quad b_L - a_L$$

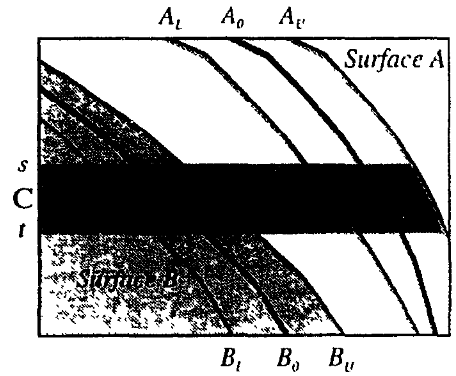
However, for any point in the x-y plane, the elevation difference between the surfaces of uncertainty and the measured surface is identical:

$$B_U - B_0 = B_0 - B_L = U_B$$
$$A_U - A_0 = A_0 - A_L = U_A$$

This allows a simplified calculation of σ_e:

$$\sigma_e = \sqrt{\frac{2}{3}\,(U_A^2 + U_B^2)}$$

(a) Calculation of vertical uncertainty



The volumetric change between surfaces A and B in the elevation zone *s-t* and within *C*, the boundary of the DEM, is represented by the mean of all the possible volumetric change figures:

$$b_U - a_U \quad b_U - a_0 \quad b_U - a_L$$
$$b_0 - a_U \quad b_0 - a_0 \quad b_0 - a_L$$
$$b_L - a_U \quad b_L - a_0 \quad b_L - a_L$$

The uncertainty of volumetric change (σ_v) is the standard deviation of those nine values.

(b) Calculation of volumetric uncertainty

Figure 3.19: Quantification of uncertainty measurement for a hypothetical glacier surface. Given two surfaces A_0 and B_0, (B later than A), having an uncertainty of vertical estimation represented by A_L, A_U, B_L, B_U, respectively, uncertainties of elevation and volumetric change are calculated as indicated in figures *(a)* and *(b)*.

possible combinations of surfaces. *SURELEV.BAS* produced the appropriate surfaces of uncertainty and added them to or subtracted them from surfaces of elevation produced. The resulting surfaces were then blanked with the appropriate blanking files as discussed in section 3.3.3.2 and raw values were produced for those DEMs in the same manner as they were produced for the unmodified surfaces of elevation. Those files were among the information read into the program *DVOL.BAS* for the calculation of volumetric change. σ_v figures appear for 10 m and 50 m zones in tables 4.1 and 4.2 in chapter four. Graphs which illustrate the magnitude of σ_v for consecutive intervals are presented in figures 4.4 and 4.5.

3.4 Uncertainty Summary

In the previous sections of this chapter, the methodology of this thesis was discussed. Each section described sources of error or imprecision inherent in each data source or data treatment. Table 3.12 lists the data sources and means of data generation used, the types of error which each introduces, and its associated magnitude of error, if quantified. Several types of error which were not considered in this work due to difficulties in quantification or inability to acquire the information are also listed. It is believed that those errors not considered were of small magnitude.

Table 3.12: Relative magnitudes of error sources

Data Source or Data Generating Process	Type of Error Produced	Relative Magnitude of Error
Photograph	scale of photography	Dealt with in greatest detail. Ranges from ± 0.43 m for large-scale photography to ± ≈ 14.0 m for small-scale terrestrial (§ 3.3.3 6)
	closeness of GCPs to measured objects	Not dealt with.
Topographic Map	scale	Effects of scale on horizontal estimation were discussed. At the 1 σ level for the large-scale maps, horizontal uncertainty was < 2 m (table 3.8)
	contour interval	A surface portrayed using a small contour interval would be better able to portray fine detail.
	proficiency of operator	Not dealt with.
	tracing or interpolation of contour lines	Not dealt with.
	georeferencing	Each map series georeferenced differently (§ 3.1.3). Problems due to this can be substantial (§ 4.1.1.1, figure 4.1)
	location of borders	assumed to have a small effect in § 3.1.1
Digitizing	proficiency of operator	Tests described in § 3.3.1.3 and table 3.7. These suggest that digitizing process was precise and accurate (average displacement from mean was 19 μm)
	registration error	Horizontal registration error measured using RMS for various maps digitized. For the large-scale maps this figure was ≈ 14 m (§ 3.3.1)
Raster DEM	residuals	A measure of how closely the surface matches the data points from which it was generated. Residuals were small in flat areas (c. 8 cm) and larger in steep areas (c. 30 cm) (table 3.11)
	interpolation algorithm	Kriging generated surfaces that had a close match with the data points digitized
	grid spacing	Finer grid spacing produces more precise measures of area (appendix 8) and volumetric change.
Correction of Georeferencing	limited number of points to interpolate from	Not dealt with.
	"minimizing differences" rather than rubber sheeting	The effects of a 10 m horizontal displacement are ≈ 0.25 the magnitude of photogrammetric uncertainty.
Area and Volumetric Calculations	depends on fineness of grid	Finer grid spacing produces more precise measures of area (appendix 8) and volumetric change.

CHAPTER FOUR: RESULTS AND ANALYSIS

4.0 Introduction

This chapter is divided into two sections. The first presents the results of volumetric and surface change of the entire ice surface as calculated using the package *Surfer* and the program DVOL.BAS. These results are discussed for 10 m and 50 m elevation zones and for the entire study area of the glacier. The second section compares *Surfer* calculations of the results of volume and surface change that occurred within the clear ice borders with previous calculations made for the same area in the report series Glacier Surveys in Alberta, the source of the large-scale maps used.

4.1 *Surfer* Results

To measure how the Athabasca Glacier has changed over the time of record, DEMs of its ablation area were prepared from contour maps representing eleven years between 1919 and 1979. These DEMs were used to calculate volumetric and surface change values for successive years in elevation zones extending up the glacier. Surface change has two aspects: change in elevation and change in planar area. Since these changes in the surface of the glacier are responsible for the volumetric change of the glacier, they will be discussed first. Change in elevation is easier to calculate and display than volumetric change is using *Surfer*, so more than consecutive DEMs could be compared. The results of these calculations are displayed in tables and figures which appear in this chapter.

Surface and volumetric change results are both affected by problems of measurement originating from the source maps. These take two forms: problems due to not all of the ice surface being contoured and problems with the accuracy of vertical estimation of contours. Both of these were discussed in previous chapters. The way these problems affect calculated results will be discussed here.

4.1.1 Surface Change

Surface change has two aspects: change in vertical elevation and change in planar area. Both of these are important in the study of how the glacier has changed over time.

4.1.1.1 Elevation Change

Elevation change is a measure of the vertical difference between surfaces. Figure 4.1 shows the distribution of elevation change over the studied portion of the glacier for consecutive DEMs. Average values of elevation change and the number of nodes over which elevation change was measured appear in tables 4.1 and 4.2 for 10 m and 50 m elevation zones.

It can be seen that many of the figures, especially 4.1*a-c*, display their most substantial elevation changes in the linear zone corresponding with the position of the terminus in the later of the years in the interval. This is because the terminus of the glacier is relatively thin and thickens with increasing altitude up the glacier. When elevation change is measured between two markedly different surfaces, it is the zone immediately around the later terminus that shows the greatest decline in elevation.

Figure 4.1*a* shows an unusual result at the terminus: part of the extreme terminus records some upgrading. This result is inconsistent with measured reality, since in fact that portion of the glacier would have downwasted substantially in that area. This suggests that in this portion of the map, the horizontal displacement due to registration errors is relatively high. The ''upgrading'' recorded in that area is probably due to that portion of the 1919 map portraying the southern part of Mt. Wilcox, while that area of

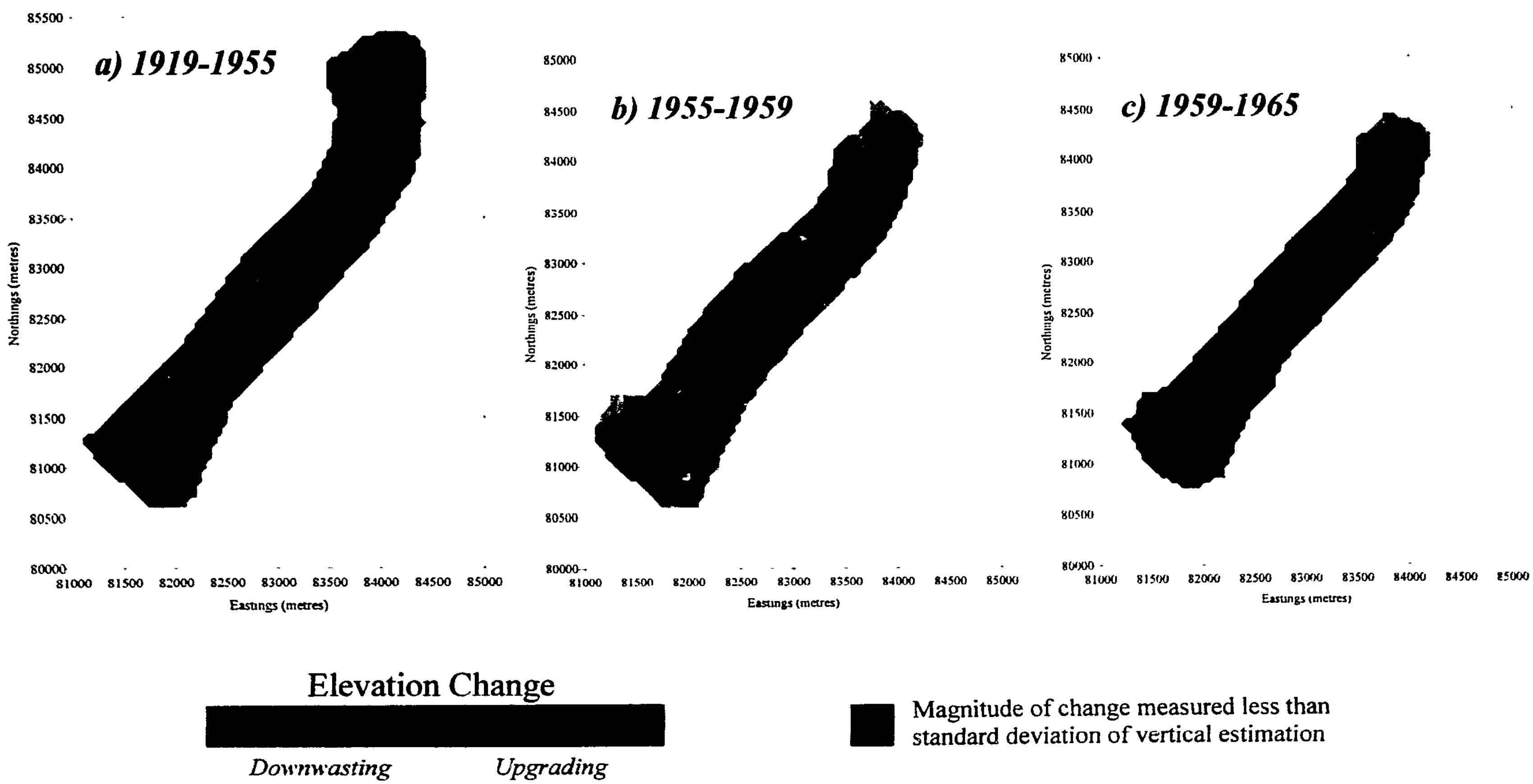


Figure 4.1: Average annual elevation change of consecutive DEMs.
Contour interval 50 m. Isoline interval 1 m/yr.

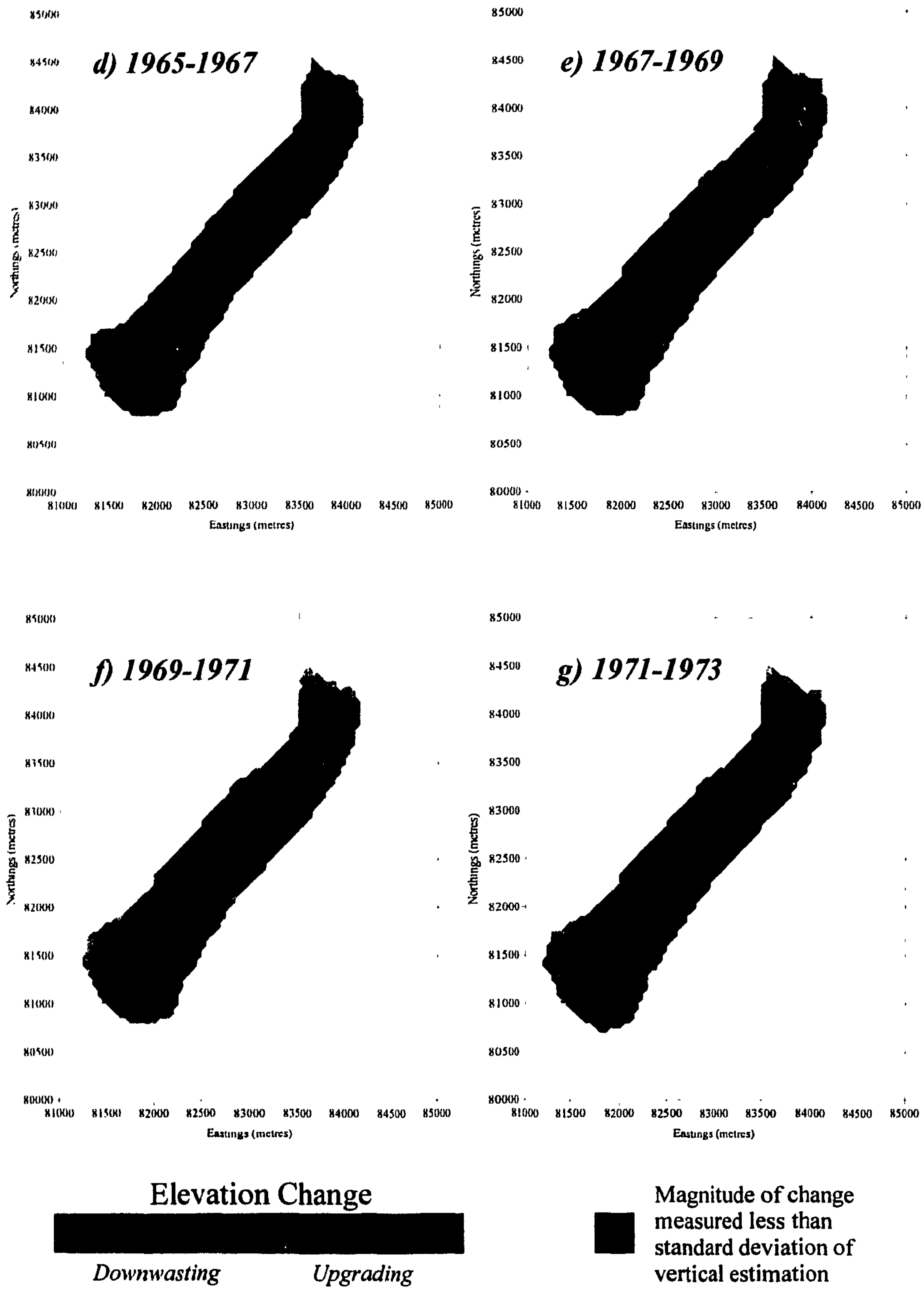


Figure 4.1: Average annual elevation change of consecutive DEMs.
Contour interval 50 m. Isoline interval 1 m/yr.

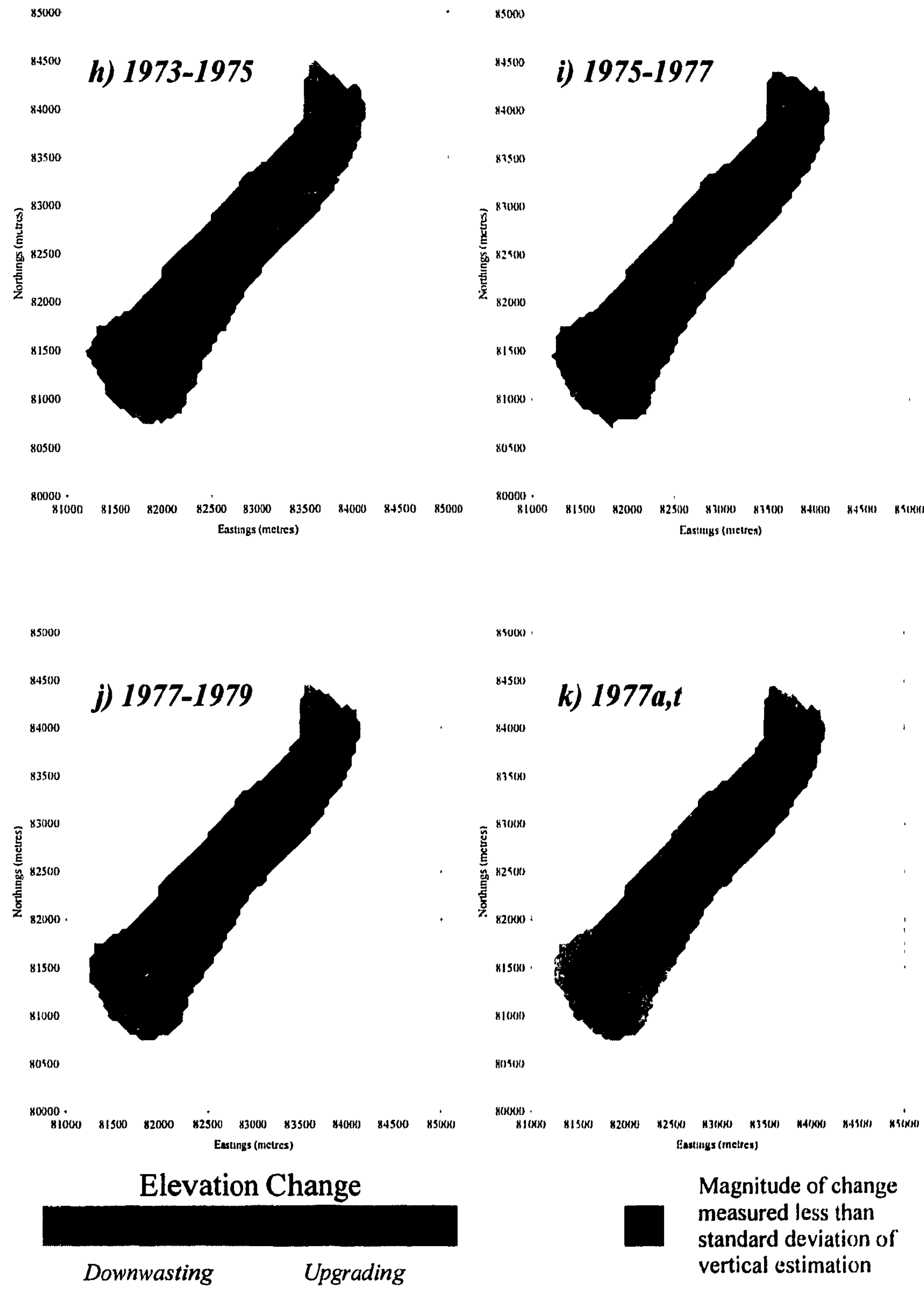


Figure 4.1: Average annual elevation change between consecutive DEMs.
Contour interval 50 m. Isoline interval 1 m/yr.

Table 4.1a: The Volumetric and Elevation Change of the Complete Ice Surface: 1919 to 1955

Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	19-55 (m^2) (,000)
1924-1930	(1,385.00)	893.20	(38.47)	24.81	0.00	0.00	0	139.40
1930-1940	(3,071.00)	481.70	(85.31)	13.38	3.05	0.08	1,826	89.21
1940-1950	(4,001.00)	599.30	(111.10)	16.65	-1.95	-0.05	2,615	107.50
1950-1960	(5,130.00)	609.60	(142.50)	16.93	-8.19	-0.23	2,072	110.20
1960-1970	(5,923.00)	367.30	(164.50)	10.20	-21.20	-0.59	1,539	65.19
1970-1980	(6,317.00)	298.50	(175.50)	8.29	-30.60	-0.85	1,540	50.57
1980-1990	(6.496.00)	277.50	(180.40)	7.71	-38.20	-1.06	1,484	45.08
1990-2000	(6,639.00)	243.00	(184.40)	6.75	-50.80	-1.41	1,259	37.42
2000-2010	(6,728.00)	240.90	(186.90)	6.69	-61.30	-1.70	1,328	36.07
2010-2020	(6,758.00)	272.50	(187.70)	7.57	-72.80	-2.02	1,593	40.83
2020-2030	(6,758.00)	310.20	(187.70)	8.62	-85.90	-2.39	1,850	45.22
2030-2040	(6,698.00)	418.30	(186.10)	11.62	-94.70	-2.63	2,353	55.33
2040-2050	(6.634.00)	496.80	(184.30)	13.80	-101.00	-2.81	3,013	70.76
2050-2060	(6,515.00)	509.40	(181.00)	14.15	-116.00	-3.22	3,038	70.00
2060-2070	(6,319.00)	639.70	(175.50)	17.77	-125.00	-3.47	3,436	77.02
2070-2080	(6,018.00)	879.40	(167.20)	24.43	-112.00	-3.11	5,579	119.30
2080-2090	(5,813.00)	943.60	(161.50)	26.21	-103.00	-2.86	5,475	129.60
2090-2100	(5.632.00)	858.80	(156.40)	23.86	-80.60	-2.24	4,608	103.80
2100-2110	(5,270.00)	825.60	(146.40)	22.93	-64.50	-1.79	5,125	107.70
2110-2120	(4,957.00)	882.90	(137.70)	24.53	-56.70	-1.58	4,998	110.70
2120-2130	(4,720.00)	840.00	(131.10)	23.33	-49.90	-1.39	4,317	96.72
2130-2140	(4,538.00)	787.00	(126.10)	21.86	-48.90	-1.36	4,219	100.60
2140-2150	(4,459.00)	709.70	(123.90)	19.71	-49.10	-1.36	3,627	84.47
2150-2160	(4,334.00)	699.80	(120.40)	19.44	-49.30	-1.37	3.403	80.56
2160-2170	(4,326.00)	778.90	(120.20)	21.64	-49.10	-1.36	3,904	103.20
2170-2180	(4,517.00)	777.50	(125.50)	21.60	-50.40	-1.40	3,584	101.60
2180-2190	(4,644.00)	855.40	(129.00)	23.76	-83.00	-2.31	8,955	97.66
2190-2200	(4,734.00)	1,468.00	(131.50)	40.78	-58.60	-1.63	11,234	182.80
2200-2210	(4,819.00)	1,642.00	(133.90)	45.61	-34.40	-0.96	9,250	187.80
2210-2220	(4,340.00)	1,546.00	(120.60)	42.94	-28.50	-0.79	5,005	127.40
2220-2230	(3,899.00)	1,871.00	(108.30)	51.97	-27.20	-0.76	10,174	217.60
2230-2240	(3,283.00)	1,982.00	(91.19)	55.06	-22.50	-0.63	8,221	147.90
2240-2250	(2,180.00)	1,554.00	(60.56)	43.17	-19.30	-0.54	6.441	117.60
2250-2260	(1,844.00)	1,456.00	(51.22)	40.44	-18.00	-0.50	5,250	168.90
2260-2270	(2,956.00)	1,347.00	(82.11)	37.42	-11.50	-0.32	5,293	173.10
2270-2280	(3,557.00)	1,096.00	(98.81)	30.44	-15.00	-0.42	3,897	130.70
2280-2290	(4,013.00)	773.60	(111.50)	21.49	-69.00	-1.92	11,771	77.15
2290-2300	(3,989.00)	587.40	(110.80)	16.32	-54.30	-1.51	13,416	49.97
2300-2310	(3,893.00)	515.40	(108.10)	14.32	-39.20	-1.09	1,909	44.27
2310-2320	(3,843.00)	461.10	(106.70)	12.81	-45.40	-1.26	1,740	43.04
2320-2330	(3,852.00)	438.40	(107.00)	12.18	-52.20	-1.45	1,614	40.94
2330-2340	(3,825.00)	460.10	(106.20)	12.78	-58.50	-1.63	1,659	42.33
2340-2350	(3.860.00)	494.50	(107.20)	13.74	-64.50	-1.79	1,845	48.18
2350-2360	(4,018.00)	584.00	(111.60)	16.22	-72.40	-2.01	1,940	67.50
2360-2370	(4,666.00)	806.90	(129.60)	22.41	-78.50	-2.18	2,221	109.00
2370-2380	(4.994.00)	1,116.00	(138.70)	31.00	-74.40	-2.07	3,475	79.05
2380-2390	(4,337.00)	1,688.00	(120.50)	46.89	-67.10	-1.86	18,456	111.40
2390-2400	(3,515.00)	1,710.00	(97.64)	47.50	-50.60	-1.41	21,140	119.90
Totals	**(225,000.00)**	**40,090.00**	**(6,250.00)**	**1,114.00**	**-54.70**	**-1.52**	**232,691**	**4,562.00**

Table 4.1b: The Volumetric and Elevation Change of the Complete Ice Surface: 1955 to 1959								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	55-59 (m^2) (,000)
1924-1930	(213.70)	57.15	(53.43)	14.29	-2.41	-0.60	655	32.74
1930-1940	(319.20)	84.66	(79.80)	21.17	-9.67	-2.42	915	22.98
1940-1950	(263.00)	82.00	(65.75)	20.50	-13.60	-3.40	1,190	24.22
1950-1960	(202.80)	66.74	(50.70)	16.68	-11.80	-2.95	838	19.91
1960-1970	(196.20)	62.13	(49.05)	15.53	-10.00	-2.50	776	18.82
1970-1980	(173.30)	65.55	(43.33)	16.39	-9.11	-2.28	800	19.39
1980-1990	(157.50)	71.61	(39.38)	17.90	-8.26	-2.07	924	22.13
1990-2000	(146.00)	77.99	(36.50)	19.50	-6.66	-1.66	975	23.68
2000-2010	(118.80)	83.93	(29.70)	20.98	-5.15	-1.29	1,043	25.01
2010-2020	(134.10)	93.29	(33.53)	23.32	-4.58	-1.15	1,167	30.79
2020-2030	(171.80)	109.20	(42.95)	27.30	-4.68	-1.17	1,365	36.77
2030-2040	(223.10)	140.50	(55.78)	35.13	-4.93	-1.23	1,680	44.15
2040-2050	(207.20)	181.20	(51.80)	45.30	-3.79	-0.95	2,403	56.21
2050-2060	(157.20)	182.70	(39.30)	45.68	-3.17	-0.79	2,296	56.15
2060-2070	(132.10)	193.10	(33.03)	48.28	-2.53	-0.63	2,368	57.94
2070-2080	(21.93)	264.20	(5.48)	66.05	-0.33	-0.08	3,433	74.65
2080-2090	301.00	342.70	75.25	85.68	4.39	1.10	4,386	88.50
2090-2100	497.90	265.60	124.50	66.40	5.63	1.41	3,255	83.95
2100-2110	433.60	257.00	108.40	64.25	4.68	1.17	3,270	89.62
2110-2120	330.00	289.30	82.50	72.32	4.23	1.06	3,661	87.80
2120-2130	429.20	250.50	107.30	62.63	6.72	1.68	3,157	75.79
2130-2140	510.90	260.40	127.70	65.10	5.75	1.44	3,366	79.52
2140-2150	507.10	218.10	126.80	54.53	6.66	1.66	2,753	71.53
2150-2160	420.10	221.10	105.00	55.28	5.06	1.27	2,745	75.47
2160-2170	457.10	309.20	114.30	77.30	4.32	1.08	4,052	93.93
2170-2180	435.00	342.60	108.70	85.65	3.57	0.89	4,383	115.40
2180-2190	183.50	336.70	45.88	84.18	1.29	0.32	4,158	122.50
2190-2200	333.60	595.50	83.40	148.90	2.22	0.56	7,866	161.80
2200-2210	299.70	459.20	74.93	114.80	1.56	0.39	5,613	189.80
2210-2220	(967.90)	437.30	(242.00)	109.30	-3.33	-0.83	5,282	197.80
2220-2230	(1,349.00)	636.70	(337.20)	159.20	-6.93	-1.73	8,642	186.60
2230-2240	(1,373.00)	326.80	(343.20)	81.70	-8.84	-2.21	4,012	133.30
2240-2250	(1.987.00)	274.30	(496.70)	68.57	-13.00	-3.25	3,476	125.00
2250-2260	(2,065.00)	728.10	(516.20)	182.00	-12.20	-3.05	9,387	182.70
2260-2270	(897.10)	652.00	(224.30)	163.00	-7.97	-1.99	8,414	174.20
2270-2280	(466.30)	476.30	(116.60)	119.10	-3.83	-0.96	6,428	133.80
2280-2290	(33.36)	274.70	(8.34)	68.68	-0.42	-0.10	3,288	69.85
2290-2300	148.10	141.10	37.03	35.28	4.53	1.13	1,797	42.09
2300-2310	182.60	126.90	45.65	31.73	8.27	2.07	1,596	39.40
2310-2320	215.00	127.60	53.75	31.90	7.62	1.91	1,677	38.54
2320-2330	309.80	145.20	77.45	36.30	5.05	1.26	1,885	43.02
2330-2340	373.80	152.70	93.45	38.18	6.22	1.55	1,981	47.12
2340-2350	247.70	159.40	61.93	39.85	6.13	1.53	2,053	65.04
2350-2360	46.14	268.00	11.54	67.00	3.77	0.94	3,519	95.07
2360-2370	306.00	480.80	76.50	120.20	3.72	0.93	6,534	125.60
2370-2380	394.90	248.50	98.73	62.13	4.05	1.01	2,936	101.70
2380-2390	133.30	181.20	33.33	45.30	4.60	1.15	2,380	62.87
2390-2400	85.75	158.40	21.44	39.60	3.22	0.81	2,045	50.08
Totals	**(4,395.00)**	**11,960.00**	**(1,099.00)**	**2,990.00**	**-0.91**	**-0.23**	**152,825**	**3,815.00**

Table 4.1c: The Volumetric and Elevation Change of the Complete Ice Surface: 1959 to 1965

Elevation Zone (m)	Volumetric Change: Total dV (m^3) (,000)	Total σ (m^3) (,000)	Mean Annual dV (m^3/yr) (,000)	Mean Annual σ (m^3/yr) (,000)	Elevation Change: Total dZ (m)	Annual dZ (m/yr)	Nodes *n*	Mean Area 59-65 (m^2) (,000)
1924-1930	(78.99)	6.81	(13.17)	1.13	-4.76	-0.79	61	7.27
1930-1940	(79.89)	5.06	(13.31)	0.84	-6.02	-1.00	358	8.68
1940-1950	(189.00)	19.01	(31.50)	3.17	-9.07	-1.51	660	24.50
1950-1960	(290.00)	11.81	(48.33)	1.97	-12.10	-2.02	721	18.42
1960-1970	(249.90)	10.70	(41.65)	1.78	-15.30	-2.55	714	15.84
1970-1980	(242.80)	13.74	(40.47)	2.29	-15.60	-2.60	726	18.97
1980-1990	(254.20)	14.31	(42.37)	2.39	-12.60	-2.10	839	20.57
1990-2000	(237.80)	14.32	(39.63)	2.39	-11.60	-1.93	896	21.48
2000-2010	(227.30)	15.83	(37.88)	2.64	-10.60	-1.77	927	23.02
2010-2020	(234.90)	24.90	(39.15)	4.15	-8.05	-1.34	1,274	33.41
2020-2030	(258.80)	27.46	(43.13)	4.58	-6.82	-1.14	1,545	38.95
2030-2040	(223.70)	31.02	(37.28)	5.17	-5.62	-0.94	1,838	43.63
2040-2050	(182.40)	33.20	(30.40)	5.53	-4.55	-0.76	1,901	44.52
2050-2060	(185.10)	41.73	(30.85)	6.96	-3.39	-0.57	2,021	54.48
2060-2070	(221.20)	34.81	(36.87)	5.80	-4.65	-0.78	1,890	46.32
2070-2080	(237.60)	46.97	(39.60)	7.83	-4.20	-0.70	2,257	57.91
2080-2090	(233.20)	47.88	(38.87)	7.98	-3.88	-0.65	2,426	61.16
2090-2100	(238.40)	71.74	(39.73)	11.96	-2.90	-0.48	3,235	85.52
2100-2110	(359.30)	75.96	(59.88)	12.66	-3.95	-0.66	3,646	87.52
2110-2120	(243.30)	60.61	(40.55)	10.10	-3.25	-0.54	3,170	76.57
2120-2130	(182.70)	61.25	(30.45)	10.21	-2.81	-0.47	2,801	69.65
2130-2140	(245.80)	70.03	(40.97)	11.67	-3.18	-0.53	2,857	74.43
2140-2150	(232.30)	55.94	(38.72)	9.32	-3.59	-0.60	2,799	66.03
2150-2160	(153.50)	64.18	(25.58)	10.70	-2.30	-0.38	3,067	72.67
2160-2170	(110.40)	76.75	(18.40)	12.79	-1.33	-0.22	3,253	80.91
2170-2180	(142.20)	112.50	(23.70)	18.75	-1.21	-0.20	4,333	114.00
2180-2190	(108.60)	109.20	(18.10)	18.20	-1.02	-0.17	5,015	116.40
2190-2200	(76.15)	122.70	(12.69)	20.45	-0.62	-0.10	4,512	116.10
2200-2210	65.04	199.10	10.84	33.18	0.37	0.06	8,350	194.80
2210-2220	81.29	227.70	13.55	37.95	0.35	0.06	8,054	206.60
2220-2230	25.22	151.00	4.20	25.17	0.20	0.03	5,043	127.90
2230-2240	131.60	160.10	21.93	26.68	1.05	0.18	5,771	137.70
2240-2250	194.70	173.10	32.45	28.85	1.43	0.24	5,762	141.90
2250-2260	(5.24)	193.60	(0.87)	32.27	-0.07	-0.01	4,843	139.20
2260-2270	(99.52)	154.30	(16.59)	25.72	-0.81	-0.13	5,336	124.50
2270-2280	51.02	130.50	8.50	21.75	0.48	0.08	4,186	100.90
2280-2290	124.30	79.21	20.72	13.20	2.18	0.36	2,317	57.72
2290-2300	91.61	56.19	15.27	9.37	2.40	0.40	1,565	40.14
2300-2310	99.73	46.53	16.62	7.76	3.16	0.53	1,496	35.31
2310-2320	134.50	46.50	22.42	7.75	4.26	0.71	1,350	33.65
2320-2330	143.60	43.61	23.93	7.27	3.96	0.66	1,499	34.12
2330-2340	142.00	68.26	23.67	11.38	2.87	0.48	1,682	45.93
2340-2350	172.80	94.40	28.80	15.73	2.57	0.43	2,827	69.37
2350-2360	255.20	90.53	42.53	15.09	3.76	0.63	3,591	74.88
2360-2370	239.80	154.60	39.97	25.77	2.37	0.40	3,094	93.27
2370-2380	255.90	163.70	42.65	27.28	2.22	0.37	4,983	118.40
2380-2390	251.10	87.97	41.85	14.66	4.38	0.73	2,478	60.05
2390-2400	239.80	72.67	39.97	12.11	5.31	0.89	1,730	45.89
Totals	**(3,125.00)**	**3,674.00**	**(520.80)**	**612.30**	**-0.90**	**-0.15**	**135,699**	**3,381.00**

Table 4.1d: The Volumetric and Elevation Change of the Complete Ice Surface: 1965 to 1967									
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	65-67 (m^2) (,000)
1928-1930	(31.12)	2.48	(15.56)	1.24	-3.91	-1.96	3	1.09
1930-1940	(67.52)	4.62	(33.76)	2.31	-7.86	-3.93	83	5.40
1940-1950	(32.06)	10.94	(16.03)	5.47	-5.78	-2.89	757	16.12
1950-1960	(17.82)	9.83	(8.91)	4.92	-1.23	-0.62	624	15.21
1960-1970	(31.36)	10.85	(15.68)	5.43	-1.99	-1.00	530	13.84
1970-1980	(29.63)	14.24	(14.81)	7.12	-1.68	-0.84	782	18.84
1980-1990	(12.29)	17.48	(6.15)	8.74	-0.65	-0.32	909	22.05
1990-2000	(12.10)	20.72	(6.05)	10.36	-0.45	-0.22	1,062	27.29
2000-2010	(24.47)	22.22	(12.23)	11.11	-0.87	-0.44	1,104	27.54
2010-2020	(16.80)	32.41	(8.40)	16.21	-0.49	-0.25	1,633	40.58
2020-2030	(7.97)	32.05	(3.99)	16.02	-0.17	-0.09	1,594	38.78
2030-2040	6.60	36.52	3.30	18.26	0.14	0.07	1,632	42.08
2040-2050	(23.82)	35.41	(11.91)	17.71	-0.54	-0.27	1,594	39.66
2050-2060	11.35	45.78	5.68	22.89	0.18	0.09	2,268	53.94
2060-2070	42.17	40.83	21.08	20.42	0.96	0.48	1,774	44.10
2070-2080	34.50	57.47	17.25	28.74	0.68	0.34	2,263	59.18
2080-2090	10.06	56.04	5.03	28.02	0.07	0.03	2,425	59.29
2090-2100	34.53	85.87	17.26	42.94	0.41	0.20	3,567	85.79
2100-2110	88.42	88.95	44.21	44.47	1.17	0.59	3,289	80.48
2110-2120	62.91	78.61	31.46	39.31	0.78	0.39	2,919	77.92
2120-2130	5.95	73.26	2.97	36.63	0.18	0.09	2,740	66.33
2130-2140	129.20	82.82	64.60	41.41	1.93	0.97	3,047	71.29
2140-2150	168.70	69.61	84.35	34.81	2.69	1.35	2,481	61.85
2150-2160	134.10	83.76	67.05	41.88	1.79	0.90	2,745	71.86
2160-2170	122.90	91.68	61.45	45.84	1.76	0.88	3,207	75.65
2170-2180	218.50	146.80	109.20	73.40	1.84	0.92	4,844	118.40
2180-2190	191.30	144.00	95.65	72.00	1.67	0.84	4,322	111.60
2190-2200	127.90	170.20	63.95	85.10	0.99	0.50	4,706	125.60
2200-2210	16.94	253.60	8.47	126.80	0.11	0.05	7,156	177.20
2210-2220	(64.64)	311.70	(32.32)	155.90	-0.31	-0.16	8,433	217.50
2220-2230	(40.96)	199.30	(20.48)	99.65	-0.36	-0.18	5,124	127.00
2230-2240	(97.22)	213.10	(48.61)	106.60	-0.73	-0.37	5,080	131.50
2240-2250	(189.00)	237.30	(94.50)	118.60	-1.32	-0.66	5,547	141.50
2250-2260	(186.10)	268.10	(93.05)	134.10	-1.19	-0.60	6,371	158.10
2260-2270	(208.80)	209.80	(104.40)	104.90	-1.67	-0.84	4,659	117.40
2270-2280	(177.00)	170.20	(88.50)	85.10	-1.97	-0.99	3,882	93.28
2280-2290	(80.90)	99.23	(40.45)	49.62	-1.55	-0.78	2,284	52.67
2290-2300	(35.49)	75.35	(17.75)	37.68	-0.82	-0.41	1,615	40.48
2300-2310	(60.14)	62.24	(30.07)	31.12	-1.70	-0.85	1,300	32.97
2310-2320	(50.04)	61.71	(25.02)	30.86	-1.66	-0.83	1,295	32.30
2320-2330	(47.66)	56.11	(23.83)	28.06	-1.82	-0.91	1,220	29.76
2330-2340	(66.76)	97.49	(33.38)	48.75	-1.14	-0.57	1,960	51.16
2340-2350	(23.99)	114.30	(12.00)	57.15	-0.33	-0.17	2,698	59.88
2350-2360	28.97	130.60	14.48	65.30	0.25	0.13	2,443	65.94
2360-2370	22.17	211.40	11.09	105.70	0.29	0.14	4,430	107.10
2370-2380	28.76	235.70	14.38	117.80	0.29	0.14	4,630	117.70
2380-2390	(16.26)	136.20	(8.13)	68.10	-0.24	-0.12	2,536	66.59
2390-2400	(49.05)	92.48	(24.53)	46.24	-1.26	-0.63	1,899	45.59
Totals	**(215.00)**	**4,801.00**	**(107.50)**	**2,401.00**	**-0.06**	**-0.03**	**133,466**	**3,337.00**

Table 4.1e: The Volumetric and Elevation Change of the Complete Ice Surface: 1967 to 1969

Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	67-69 (m^2) (,000)
1928-1930	6.49	3.02	3.25	1.51	2.08	1.04	6	0.10
1930-1940	0.61	5.20	0.31	2.60	1.71	0.86	233	6.01
1940-1950	2.67	7.34	1.33	3.67	1.12	0.56	529	12.26
1950-1960	7.33	7.96	3.67	3.98	0.42	0.21	553	14.57
1960-1970	2.93	11.35	1.46	5.68	-0.10	-0.05	578	14.55
1970-1980	(3.82)	13.61	(1.91)	6.81	-0.41	-0.20	734	17.61
1980-1990	4.27	17.10	2.14	8.55	-0.02	-0.01	846	22.20
1990-2000	13.59	20.87	6.80	10.44	0.56	0.28	1,125	26.91
2000-2010	11.35	23.19	5.68	11.60	0.36	0.18	1,094	28.74
2010-2020	(16.56)	32.54	(8.28)	16.27	-0.45	-0.23	1,616	40.97
2020-2030	(44.27)	33.40	(22.14)	16.70	-1.05	-0.53	1,511	39.77
2030-2040	(53.52)	36.04	(26.76)	18.02	-1.35	-0.68	1,739	40.80
2040-2050	(25.74)	36.57	(12.87)	18.29	-0.51	-0.25	1,573	42.02
2050-2060	(59.64)	43.74	(29.82)	21.87	-1.25	-0.63	2,044	50.26
2060-2070	(46.83)	41.53	(23.42)	20.76	-0.97	-0.48	1,755	44.55
2070-2080	(24.07)	55.78	(12.04)	27.89	-0.48	-0.24	2,475	58.74
2080-2090	(54.75)	60.32	(27.38)	30.16	-0.85	-0.42	2,316	63.24
2090-2100	(81.82)	78.47	(40.91)	39.24	-1.05	-0.53	3,302	77.07
2100-2110	1.79	88.20	0.89	44.10	0.02	0.01	3,154	81.70
2110-2120	(15.42)	81.31	(7.71)	40.65	-0.27	-0.14	3,301	80.03
2120-2130	8.06	68.75	4.03	34.38	0.20	0.10	2,587	63.68
2130-2140	2.69	76.72	1.35	38.36	-0.04	-0.02	2,734	67.93
2140-2150	(31.49)	80.04	(15.75)	40.02	-0.36	-0.18	2,605	70.07
2150-2160	3.63	83.20	1.82	41.60	0.15	0.07	3,165	71.17
2160-2170	45.61	99.77	22.81	49.88	0.56	0.28	3,064	82.36
2170-2180	7.50	143.60	3.75	71.80	0.04	0.02	4,912	116.60
2180-2190	100.10	153.00	50.05	76.50	0.90	0.45	4,754	118.00
2190-2200	65.23	184.70	32.62	92.35	0.47	0.24	5,705	138.80
2200-2210	263.00	268.00	131.50	134.00	1.37	0.69	7,645	188.80
2210-2220	460.80	352.60	230.40	176.30	2.03	1.02	10,344	246.00
2220-2230	314.30	256.30	157.10	128.10	1.78	0.89	5,976	163.60
2230-2240	242.80	245.00	121.40	122.50	1.67	0.84	6,348	153.10
2240-2250	383.60	263.40	191.80	131.70	2.55	1.28	6,479	156.50
2250-2260	468.00	292.00	234.00	146.00	2.77	1.39	7,037	171.80
2260-2270	419.30	237.60	209.60	118.80	3.23	1.62	5,012	135.10
2270-2280	342.20	173.70	171.10	86.85	3.68	1.84	3,757	94.75
2280-2290	182.40	120.70	91.20	60.35	2.56	1.28	2,082	63.48
2290-2300	60.07	81.36	30.04	40.68	1.75	0.88	1,809	43.65
2300-2310	117.30	70.21	58.65	35.11	3.12	1.56	1,535	37.37
2310-2320	124.00	70.58	62.00	35.29	3.53	1.77	1,529	37.21
2320-2330	116.80	68.13	58.40	34.06	2.70	1.35	1,339	35.72
2330-2340	161.10	97.43	80.55	48.72	3.60	1.80	2,281	51.23
2340-2350	188.60	105.90	94.30	52.95	3.21	1.61	2,194	54.53
2350-2360	149.10	150.30	74.55	75.15	1.62	0.81	2,873	77.96
2360-2370	276.60	176.80	138.30	88.40	3.80	1.90	4,184	88.32
2370-2380	457.10	236.60	228.50	118.30	3.65	1.83	4,854	118.40
2380-2390	236.30	188.70	118.10	94.35	2.05	1.02	2,853	90.86
2390-2400	73.51	104.80	36.76	52.40	1.27	0.64	1,879	51.53
Totals	**4,863.00**	**5,147.00**	**2,432.00**	**2,574.00**	**1.38**	**0.69**	**142,020**	**3,551.00**

Table 4.1f: The Volumetric and Elevation Change of the Complete Ice Surface: 1969 to 1971								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 69-71 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
1929-1930	1.82	0.62	0.91	0.31	3.20	1.60	1	0.01
1930-1940	(1.91)	4.69	(0.96)	2.34	0.32	0.16	107	3.31
1940-1950	(27.56)	6.38	(13.78)	3.19	-2.34	-1.17	331	9.57
1950-1960	(59.94)	9.94	(29.97)	4.97	-3.31	-1.66	498	14.31
1960-1970	(91.10)	10.72	(45.55)	5.36	-4.59	-2.30	583	15.37
1970-1980	(92.49)	14.06	(46.24)	7.03	-4.61	-2.31	685	18.16
1980-1990	(85.94)	17.41	(42.97)	8.71	-5.55	-2.78	1,012	23.80
1990-2000	(77.49)	21.78	(38.74)	10.89	-2.99	-1.50	1,125	28.14
2000-2010	(91.84)	27.83	(45.92)	13.92	-2.81	-1.41	1,304	33.25
2010-2020	(96.75)	31.93	(48.38)	15.97	-2.46	-1.23	1,672	40.90
2020-2030	(70.06)	34.96	(35.03)	17.48	-1.75	-0.88	1,655	40.60
2030-2040	(78.42)	35.83	(39.21)	17.92	-1.82	-0.91	1,519	39.79
2040-2050	(101.80)	37.09	(50.90)	18.55	-2.36	-1.18	1,783	44.22
2050-2060	(73.54)	44.40	(36.77)	22.20	-1.64	-0.82	1,951	48.51
2060-2070	(114.60)	43.72	(57.30)	21.86	-2.21	-1.11	1,809	48.07
2070-2080	(158.10)	53.43	(79.05)	26.72	-2.90	-1.45	2,242	57.04
2080-2090	(170.00)	66.81	(85.00)	33.40	-2.53	-1.27	2,731	68.06
2090-2100	(218.60)	77.99	(109.30)	38.99	-2.73	-1.37	2,849	76.35
2100-2110	(246.40)	90.91	(123.20)	45.46	-2.96	-1.48	3,373	83.89
2110-2120	(212.90)	77.87	(106.40)	38.94	-2.82	-1.41	3,137	76.42
2120-2130	(225.30)	70.39	(112.60)	35.20	-3.15	-1.58	2,525	66.31
2130-2140	(259.00)	76.16	(129.50)	38.08	-3.77	-1.89	2,738	67.92
2140-2150	(218.30)	82.17	(109.10)	41.09	-3.43	-1.72	2,999	71.22
2150-2160	(257.50)	84.82	(128.70)	42.41	-3.30	-1.65	2,532	72.26
2160-2170	(366.50)	107.80	(183.20)	53.90	-4.24	-2.12	3,540	89.03
2170-2180	(472.10)	154.60	(236.00)	77.30	-3.69	-1.85	4,493	123.20
2180-2190	(673.20)	162.60	(336.60)	81.30	-5.09	-2.55	4,695	125.20
2190-2200	(755.30)	194.20	(377.60)	97.10	-5.41	-2.70	5,470	143.60
2200-2210	(1,404.00)	333.80	(702.00)	166.90	-5.82	-2.91	7,545	232.00
2210-2220	(1,490.00)	304.90	(745.00)	152.40	-6.83	-3.42	9,413	206.10
2220-2230	(1,053.00)	283.70	(526.50)	141.90	-6.37	-3.19	7,117	177.60
2230-2240	(1,193.00)	254.50	(596.50)	127.20	-7.21	-3.61	5,910	159.60
2240-2250	(1,503.00)	264.10	(751.50)	132.10	-8.71	-4.36	6,026	155.80
2250-2260	(1,443.00)	277.90	(721.50)	138.90	-9.23	-4.61	6,709	159.20
2260-2270	(1,192.00)	234.00	(596.00)	117.00	-8.96	-4.48	5,801	130.90
2270-2280	(964.30)	154.90	(482.10)	77.45	-10.40	-5.20	3,823	82.96
2280-2290	(611.00)	112.80	(305.50)	56.40	-11.30	-5.65	3,008	58.01
2290-2300	(438.20)	77.44	(219.10)	38.72	-10.80	-5.40	1,689	41.42
2300-2310	(438.60)	68.72	(219.30)	34.36	-11.20	-5.60	1,442	36.63
2310-2320	(485.10)	74.97	(242.50)	37.49	-12.00	-6.00	1,458	39.78
2320-2330	(524.80)	73.15	(262.40)	36.58	-12.70	-6.35	1,495	37.95
2330-2340	(578.40)	100.40	(289.20)	50.20	-12.80	-6.40	1,843	52.29
2340-2350	(770.80)	130.20	[illegible].40)	65.10	-11.60	-5.80	2,152	65.95
2350-2360	(944.10)	171.20	[illegible]2.00)	85.60	-11.10	-5.55	3,365	87.84
2360-2370	(1,163.00)	196.30	(581.50)	98.15	-12.30	-6.15	2,873	95.14
2370-2380	(1,492.00)	223.40	(746.00)	111.70	-11.90	-5.95	4,609	109.30
2380-2390	(921.20)	166.30	(460.60)	83.15	-12.60	-6.30	4,418	72.77
2390-2400	(259.90)	80.60	(129.90)	40.30	-13.20	-6.60	2,348	29.63
Totals	**(24,160.00)**	**5,224.00**	**(12,080.00)**	**2,612.00**	**-6.84**	**-3.42**	**142,403**	**3,559.00**

Table 4.1g: The Volumetric and Elevation Change of the Complete Ice Surface: 1971 to 1973								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 71-73 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
1938-1940	(20.79)	5.58	(10.40)	2.79	-1.50	-0.75	6	0.84
1940-1950	(20.80)	3.64	(10.40)	1.82	-3.86	-1.93	203	6.68
1950-1960	(30.78)	6.48	(15.39)	3.24	-3.86	-1.93	434	10.17
1960-1970	(33.17)	9.32	(16.58)	4.66	-1.51	-0.76	517	12.93
1970-1980	(27.02)	11.47	(13.51)	5.74	-1.64	-0.82	699	17.47
1980-1990	(32.66)	17.84	(16.33)	8.92	-2.00	-1.00	902	22.94
1990-2000	(59.75)	22.24	(29.88)	11.12	-2.18	-1.09	1,076	28.47
2000-2010	(67.42)	28.12	(33.71)	14.06	-2.07	-1.03	1,345	33.71
2010-2020	(70.51)	31.33	(35.26)	15.67	-1.86	-0.93	1,621	39.95
2020-2030	(50.41)	33.55	(25.21)	16.77	-1.38	-0.69	1,582	39.42
2030-2040	(59.81)	36.73	(29.91)	18.37	-1.47	-0.74	1,673	41.86
2040-2050	(43.93)	36.71	(21.97)	18.35	-1.05	-0.53	1,745	42.57
2050-2060	(26.69)	42.52	(13.35)	21.26	-0.58	-0.29	1,893	45.12
2060-2070	4.57	44.85	2.29	22.42	0.10	0.05	2,012	50.09
2070-2080	5.30	52 93	2.65	26.47	0.13	0.06	2,296	56.35
2080-2090	33.78	66.85	16.89	33.42	0.53	0.26	2,689	67.05
2090-2100	53.68	79.94	26.84	39.97	0.65	0.32	3,198	79.26
2100-2110	65.48	81.40	32.74	40.70	0.82	0.41	3,275	79.49
2110-2120	97.80	81.15	48.90	40.58	1.37	0.69	2,955	73.40
2120-2130	80.00	71.83	40.00	35.92	1.21	0.61	2,727	69.07
2130-2140	80.73	76.71	40.37	38.36	1.24	0.62	2,676	67.09
2140-2150	74.28	76.68	37.14	38.34	1.06	0.53	2,670	67.31
2150-2160	79.87	91.46	39.94	45.73	1.07	0.54	3,221	78.05
2160-2170	123.00	107.70	61.50	53.85	1.37	0.69	3,558	88.12
2170-2180	206.30	153.20	103.10	76.60	1.85	0.93	5,327	122.10
2180-2190	411.50	161.60	205.70	80.80	3.42	1.71	5,246	125.40
2190-2200	487.70	195.00	243.90	97.50	3.43	1.72	5,913	142.70
2200-2210	969.70	340.20	484.80	170.10	4.34	2.17	10,845	238.80
2210-2220	1,044.00	292.00	522.00	146.00	5.45	2.72	6,986	195.20
2220-2230	695.70	272.10	347.90	136.10	4.11	2.06	6,799	172.80
2230-2240	762.10	264.20	381.00	132.10	5.29	2.65	6,844	161.40
2240-2250	957.80	264.90	478.90	132.40	6.15	3.08	6,303	155.80
2250-2260	943.70	268.90	471.80	134.40	6.19	3.10	5,958	154.60
2260-2270	811.70	217.40	405.90	108.70	6.81	3.41	4,660	121.90
2270-2280	589.40	161.40	294.70	80.70	6.87	3.44	2,789	86.26
2280-2290	322.20	95.19	161.10	47.60	6.07	3.04	1,619	50.28
2290-2300	229.20	75.81	114.60	37.90	6.17	3.09	1,619	40.10
2300-2310	225.30	71.08	112.60	35.54	5.66	2.83	1,483	37.42
2310-2320	242.80	77.17	121.40	38.58	6.41	3.20	1,724	41.34
2320-2330	258.70	74.70	129.40	37.35	6.10	3.05	1,512	39.04
2330-2340	297.10	101.10	148.60	50.55	6.35	3.18	2,351	51.55
2340-2350	467.40	133.20	233.70	66.60	6.34	3.17	3,055	68.70
2350-2360	553.10	174.30	276.50	87.15	6.63	3.32	3,642	89.73
2360-2370	670.70	203.00	335.40	101.50	7.33	3.67	4,707	100.80
2370-2380	857.70	240.20	428.80	120.10	7.20	3.60	4,168	120.50
2380-2390	458.50	131.50	229.20	65.75	6.61	3.31	1,972	60.07
2390-2400	326.10	110.00	163.10	55.00	5.33	2.67	2,099	53.76
Totals	**12,940.00**	**5,195.00**	**6,470.00**	**2,598.00**	**3.69**	**1.84**	**142,594**	**3,548.00**

Table 4.1h: The Volumetric and Elevation Change of the Complete Ice Surface: 1973 to 1975								
	Volumetric Change				Elevation Change			Mean
Elevation	Total		Mean Annual		Total	Annual	Nodes	Area
Zone (m)	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	73-75 (m^2) (,000)
1938-1940	(5.31)	5.16	(2.66)	2.58	-1.55	-0.78	6	0.49
1940-1950	(31.52)	4.46	(15.76)	2.23	-3.64	-1.82	180	6.70
1950-1960	(55.27)	5.86	(27.64)	2.93	-5.35	-2.68	339	9.36
1960-1970	(67.49)	7.95	(33.74)	3.97	-6.18	-3.09	486	12.63
1970-1980	(86.69)	13.38	(43.35)	6.69	-5.36	-2.68	652	17.73
1980-1990	(107.90)	18.61	(53.95)	9.31	-4.57	-2.29	936	23.73
1990-2000	(101.60)	23.39	(50.80)	11.69	-3.79	-1.90	1,197	29.77
2000-2010	(121.30)	28.52	(60.65)	14.26	-3.59	-1.79	1,327	33.93
2010-2020	(122.10)	31.92	(61.05)	15.96	-3.05	-1.53	1,567	39.72
2020-2030	(134.10)	33.03	(67.05)	16.51	-3.55	-1.78	1,580	39.13
2030-2040	(133.50)	36.50	(66.75)	18.25	-3.19	-1.60	1,668	41.81
2040-2050	(121.00)	36.88	(60.50)	18.44	-2.95	-1.48	1,662	41.65
2050-2060	(133.50)	43.35	(66.75)	21.67	-3.00	-1.50	1,749	44.90
2060-2070	(156.00)	45.56	(78.00)	22.78	-3.17	-1.59	2,017	50.47
2070-2080	(166.60)	52.41	(83.30)	26.21	-2.97	-1.49	2,231	57.14
2080-2090	(188.50)	66.55	(94.25)	33.28	-2.85	-1.42	2,674	67.38
2090-2100	(179.90)	79.07	(89.95)	39.54	-2.40	-1.20	3,162	77.57
2100-2110	(153.80)	77.42	(76.90)	38.71	-1.98	-0.99	3,108	76.93
2110-2120	(146.60)	83.93	(73.30)	41.97	-2.03	-1.02	2,907	73 11
2120-2130	(152.00)	74.63	(76.00)	37.31	-2.13	-1.07	2,824	70.84
2130-2140	(149.50)	76.86	(74.75)	38.43	-2.24	-1.12	2,729	67.98
2140-2150	(175.90)	80.69	(87.95)	40.35	-2.39	-1.20	2,759	70.68
2150-2160	(196.60)	90.93	(98.30)	45.47	-2.58	-1.29	3,044	77.42
2160-2170	(221.30)	110.80	(110.60)	55.40	-2.48	-1.24	3,542	89.56
2170-2180	(218.90)	139.10	(109.40)	69.55	-2.03	-1.02	4,499	112.40
2180-2190	(200.00)	154.70	(100.00)	77.35	-1.73	-0.87	4,853	119.30
2190-2200	(252.80)	190.30	(126.40)	95.15	-1.81	-0.91	5,567	142.00
2200-2210	(167.00)	282.30	(83.50)	141.10	-0.90	-0.45	8,368	201.70
2210-2220	(42.24)	313.60	(21.12)	156.80	-0.18	-0.09	8,727	212.70
2220-2230	57.77	273.30	28.89	136.60	0.34	0.17	7,197	176.20
2230-2240	110.30	251.60	55.15	125.80	0.75	0.37	6,246	154.30
2240-2250	111.50	260.70	55.75	130.40	0.73	0.37	6,231	154.60
2250-2260	190.90	269.00	95.45	134.50	1.32	0.66	6,454	157.40
2260-2270	271.20	220.20	135.60	110.10	2.23	1.12	5,136	125.70
2270-2280	222.20	197.80	111.10	98.90	2.09	1.04	4,161	108.60
2280-2290	161.30	122.70	80.65	61.35	2.36	1.18	2,498	65.63
2290-2300	87.69	83.47	43.85	41.74	1.85	0.93	1,660	44.09
2300-2310	46.07	78.10	23.04	39.05	1.16	0.58	1,564	41.12
2310-2320	26.61	78.75	13.31	39.38	0.51	0.26	1,626	41.69
2320-2330	10.71	80.75	5.36	40.38	0.30	0.15	1,681	42.73
2330-2340	(4.50)	90.08	(2.25)	45.04	-0.08	-0.04	1,856	46.30
2340-2350	15.57	118.80	7.79	59.40	0.25	0.12	2,547	61.66
2350-2360	84.81	167.60	42.40	83.80	1.00	0.50	3,610	87.42
2360-2370	87.94	170.10	43.97	85.05	1.08	0.54	3,403	84.56
2370-2380	206.50	253.40	103.20	126.70	1.64	0.82	5,509	131.20
2380-2390	98.91	178.10	49.46	89.05	1.13	0.57	2,919	82.58
2390-2400	40.85	113.70	20.42	56.85	0.65	0.32	2,384	56.06
Totals	**(2,163.00)**	**5,216.00**	**(1,081.00)**	**2,608.00**	**-0.61**	**-0.30**	**143,042**	**3,571.00**

Table 4.1i: The Volumetric and Elevation Change of the Complete Ice Surface: 1975 to 1977								
Elevation	Volumetric Change				Elevation Change			Mean
	Total		Mean Annual		Total	Annual	Nodes	Area
Zone (m)	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	75-77 (m^2) (,000)
1938-1940	6.34	1.14	3.17	0.57	4.83	2.42	12	0.14
1940-1950	25.75	3.77	12.88	1.88	6.28	3.14	164	2.55
1950-1960	32.74	3.82	16.37	1.91	7.10	3.55	307	6.57
1960-1970	62.09	6.88	31.05	3.44	7.13	3.57	365	8.57
1970-1980	79.02	11.07	39.51	5.54	6.50	3.25	644	13.71
1980-1990	117.00	15.86	58.50	7.93	5.18	2.59	929	22.67
1990-2000	122.10	23.36	61.05	11.68	4.09	2.05	1,181	28.81
2000-2010	145.50	27.33	72.75	13.67	4.49	2.24	1,364	33.52
2010-2020	162.60	31.53	81.30	15.77	3.89	1.95	1,601	39.46
2020-2030	154.90	33.12	77.45	16.56	3.96	1.98	1,558	39.31
2030-2040	159.60	36.09	79.80	18.05	3.79	1.90	1,683	42.32
2040-2050	157.10	37.62	78.55	18.81	3.74	1.87	1,676	41.37
2050-2060	151.70	42.52	75.85	21.26	3.24	1.62	1,868	46.65
2060-2070	156.00	47.43	78.00	23.72	3.04	1.52	2,026	51.20
2070-2080	168.80	51.31	84.40	25.66	3.01	1.51	2,314	56.18
2080-2090	212.90	65.95	106.40	32.97	3.10	1.55	2,704	66.41
2090-2100	190.80	76.91	95.40	38.46	2.61	1.30	3,019	76.02
2100-2110	185.80	82.14	92.90	41.07	2.42	1.21	3,043	77.77
2110-2120	146.80	75.81	73.40	37.90	2.03	1.02	2,894	72.21
2120-2130	163.40	80.87	81.70	40.44	2.18	1.09	2,844	71.57
2130-2140	137.00	75.44	68.50	37.72	2.11	1.05	2,728	67.87
2140-2150	163.90	82.29	81.95	41.15	2.36	1.18	2,903	72.49
2150-2160	192.30	90.15	96.15	45.08	2.60	1.30	3,175	76.20
2160-2170	278.90	107.90	139.40	53.95	3.22	1.61	3,669	86.93
2170-2180	346.40	139.00	173.20	69.50	3.03	1.52	4,501	111.80
2180-2190	268.00	161.00	134.00	80.50	2.03	1.02	4,711	124.50
2190-2200	205.70	192.10	102.80	96.05	1.45	0.73	5,820	144.50
2200-2210	253.20	272.90	126.60	136.40	1.31	0.66	7,734	192.70
2210-2220	132.60	324.60	66.30	162.30	0.57	0.29	8,337	220.50
2220-2230	(43.67)	270.10	(21.83)	135.10	-0.28	-0.14	6,876	174.70
2230-2240	(85.50)	247.70	(42.75)	123.80	-0.56	-0.28	6,061	152.90
2240-2250	(45.69)	265.80	(22.84)	132.90	-0.27	-0.14	6,225	157.60
2250-2260	(165.10)	270.30	(82.55)	135.10	-1.00	-0.50	6,134	157.50
2260-2270	(321.30)	232.10	(160.60)	116.10	-2.14	-1.07	4,936	132.30
2270-2280	(292.70)	193.20	(146.40)	96.60	-2.85	-1.42	4,518	105.70
2280-2290	(188.90)	118.70	(94.45)	59.35	-3.05	-1.53	2,736	63.41
2290-2300	(103.00)	82.29	(51.50)	41.15	-2.42	-1.21	1,858	43.43
2300-2310	(44.24)	74.54	(22.12)	37.27	-1.29	-0.65	1,684	39.46
2310-2320	(14.93)	78.32	(7.47)	39.16	-0.34	-0.17	1,676	41.21
2320-2330	11.24	76.29	5.62	38.15	0.32	0.16	1,705	40.49
2330-2340	37.10	87.12	18.55	43.56	0.83	0.42	1,824	45.53
2340-2350	23.81	117.00	11.90	58.50	0.36	0.18	2,312	59.72
2350-2360	16.84	153.10	8.42	76.55	0.24	0.12	3,369	80.02
2360-2370	82.80	168.50	41.40	84.25	1.06	0.53	3,366	84.90
2370-2380	93.70	231.90	46.85	115.90	0.89	0.45	4,986	117.90
2380-2390	131.70	208.50	65.85	104.20	1.32	0.66	3,722	100.30
2390-2400	47.48	116.70	23.74	58.35	0.57	0.29	2,373	57.84
Totals	**3,719.00**	**5,192.00**	**1,860.00**	**2,596.00**	**1.06**	**0.53**	**142,135**	**3,549.00**

Table 4.1j: The Volumetric and Elevation Change of the Complete Ice Surface: 1977 to 1979								
	Volumetric Change				**Elevation Change**			**Mean**
Elevation	**Total**		**Mean Annual**		**Total**	**Annual**	**Nodes**	**Area**
Zone (m)	**dV (m^3) (,000)**	**σ (m^3) (,000)**	**dV (m^3/yr) (,000)**	**σ (m^3/yr) (,000)**	**dZ (m)**	**dZ (m/yr)**	***n***	**77-79 (m^2) (,000)**
1944-1950	(9.88)	4.64	(4.94)	2.32	-1.59	-0.80	50	2.00
1950-1960	(11.46)	3.85	(5.73)	1.92	-2.37	-1.18	275	6.97
1960-1970	(21.71)	7.33	(10.85)	3.67	-1.75	-0.88	376	9.76
1970-1980	(26.31)	10.36	(13.16)	5.18	-1.84	-0.92	550	14.54
1980-1990	(36.59)	16.67	(18.30)	8.34	-1.85	-0.93	953	23.56
1990-2000	(29.24)	22.93	(14.62)	11.47	-1.03	-0.52	1,139	28.04
2000-2010	(37.04)	27.22	(18.52)	13.61	-1.15	-0.58	1,311	33.30
2010-2020	(43.23)	31.29	(21.62)	15.65	-1.14	-0.57	1,582	39.72
2020-2030	(36.48)	33.89	(18.24)	16.95	-0.93	-0.47	1,625	40.38
2030-2040	(30.95)	36.94	(15.48)	18.47	-0.75	-0.38	1,725	42.29
2040-2050	(20.84)	37.03	(10.42)	18.51	-0.49	-0.25	1,625	41.29
2050-2060	(23.15)	40.86	(11.58)	20.43	-0.53	-0.27	1,859	45.83
2060-2070	(19.62)	48.57	(9.81)	24.29	-0.34	-0.17	2,053	51.54
2070-2080	(27.49)	48.36	(13.75)	24.18	-0.50	-0.25	2,153	54.26
2080-2090	(34.01)	65.65	(17.00)	32.83	-0.60	-0.30	2,583	64.85
2090-2100	(43.05)	77.46	(21.53)	38.73	-0.51	-0.25	3,030	77.14
2100-2110	(82.93)	79.33	(41.47)	39.67	-1.01	-0.51	3,116	78.59
2110-2120	(43.13)	73.56	(21.57)	36.78	-0.67	-0.33	2,857	69.64
2120-2130	(38.68)	79.19	(19.34)	39.60	-0.58	-0.29	2,833	70.41
2130-2140	(28.26)	74.36	(14.13)	37.18	-0.44	-0.22	2,694	67.44
2140-2150	(37.28)	82.13	(18.64)	41.06	-0.51	-0.25	2,895	72.33
2150-2160	(15.25)	86.08	(7.63)	43.04	-0.22	-0.11	2,916	72.78
2160-2170	(49.70)	104.20	(24.85)	52.10	-0.59	-0.30	3,293	83.83
2170-2180	(71.03)	139.20	(35.52)	69.60	-0.64	-0.32	4,450	112.10
2180-2190	(68.64)	165.80	(34.32)	82.90	-0.54	-0.27	5,220	129.20
2190-2200	(28.49)	191.20	(14.25)	95.60	-0.19	-0.09	5,727	143.60
2200-2210	(146.90)	281.00	(73.45)	140.50	-0.75	-0.38	7,633	199.40
2210-2220	(144.70)	332.90	(72.35)	166.40	-0.67	-0.33	9,315	226.00
2220-2230	(56.76)	277.00	(28.38)	138.50	-0.28	-0.14	7,117	178.30
2230-2240	(60.69)	254.80	(30.34)	127.40	-0.37	-0.19	6,278	156.70
2240-2250	(122.60)	275.90	(61.30)	137.90	-0.76	-0.38	6,534	163.50
2250-2260	(72.46)	276.90	(36.23)	138.40	-0.46	-0.23	6,515	160.80
2260-2270	(9.15)	241.80	(4.57)	120.90	-0.08	-0.04	5,649	138.10
2270-2280	7.63	181.10	3.81	90.55	0.06	0.03	3,947	98.83
2280-2290	3.00	110.00	1.50	55.00	0.08	0.04	2,346	58.71
2290-2300	(2.56)	78.15	(1.28)	39.08	-0.03	-0.02	1,634	41.36
2300-2310	(0.27)	66.62	(0.14)	33.31	0.09	0.05	1,473	35.54
2310-2320	33.90	74.59	16.95	37.29	0.94	0.47	1,616	39.13
2320-2330	30.40	74.47	15.20	37.24	0.79	0.39	1,531	39.93
2330-2340	13.75	88.38	6.88	44.19	0.19	0.10	1,836	45.85
2340-2350	9.02	118.60	4.51	59.30	0.20	0.10	2,443	61.15
2350-2360	14.86	147.80	7.43	73.90	0.17	0.08	3,025	76.92
2360-2370	(52.25)	178.80	(26.13)	89.40	-0.57	-0.28	3,421	89.93
2370-2380	(246.60)	245.80	(123.30)	122.90	-1.73	-0.87	4,423	124.90
2380-2390	(190.40)	193.20	(95.20)	96.60	-2.25	-1.13	4,256	91.51
2390-2400	(45.42)	111.80	(22.71)	55.90	-1.08	-0.54	2,530	55.42
Totals	**(1,953.00)**	**5,198.00**	**(976.50)**	**2,599.00**	**-0.55**	**-0.28**	**142,412**	**3,557.00**

Table 4.1k: The Volumetric and Elevation Differences of the Complete Ice Surface: 1977a,t					
	Volumetric Change		Elevation Change		Mean
Elevation	Total		Total	Nodes	Area
Zone (m)	dV (m^3) (,000)	σ (m^3) (,000)	dZ (m)	*n*	1977a,t (m^2) (,000)
1940-1950	32.57	3.08	13.80	50	0.64
1950-1960	57.93	13.39	10.50	275	3.50
1960-1970	45.37	34.41	6.19	376	10.26
1970-1980	(2.07)	60.96	1.02	550	19.38
1980-1990	(65.19)	67.85	-2.07	953	24.29
1990-2000	(45.61)	80.46	-1.91	1,139	26.99
2000-2010	(45.11)	96.71	-1.49	1,311	33.30
2010-2020	(19.16)	108.40	-0.54	1,582	37.49
2020-2030	(14.30)	121.50	-0.55	1,625	41.54
2030-2040	7.09	123.10	0.41	1,725	41.53
2040-2050	(13.35)	125.10	-0.17	1,625	42.10
2050-2060	12.03	131.70	0.11	1,859	44.75
2060-2070	17.30	151.60	0.32	2,053	51.68
2070-2080	(7.19)	179.80	0.12	2,153	56.66
2080-2090	(108.80)	207.30	-1.31	2,583	68.54
2090-2100	(82.43)	233.10	-1.00	3,030	76.91
2100-2110	(100.50)	193.70	-1.56	3,116	71.46
2110-2120	88.88	178.00	1.63	2,857	63.96
2120-2130	92.88	222.60	0.99	2,833	73.87
2130-2140	64.76	217.40	1.02	2,694	69.38
2140-2150	(9.55)	218.00	-0.21	2,895	74.29
2150-2160	15.08	218.10	0.41	2,916	71.83
2160-2170	(83.14)	282.10	-0.89	3,293	90.43
2170-2180	(117.90)	350.50	-0.95	4,450	111.40
2180-2190	(382.40)	483.90	-2.32	5,220	150.80
2190-2200	(607.80)	535.60	-3.58	5,727	162.00
2200-2210	(1,093.00)	534.80	-5.01	7,633	185.20
2210-2220	(436.40)	604.20	-2.99	9,315	209.70
2220-2230	(748.60)	685.30	-3.11	7,117	207.10
2230-2240	(918.30)	500.30	-5.58	6,278	161.20
2240-2250	(1,042.00)	473.80	-6.28	6,534	158.70
2250-2260	(697.10)	420.40	-5.34	6,515	145.50
2260-2270	(577.80)	420.30	-4.19	5,649	139.70
2270-2280	(367.70)	236.90	-4.98	3,947	84.36
2280-2290	(210.80)	163.10	-3.42	2,346	53.54
2290-2300	(160.00)	130.40	-3.71	1,634	41.48
2300-2310	(196.70)	115.70	-4.02	1,473	37.94
2310-2320	(172.60)	118.50	-6.04	1,616	38.69
2320-2330	(197.20)	138.10	-4.48	1,531	42.33
2330-2340	(260.00)	158.80	-5.33	1,836	49.02
2340-2350	(312.30)	196.30	-4.29	2,443	61.89
2350-2360	(365.70)	278.20	-3.06	3,025	85.41
2360-2370	(564.40)	299.10	-4.98	3,421	90.69
2370-2380	(553.70)	302.70	-6.19	4,423	102.50
2380-2390	(254.70)	260.00	-3.19	4,256	90.10
2390-2400	(17.30)	133.00	-4.54	2,530	49.50
Totals	**(10,420.00)**	**10,810.00**	**-2.94**	**142,412**	**3,554.00**

Table 4.2a: The Volumetric and Elevation Change of the Complete Ice Surface: 1919 to 1955								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 19-55 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(38,962.0)	3,770.1	(1,082.2)	104.7	-17.6	-0.49	12,335	644.6
2000-2050	(33,576.0)	1,738.7	(932.7)	48.3	-87.1	-2.42	10,137	248.2
2050-2100	(30,297.0)	3,830.9	(841.6)	106.4	-106.0	-2.94	22,136	499.7
2100-2150	(23,944.0)	4,045.2	(665.2)	112.4	-54.5	-1.51	22,286	500.2
2150-2200	(22,555.0)	4,579.6	(626.6)	127.2	-62.5	-1.74	31,080	565.8
2200-2250	(18,521.0)	8,595.0	(514.6)	238.8	-26.8	-0.74	39,091	798.3
2250-2300	(16,359.0)	5,260.0	(454.4)	146.1	-44.3	-1.23	39,627	599.8
2300-2350	(19,273.0)	2,369.5	(535.2)	65.8	-51.8	-1.44	8,767	218.8
2350-2400	(21,530.0)	5,904.9	(598.0)	164.0	-61.0	-1.69	47,232	486.9
Totals	**(225,000.0)**	**40,090.0**	**(6,250.0)**	**1,114.0**	**-54.7**	**-1.52**	**232,691**	**4,562.0**

Table 4.2b: The Volumetric and Elevation Change of the Complete Ice Surface: 1955 to 1959								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 55-59 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(1,671.7)	567.8	(417.9)	142.0	-9.28	-2.32	7,073	183.9
2000-2050	(855.0)	608.1	(213.8)	152.0	-4.50	-1.13	7,658	192.9
2050-2100	487.7	1,248.3	121.9	312.1	1.47	0.37	15,738	361.2
2100-2150	2,210.8	1,275.3	552.7	318.8	5.53	1.38	16,207	404.3
2150-2200	1,829.3	1,805.1	457.3	451.3	3.01	0.75	23,204	569.1
2200-2250	(5,377.2)	2,134.3	(1,344.3)	533.6	-5.53	-1.38	27,025	832.5
2250-2300	(3,313.7)	2,272.2	(828.4)	568.1	-6.80	-1.70	29,314	602.6
2300-2350	1,328.9	711.8	332.2	178.0	6.57	1.64	9,192	233.1
2350-2400	966.1	1,336.9	241.5	334.2	3.85	0.96	17,414	435.3
Totals	**(4,395.0)**	**11,960.0**	**(1,099.0)**	**2,990.0**	**-0.91**	**-0.23**	**152,825**	**3,815.0**

Table 4.2c: The Volumetric and Elevation Change of the Complete Ice Surface: 1959 to 1965								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 59-65 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(1,622.6)	95.8	(270.4)	16.0	-12.10	-2.02	4,975	135.7
2000-2050	(1,127.1)	132.4	(187.8)	22.1	-6.63	-1.10	7,485	183.5
2050-2100	(1,115.5)	243.1	(185.9)	40.5	-3.71	-0.62	11,829	305.4
2100-2150	(1,263.4)	323.8	(210.6)	54.0	-3.39	-0.57	15,273	374.2
2150-2200	(590.9)	485.3	(98.5)	80.9	-1.22	-0.20	20,180	500.1
2200-2250	497.9	911.0	83.0	151.8	0.64	0.11	32,980	808.9
2250-2300	162.2	613.8	27.0	102.3	0.34	0.06	18,247	462.5
2300-2350	692.6	299.3	115.4	49.9	3.22	0.54	8,854	218.4
2350-2400	1,241.8	569.5	207.0	94.9	3.27	0.55	15,876	392.5
Totals	**(3,125.0)**	**3,674.0**	**(520.8)**	**612.3**	**-0.90**	**-0.15**	**135,699**	**3,381.0**

Table 4.2d: The Volumetric and Elevation Change of the Complete Ice Surface: 1965 to 1967								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	65-67 (m^2) (,000)
Term.-2000	(233.9)	91.2	(116.9)	45.6	-1.95	-0.98	4,750	119.8
2000-2050	(66.5)	158.6	(33.2)	79.3	-0.35	-0.18	7,557	188.6
2050-2100	132.6	286.0	66.3	143.0	0.43	0.21	12,297	302.3
2100-2150	455.2	393.3	227.6	196.6	1.32	0.66	14,476	357.9
2150-2200	794.7	636.4	397.3	318.2	1.58	0.79	19,824	503.1
2200-2250	(374.9)	1,215.0	(187.4)	607.6	-0.47	-0.24	31,340	794.7
2250-2300	(688.3)	822.7	(344.3)	411.4	-1.48	-0.74	18,811	461.9
2300-2350	(248.6)	391.9	(124.3)	195.9	-1.15	-0.58	8,473	206.1
2350-2400	14.6	806.4	7.3	403.1	0.01	0.01	15,938	402.9
Totals	**(215.0)**	**4,801.0**	**(107.5)**	**2,401.0**	**-0.06**	**-0.03**	**133,466**	**3,337.0**

Table 4.2e: The Volumetric and Elevation Change of the Complete Ice Surface: 1967 to 1969								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	67-69 (m^2) (,000)
Term.-2000	34.1	86.4	17.0	43.2	0.33	0.16	4,604	114.2
2000-2050	(128.7)	161.7	(64.4)	80.9	-0.67	-0.34	7,533	192.3
2050-2100	(267.1)	279.8	(133.6)	139.9	-0.91	-0.46	11,892	293.9
2100-2150	(34.4)	395.0	(17.2)	197.5	-0.09	-0.05	14,381	363.4
2150-2200	222.1	664.3	111.0	332.1	0.43	0.22	21,600	526.9
2200-2250	1,664.5	1,385.3	832.2	692.6	1.88	0.94	36,792	908.0
2250-2300	1,472.0	905.4	735.9	452.7	2.94	1.47	19,697	508.8
2300-2350	707.8	412.3	353.9	206.1	3.27	1.64	8,878	216.1
2350-2400	1,192.6	857.2	596.2	428.6	2.79	1.40	16,643	427.1
Totals	**4,863.0**	**5,147.0**	**2,432.0**	**2,574.0**	**1.38**	**0.69**	**142,020**	**3,551.0**

Table 4.2f: The Volumetric and Elevation Change of the Complete Ice Surface: 1969 to 1971								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	69-71 (m^2) (,000)
Term.-2000	(434.6)	85.6	(217.3)	42.8	-3.96	-1.98	4,342	112.7
2000-2050	(438.9)	167.6	(219.4)	83.8	-2.22	-1.11	7,933	198.8
2050-2100	(734.8)	286.4	(367.4)	143.2	-2.45	-1.23	11,582	298.0
2100-2150	(1,161.9)	397.5	(580.8)	198.8	-3.21	-1.61	14,772	365.8
2150-2200	(2,524.6)	704.0	(1,262.1)	352.0	-4.51	-2.25	20,730	553.3
2200-2250	(6,643.0)	1,441.0	(3,321.5)	720.5	-6.90	-3.45	36,011	931.1
2250-2300	(4,648.5)	857.0	(2,324.2)	428.5	-9.79	-4.90	21,030	472.5
2300-2350	(2,797.7)	447.4	(1,398.8)	223.7	-12.10	-6.05	8,390	232.6
2350-2400	(4,780.2)	837.8	(2,390.0)	418.9	-12.20	-6.10	17,613	394.7
Totals	**(24,160.0)**	**5,224.0**	**(12,080.0)**	**2,612.0**	**-6.84**	**-3.42**	**142,403**	**3,559.0**

Table 4.2g: The Volumetric and Elevation Change of the Complete Ice Surface: 1971 to 1973								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 71-73 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(225.0)	76.6	(112.5)	38.3	-2.23	-1.12	3,837	99.5
2000-2050	(292.1)	166.4	(146.1)	83.2	-1.54	-0.77	7,966	197.5
2050-2100	70.6	287.1	35.3	143.5	0.24	0.12	12,088	297.9
2100-2150	398.3	387.8	199.2	193.9	1.13	0.57	14,303	356.4
2150-2200	1,308.4	709.0	654.2	354.5	2.42	1.21	23,265	556.4
2200-2250	4,429.3	1,433.4	2,214.6	716.7	4.98	2.49	37,777	924.0
2250-2300	2,896.2	818.7	1,448.1	409.3	6.46	3.23	16,645	453.1
2300-2350	1,491.3	457.3	745.7	228.6	6.22	3.11	10,125	238.1
2350-2400	2,866.1	859.0	1,433.0	429.5	6.80	3.40	16,588	424.9
Totals	**12,940.0**	**5,195.0**	**6,470.0**	**2,598.0**	**3.69**	**1.84**	**142,594**	**3,548.0**

Table 4.2h: The Volumetric and Elevation Change of the Complete Ice Surface: 1973 to 1975								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 73-75 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(455.8)	78.8	(227.9)	39.4	-4.69	-2.35	3,796	100.4
2000-2050	(632.0)	166.9	(316.0)	83.4	-3.25	-1.63	7,804	196.2
2050-2100	(824.5)	286.9	(412.3)	143.5	-2.83	-1.41	11,833	297.5
2100-2150	(777.8)	393.5	(388.9)	196.8	-2.15	-1.07	14,327	359.5
2150-2200	(1,089.6)	685.8	(544.7)	342.9	-2.06	-1.03	21,505	540.7
2200-2250	70.3	1,381.5	35.2	690.7	0.07	0.04	36,769	899.5
2250-2300	933.3	893.2	466.7	446.6	1.89	0.95	19,909	501.4
2300-2350	94.5	446.5	47.2	223.3	0.39	0.20	9,274	233.5
2350-2400	519.0	882.9	259.5	441.5	1.19	0.60	17,825	441.8
Totals	**(2,163.0)**	**5,216.0**	**(1,081.0)**	**2,608.0**	**-0.61**	**-0.30**	**143,042**	**3,571.0**

Table 4.2i: The Volumetric and Elevation Change of the Complete Ice Surface: 1975 to 1977								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 75-77 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	445.0	65.9	222.5	32.9	5.47	2.73	3,602	83.0
2000-2050	779.7	165.7	389.9	82.9	3.95	1.97	7,882	196.0
2050-2100	880.2	284.2	440.1	142.1	2.97	1.49	11,931	296.5
2100-2150	796.9	396.6	398.5	198.3	2.22	1.11	14,412	361.9
2150-2200	1,291.3	690.2	645.6	345.1	2.36	1.18	21,876	543.9
2200-2250	210.9	1,381.1	105.5	690.5	0.22	0.11	35,233	898.4
2250-2300	(1,071.0)	896.6	(535.5)	448.3	-2.10	-1.05	20,182	502.3
2300-2350	13.0	433.3	6.5	216.6	0.02	0.01	9,201	226.4
2350-2400	372.5	878.7	186.3	439.3	0.85	0.42	17,816	441.0
Totals	**3,719.0**	**5,192.0**	**1,860.0**	**2,596.0**	**1.06**	**0.53**	**142,135**	**3,549.0**

Table 4.2j: The Volumetric and Elevation Change of the Complete Ice Surface: 1977 to 1979								
Elevation Zone (m)	Volumetric Change				Elevation Change			Mean Area 77-79 (m^2) (,000)
	Total		Mean Annual		Total	Annual	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dV (m^3/yr) (,000)	σ (m^3/yr) (,000)	dZ (m)	dZ (m/yr)	*n*	
Term.-2000	(135.2)	65.8	(67.6)	32.9	-1.60	-0.80	3,343	84.9
2000-2050	(168.5)	166.4	(84.3)	83.2	-0.88	-0.44	7,868	197.0
2050-2100	(147.3)	280.9	(73.7)	140.5	-0.50	-0.25	11,678	293.6
2100-2150	(230.3)	388.6	(115.2)	194.3	-0.65	-0.33	14,395	358.4
2150-2200	(233.1)	686.5	(116.6)	343.2	-0.43	-0.22	21,606	541.5
2200-2250	(531.7)	1,421.6	(265.8)	710.7	-0.58	-0.29	36,877	923.9
2250-2300	(73.5)	888.0	(36.8)	443.9	-0.15	-0.08	20,091	497.8
2300-2350	86.8	422.7	43.4	211.3	0.42	0.21	8,899	221.6
2350-2400	(519.8)	877.4	(259.9)	438.7	-1.21	-0.61	17,655	438.7
Totals	**(1,953.0)**	**5,198.0**	**(976.5)**	**2,599.0**	**-0.55**	**-0.28**	**142,412**	**3,557.0**

Table 4.2k: The Volumetric and Elevation Differences of the Complete Ice Surface: 1977a,t					
Elevation Zone (m)	Volumetric Change		Elevation Change		Mean Area 1977a,t (m^2) (,000)
	Total		Total	Nodes	
	dV (m^3) (,000)	σ (m^3) (,000)	dZ (m)	*n*	
Term.-2000	23.0	260.3	0.69	3,343	85.1
2000-2050	(84.8)	574.8	-0.42	7,868	196.0
2050-2100	(169.1)	903.5	-0.45	11,678	298.5
2100-2150	136.5	1,029.7	0.33	14,395	353.0
2150-2200	(1,176.2)	1,870.2	-1.79	21,606	586.5
2200-2250	(4,238.3)	2,798.4	-4.46	36,877	921.9
2250-2300	(2,013.4)	1,371.1	-4.59	20,091	464.6
2300-2350	(1,138.8)	727.4	-4.81	8,899	229.9
2350-2400	(1,755.8)	1,273.0	-4.46	17,655	418.2
Totals	**(10,420.0)**	**10,810.0**	**-2.94**	**142,412**	**3,554.0**

the 1955 map shows the flat area of Sunwapta Pass.

Figure 4.1 shows that substantial portions of the glacier often have a measured elevation change that is less than one standard deviation from the mean of measurements based on the precision figures which were discussed in section 3.3.3.6, suggesting that the variability of measurement was greater than the magnitude of change. When elevation change DEMs based on large-scale maps are compared (figures 4.1*c-j*), it can be seen that these zones usually cross the width of the glacier and tend to separate zones of downwasting and upgrading.

4.1.1.2 Borders

As was noted in chapter two, the maps from 1959 and 1965 did not contour the complete glacier surface. The 1959 map ignored part of the surface from the terminus to 2040 m. The 1965 map left uncontoured part of the area from 2130-2400 m. As a result, borders representing the glacier in those years were truncated to match the contoured areas. This results in measured area and volumetric values for the intervals 1955-1959, 1959-1965, 1965-1967 that do not represent the entire area and volumetric values for those intervals.

To produce estimates of volumetric change for the entire glacier surface from values which measure volumetric change over a truncated surface, it is necessary to assume that for a given elevation zone

$$\frac{\Delta V_M}{A_M} \propto \frac{\Delta V_E}{A_E}$$

where ΔV_M = total measured volumetric change of the truncated elevation zone
ΔV_E = estimated additional volumetric change of the elevation zone
A_M = total measured area of the truncated elevation zone
A_E = additional area of the elevation zone

Thus, to produce an estimate of volumetric change over the entire glacier surface, it is necessary to estimate the amount of area in each non-truncated elevation zone.

For each of 1959 and 1965, borders of the complete ice surface were produced from comparison with the non-truncated 1955 and 1967 borders. The non-truncated borders of the glacier in 1959 and 1965 were compared with the DEMs from 1955 and 1967, respectively. From this comparison, the areas in each of the non-truncated elevation zones were estimated.

Due to the heterogenous nature of change over elevation, values of estimated additional volumetric change were produced for 10 m elevation zones over the truncated portions of the glacier surface. The total estimated volumetric change for the non-truncated map is the sum of these values over the entire uncontoured portion of the map.

$$\Delta V_e = \frac{k \cdot \Delta V_m \cdot A_e}{A_m}$$

where ΔV_m = total measured volumetric change of the truncated elevation zone
ΔV_e = estimated additional volumetric change of the elevation zone
A_m = total measured area of the truncated elevation zone
A_e = additional area of the elevation zone
k = scaling constant

Values of k were produced for other intervals by calculating the quantity

$$k = \left(\frac{\Delta V_e}{\Delta V_m}\right)\left(\frac{A_m}{A_e}\right)$$

for the intervals 1969-71, 1971-73, 1973-75, 1975-77 and 1977-79. These comparisons demonstrated that $k = 1$ was a reasonable value for that constant.

Table 4.3 shows the results of these calculations.

Table 4.3: Estimated Volumetric Change for Partially Contoured Elevation Zones

		Area (m^2) (,000)		Volumetric Change (m^3) (,000)	
Interval	Elevation Zone	Measured	Estimated	Measured	Estimated
1955-59	Terminus-2040 m	321	472	-2,319	-3,430
1959-65	Terminus-2040 m	275	439	-2,567	-4,259
	2130-2400 m	2,523	2,780	1,525	9,338
1965-67	2130-2400 m	2,502	2,755	-205	-304

The magnitude of additional estimated volumetric change is directly proportional to the magnitude of volumetric change measured over the contoured surface and the estimated additional area. The two intervals including 1965 show the same relative addition to area in the 2130-2400 m zone because the same proportion of the glacier surface was removed in both intervals from the 1965 map. Estimates of additional volumetric change for this region should be regarded as relatively good, since the magnitude of change in the corresponding contoured region does not show substantial cross-glacier variation (figures 4.1*cd*).

The estimates of volumetric change made to account for the uncontoured portion of the 1959 map should be looked upon as being less certain. Change at the terminus of the glacier from 1955 to 1965 was substantial. As discussed above, the debris-covered section is unlikely to display the same melting characteristics as the clear-ice portions. When the change is of the magnitude displayed in the contoured areas, the estimate can only be more uncertain. Tables 4.4 and 4.5 are modified to reflect the estimates of additional area and volume presented in table 4.3. Tables 4.1 and 4.2 (above) present results unmodified by border estimates.

4.1.1.3 Terminus Position

The former extent of the glacier and its rate of recession were determined by Luckman (1988) using old photos, dendrochronology and by dating recessional moraines. The terminal positions of the clear-ice portions of the glacier in this study were determined solely from the terminus positions marked on the maps. The position of the debris-covered portion of the terminus is more open to question, as discussed in section 3.3.1.2. It was necessary to estimate the ice-front position of that section of the terminus since it was marked on only one map. In addition, the western lateral border between the flowing and stagnant debris-covered ice is inexact.

When the terminal positions are compared (figure 4.2), a portion of the 1977 terminus is portrayed as being slightly (10-15 m) in advance of the 1975 terminus. Field observations at the time (Luckman, 1988) indicated that in fact retreat occurred between 1975 and 1977. This discrepancy may be because the photos for the 1977 map were taken two weeks earlier than those for 1975. Alternately, the slight distance between the 1975 and 1977 terminal positions may be the result of the horizontal difference between the maps being less than the accuracy of horizontal position estimation.

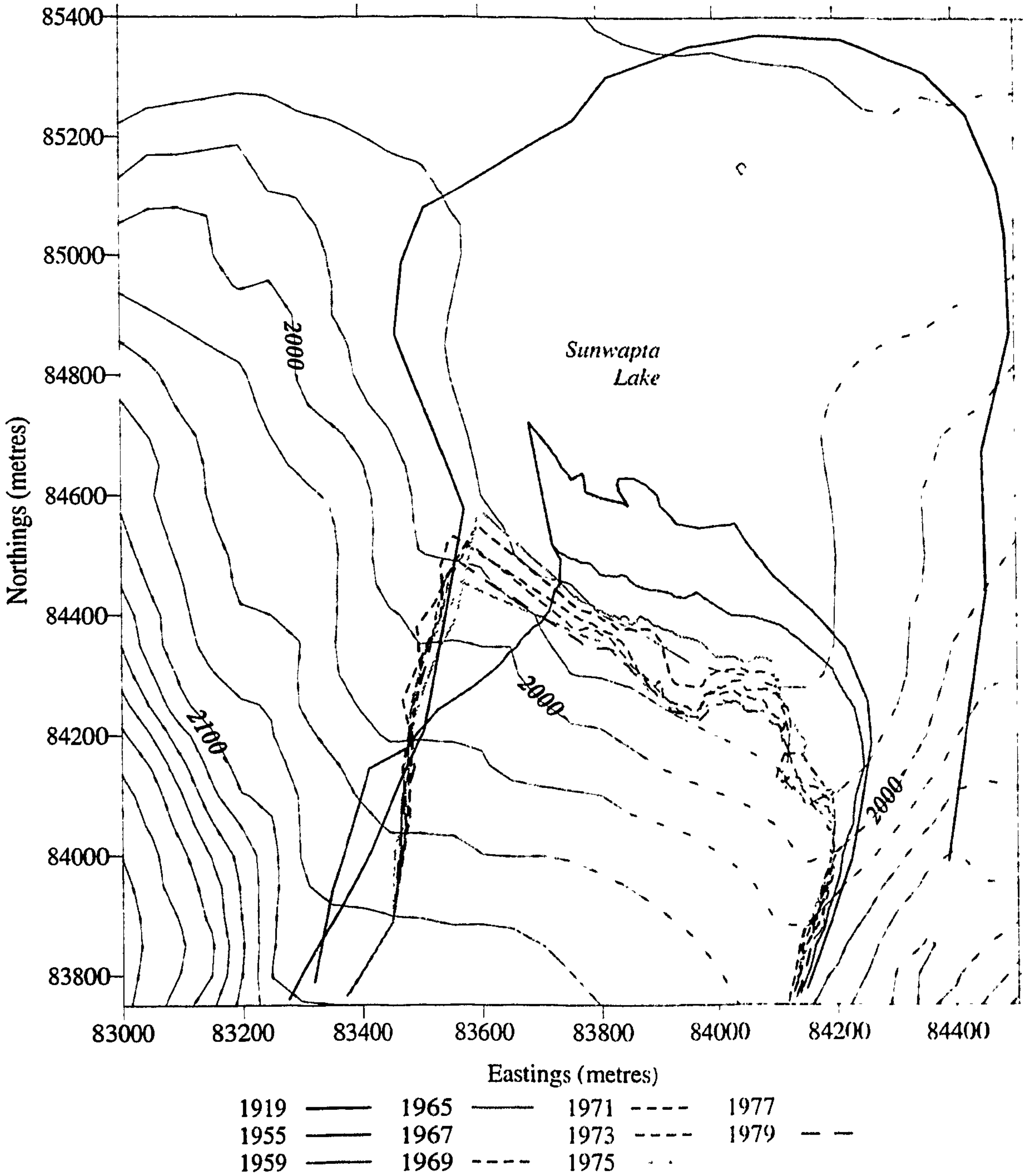
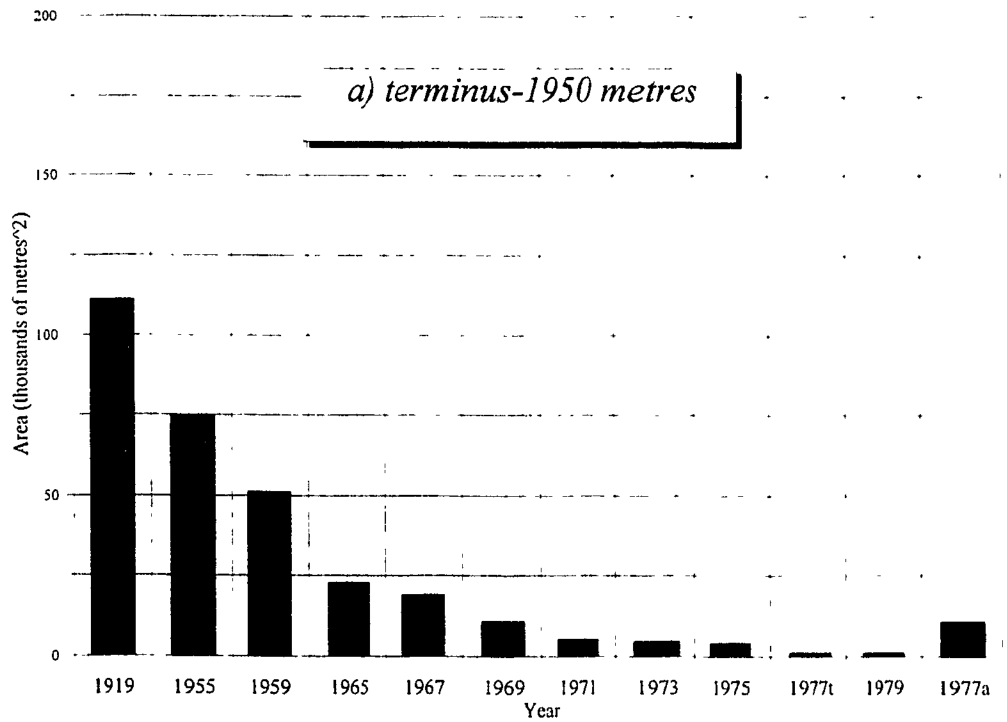


Figure 4.2: Terminus positions of glacier, 1919-1979. Termini digitized from maps of glacier. Thus, clear ice portions of ice are more precise; the position of the debris-covered portion of the terminus is estimated from the maps. The 1977 terminus is in advance of the 1975 terminus, but visual observation of the terminus in that time did not reveal any advance at that time (Luckman, 1988).

Table 4.4: Area in each Elevation Zone for Complete Ice Surface (thousands of m^2)

	1919	1955	1959	1965	1967	1969	1971	1973	1975	1977t	1979	1977a
Terminus-1950	111	75	51	23	19	11	5	5	4	1	1	11
1950-2000	197	154	173	97	96	97	91	90	86	82	82	117
2000-2050	254	217	247	189	188	198	199	195	197	197	195	181
2050-2100	553	394	295	302	297	290	302	296	298	292	296	312
2100-2150	557	405	381	366	360	369	357	358	360	360	361	341
2150-2200	525	580	505	529	540	518	582	538	547	540	538	596
2200-2250	977	676	825	908	920	901	944	919	881	922	921	824
2250-2300	486	733	456	509	492	525	416	498	504	502	494	397
2300-2350	219	230	221	221	222	210	253	232	230	223	219	217
2350-2400	551	435	397	446	416	438	413	443	443	439	446	335
Totals	**4,431**	**3,899**	**3,551**	**3,590**	**3,550**	**3,558**	**3,563**	**3,574**	**3,551**	**3,559**	**3,554**	**3,331**

Figure 4.3: Area of Glacier



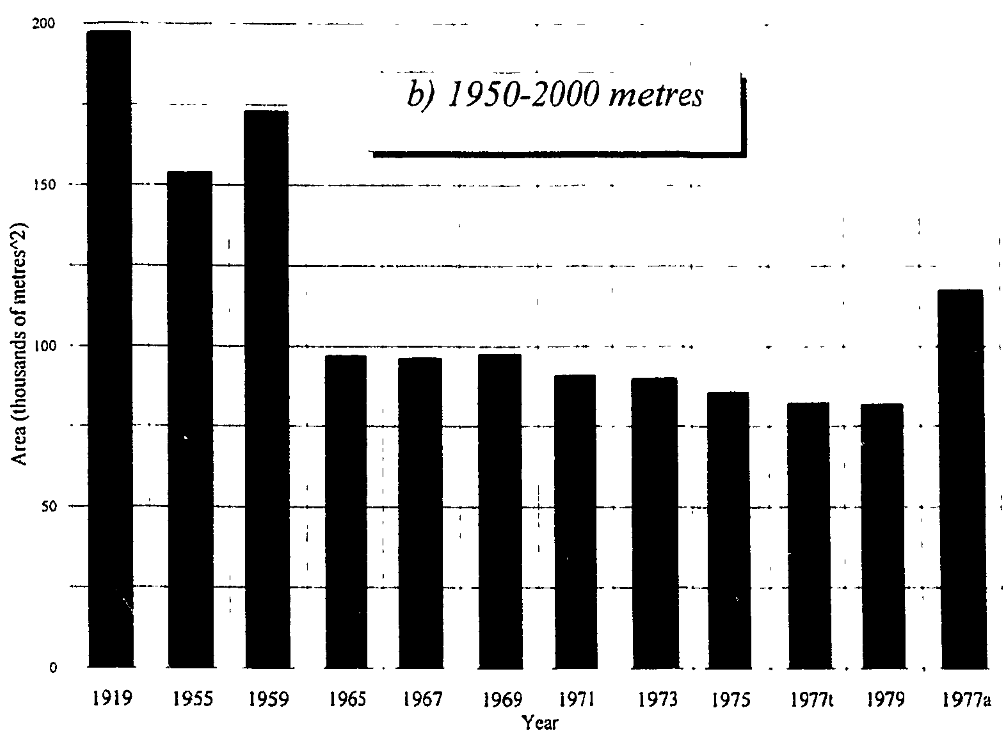


Figure 4.3: Area of Glacier

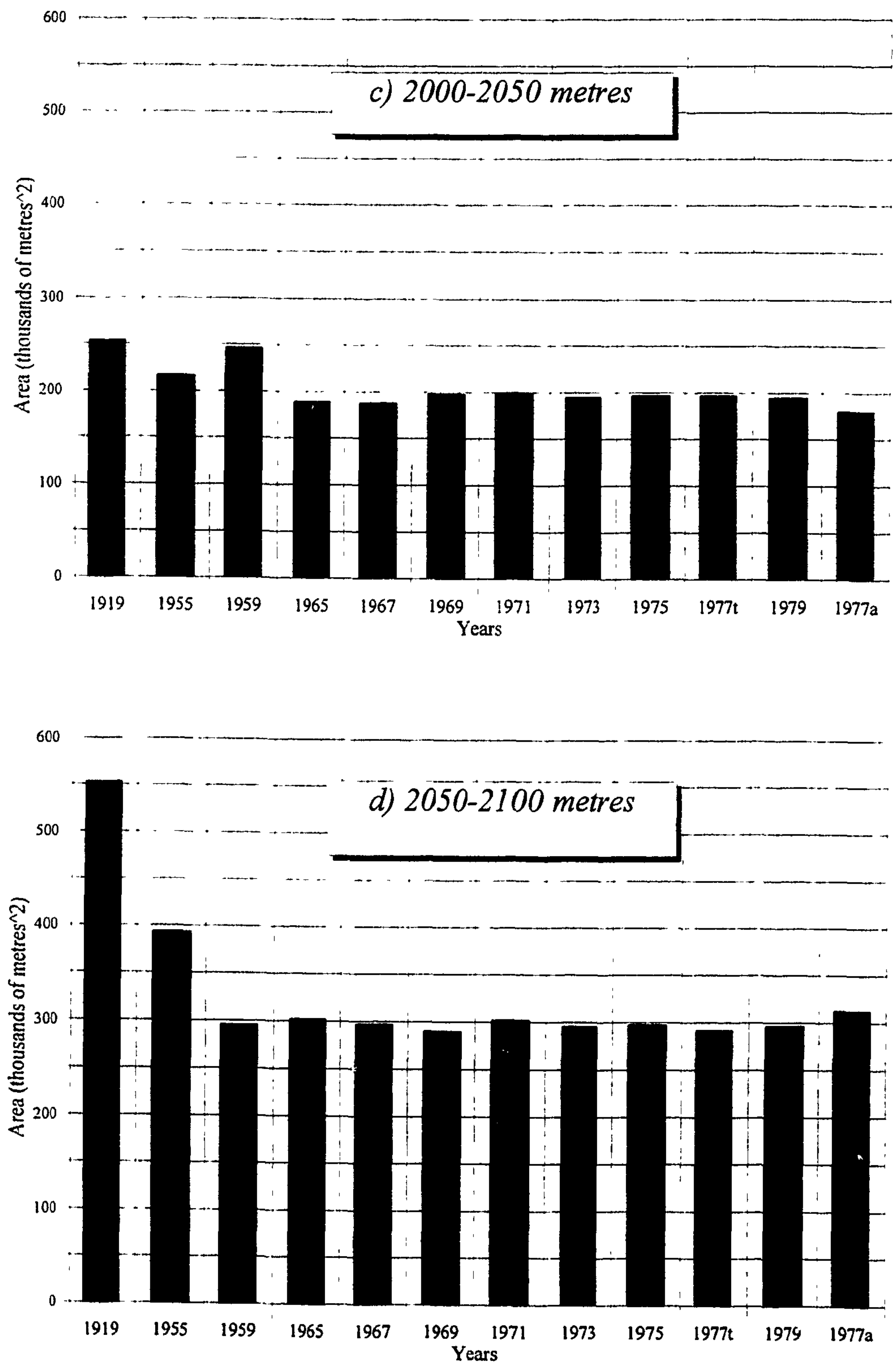
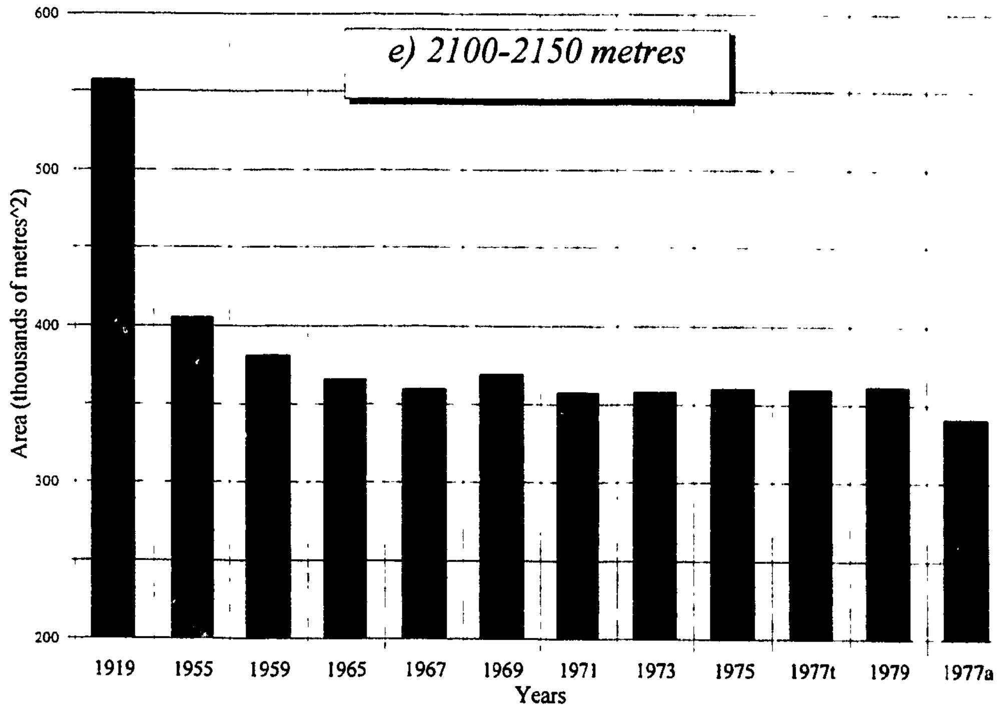


Figure 4.3: Area of Glacier



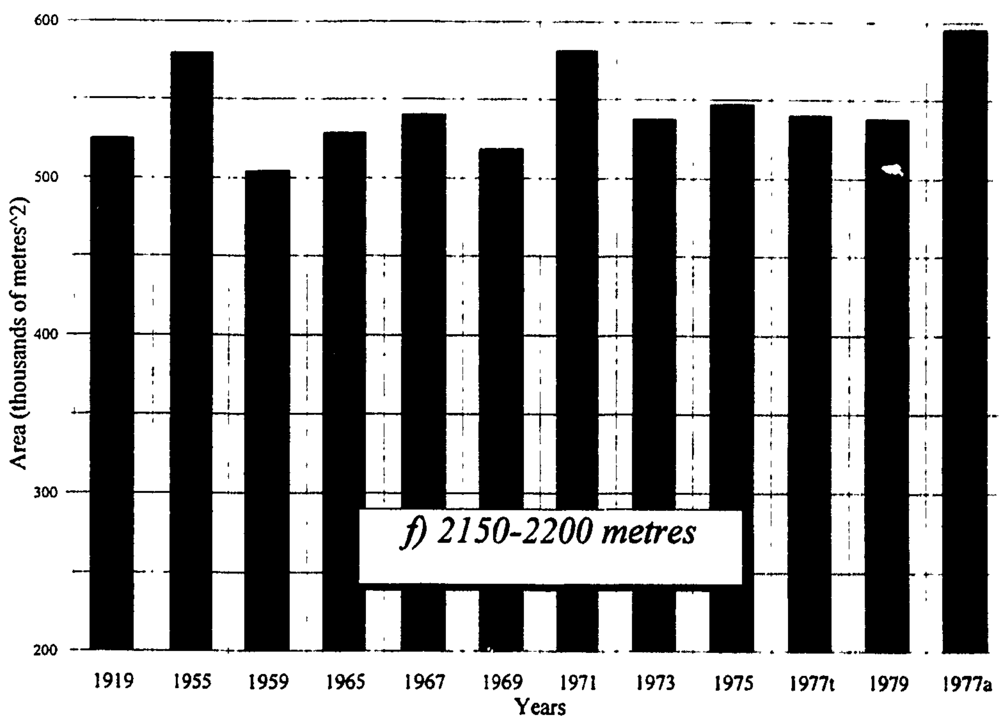
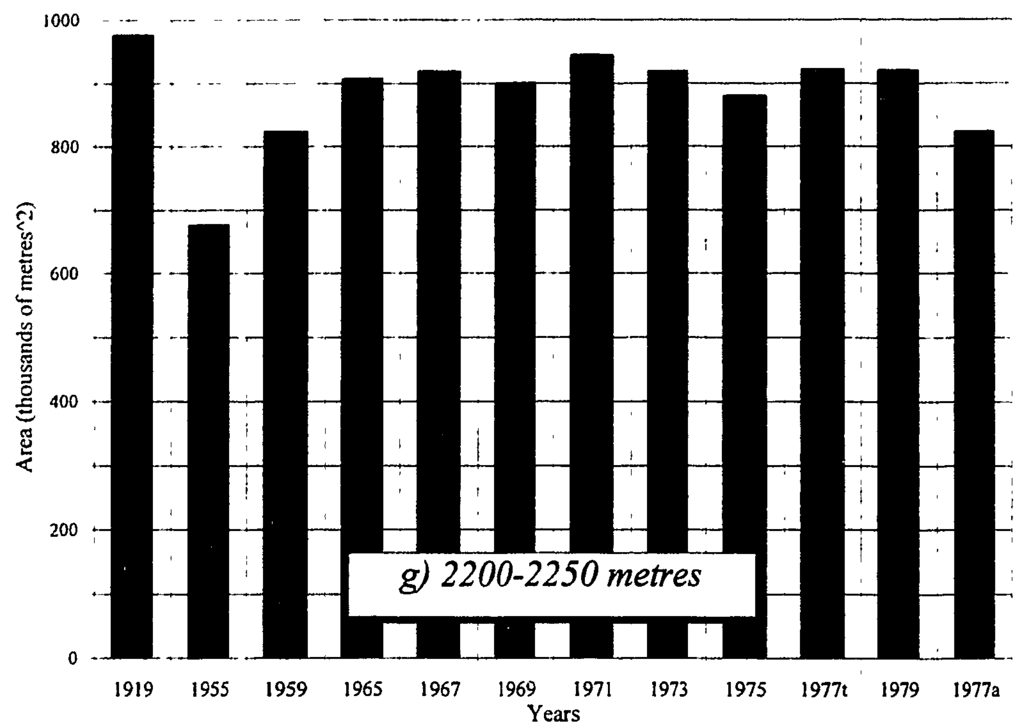


Figure 4.3: Area of Glacier



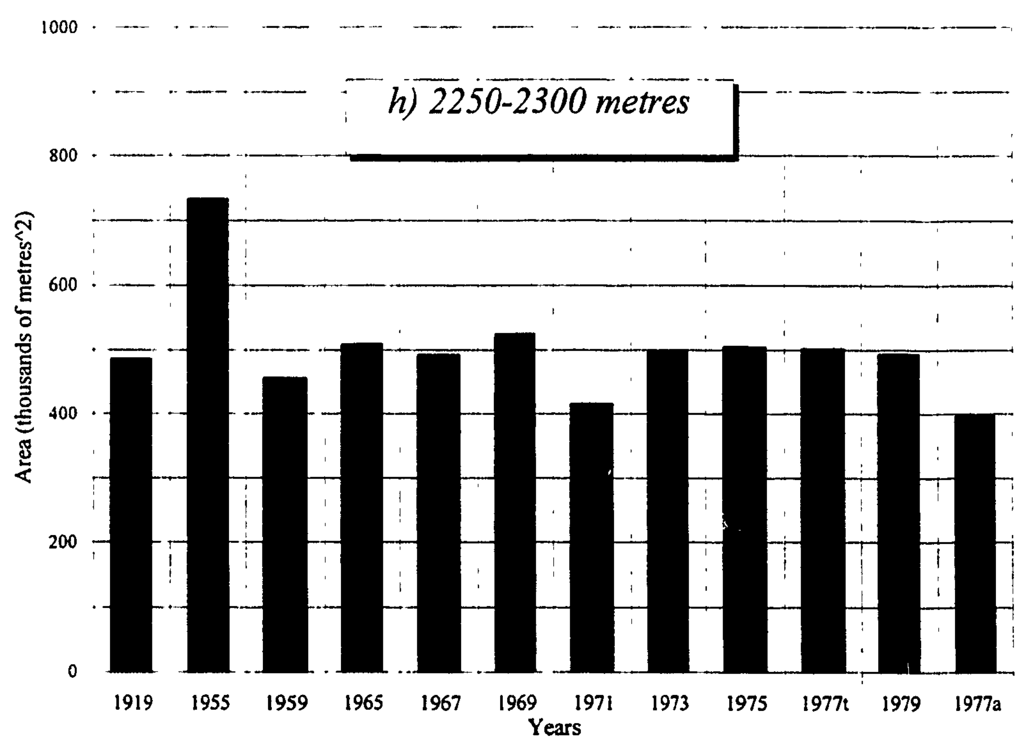
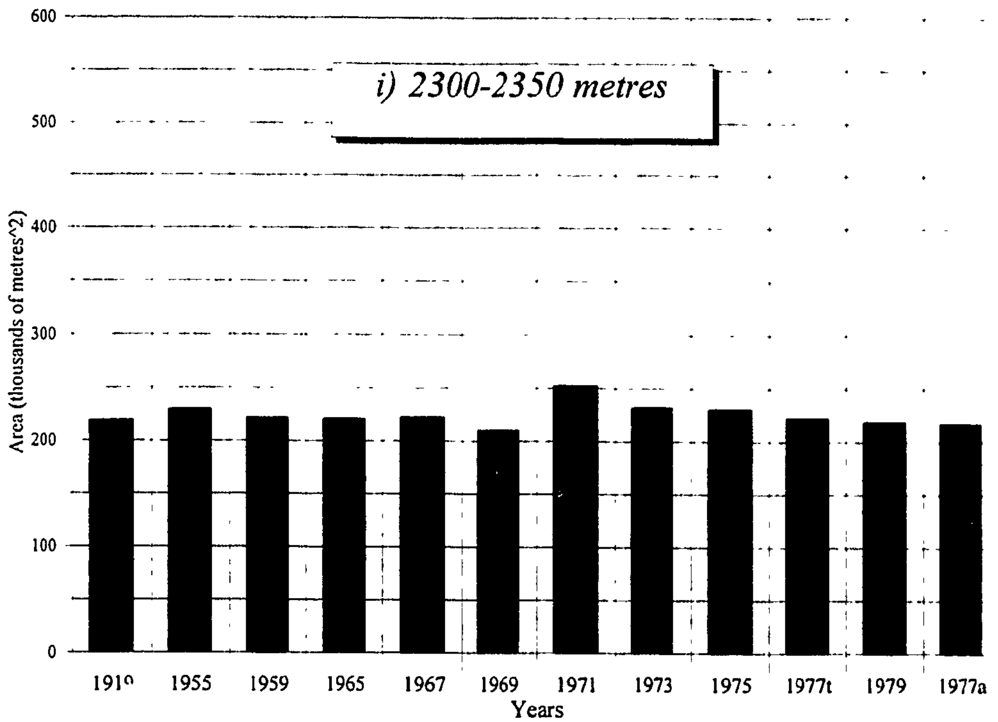


Figure 4.3: Area of Glacier



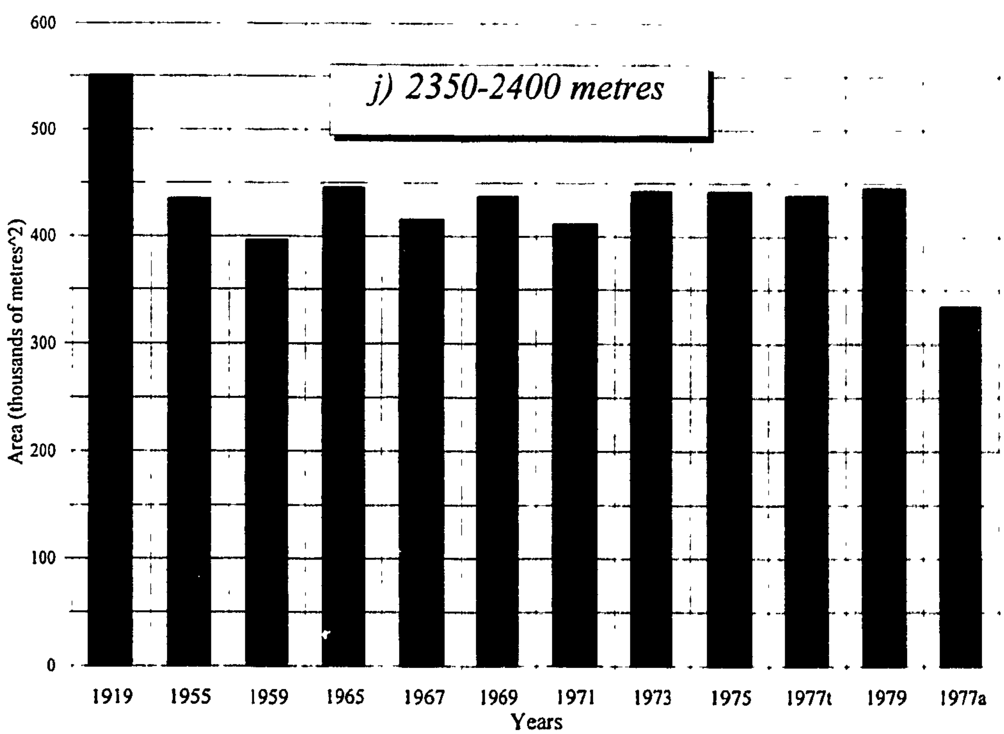
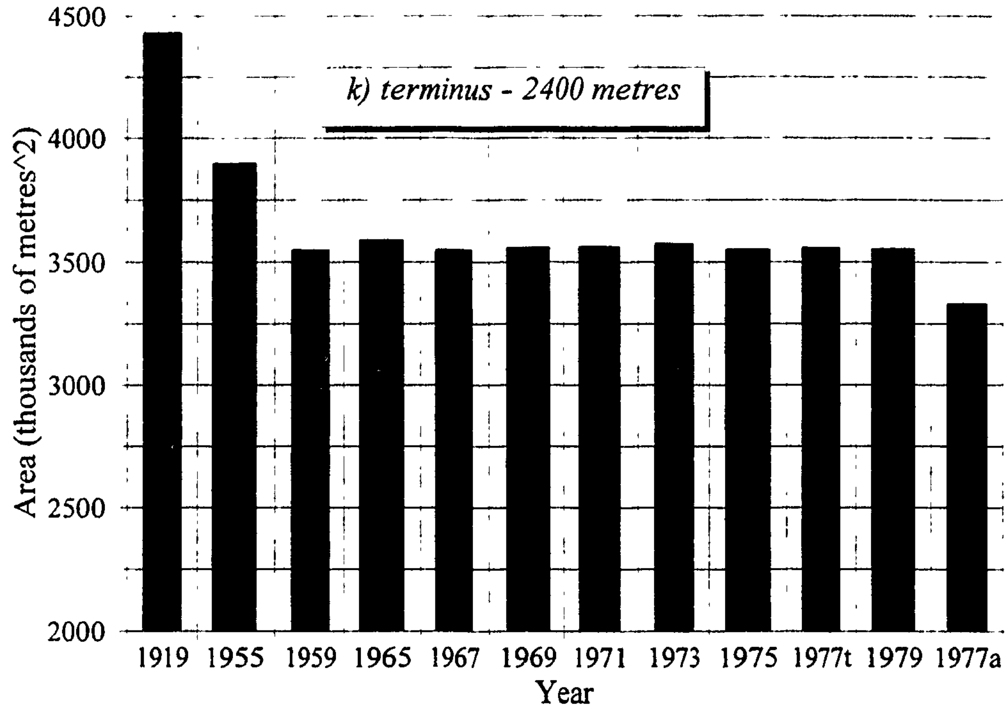


Figure 4.3: Area of Glacier



4.1.1.4 Area Changes

As the glacier contracted from 1919-1979, areas in each elevation zone also changed. This is shown in table 4.4 and figure 4.3. One of the most notable trends is the great decline in the area of the glacier below 1950 metres, shown in figure 4.3*a*. In 1919, the glacier covered more than 100,000 m^2 below the 1950 metre elevation mark. In 1979, less than 1500 m^2 was below that elevation. This decline in area was due to substantial recession of the terminus over the time of record, shown in figure 4.2. The elevation zone 1950-2000 metres shows a similar but not as precipitous decline (figure 4.3*b*). For both of the elevation zones below 2000 m, the amount of area recorded in the DEM based on 1977 aerial photography (1977A) was much larger than that recorded from the terrestrial photographic- based DEM of 1977 (1977T).

Measured areas in the three 50 m elevation zones between 2000 and 2150 metres (figure 4.3*c-e*) for the large-scale maps are quite similar from one year to the next. In those elevation zones, the small-scale maps of 1919 and 1955 show greater areas. The 1977A DEM records areas which are similar to the 1977T values.

From 2150-2400 m, measured areas in the elevation zones vary substantially from year to year. Of the large-scale based DEMs, the 1971 DEM shows the greatest variability in measurement of area. In the zones 2150-2200 m and 2300-2350 m (figures 4.3*f,i*) it has an area substantially greater than those measured in 1969, 1973, and the other large-scale DEMs. In the 2250-2300 metre zone, figure 4.3*h*, 1971 has substantially less area than either 1969 or 1973.

Finally, figure 4.3*k* shows the total area of the glacier below 2400 m in each DEM. The substantial decline in the area of the glacier from 1919-1979 is noted. Area figures for 1959 and 1965 show the effects of lack of measurement of part of their surfaces, discussed in section 4.1.3.3. In addition, the 1977A DEM displays a markedly different area than the 1977T DEM. This is due to the different definition of the borders on the 1977A,T maps, identified in section 3.3.1.2.

4.1.2 Volumetric Change

The volumetric change of the ablation area was measured between successive DEMs in a series of 10 m elevation zones extending up the surface of the glacier. The sole exception was that the DEM produced from the 1977 air photo map was compared only with the 1977 DEM produced from terrestrial photogrammetry. That comparison is discussed later in this chapter.

Table 4.1*a-k* lists total volumetric change in each elevation zone between successive DEMs. These tables show that the 10 m elevation zones that recorded the volumetric change of greatest magnitude occurred in the interval 1919-1955, shown in table 4.1*a*. This interval also shows the greatest rate of volumetric loss in the elevation zones below 2150 metres. However, in the part of the glacier above that elevation, the thirty-six year interval between measurements reduced the large absolute change to a rate similar in magnitude to mean annual changes measured over later intervals.

Another trend to note in the volumetric change results is the variability of change over elevation and between intervals. Most intervals show decline in volume in some elevation zones and increase in others. Although this is most pronounced in tables 4.1*d,h*, in which both strong gains and marked losses are evident in different series of elevation zones, it is visible in most intervals. Additionally, elevation zones that show slight volumetric loss in one interval can show strong loss or gains in other intervals.

An example of this is found in elevation zone 2080-2090 in table 4.1*b*-*d*, which shows strong volumetric gain in 4.1*b*, strong loss in 4.1*c* and mild gain in 4.1*d*; many other examples exist.

Values showing the uncertainty of volumetric change appear with volumetric change values for each elevation zone in table 4.1. These results present the uncertainty of vertical estimation of the source maps as a measure of volume, as discussed in sections 3.1.2 and 3.3.3.6 and shown in table 3.3 and figure 3.19. If the standard deviation of volumetric estimation (σ_v) is greater than the volumetric change for an elevation zone, then the variability of measurement is greater than the quantity measured.

The interval 1919-1955 has the greatest magnitude of standard deviation of volumetric change measurement in each of its 10 m elevation zones. This is due to the high degree of uncertainty of vertical estimation for both the 1919 and 1955 maps. The lowest values of standard deviation for volumetric change occur in the interval 1959-1965. This is due to the low levels of uncertainty attached to the 1959 map and the comparatively low uncertainty values of the 1965 map. When standard deviation values for the terrestrial photogrammetric-based DEMs in the intervals from 1965-1979 are compared, it can be seen that corresponding elevation zones in different intervals have very similar values. This is because all of the 1:10,000 maps have an identical function to describe the uncertainty of vertical estimation. Since the planar location of elevation zones does not grossly change between the terrestrial-based DEMs, corresponding measurements of standard deviation of volumetric change are necessarily similar.

Volumetric change values measure volumetric change in a horizontal elevation zone. Thus, elevation and volumetric change figures measure quantities at right angles to each other. Placing these values in the same table allows two different methods of calculating change to be compared. Tables 4.1 and 4.2 include values of both absolute and average annual elevation and volumetric change for each elevation zone.

Statistical tests were performed to compare values of elevation change in 10 m elevation zones with corresponding volumetric change measurements. All but one of the intervals showed correlation significant at the 99% confidence level between elevation change and volumetric change. The exception was 1977-1979; in that interval elevation change was minimal and fairly homogenous over the surface. This contributed to the low correlation between elevation and volumetric change in that interval.

It is easier to see overall patterns when volumetric change and σ_v are measured over 50 m zones. It becomes possible to produce figures which can graphically compare the same elevation zones in different intervals. Table 4.2*a-k* summarizes the results of table 4.1*a-k* over 50 m elevation zones. Table 4.2 shows that over many elevation zones for numerous intervals, σ_v is greater than volumetric change. This is most apparent in the intervals 1965-1967, 1975-1977 and 1977-1979.

Figure 4.4, produced from table 4.2, shows the average annual volumetric change that occurred in the intervals between consecutive mappings for each elevation zone. Volumetric change is represented as the range of values that falls between the mean value plus and minus one standard deviation of volumetric estimation. Representing volumetric change as a range of values provides a visual measure of the relative magnitude of the uncertainty of measurement in each interval.

In figure 4.4, zones above 2000 m were 50 m deep. Below this, measurement went from the terminus to 2000 m. From the terminus to 2050 metres, shown in figures

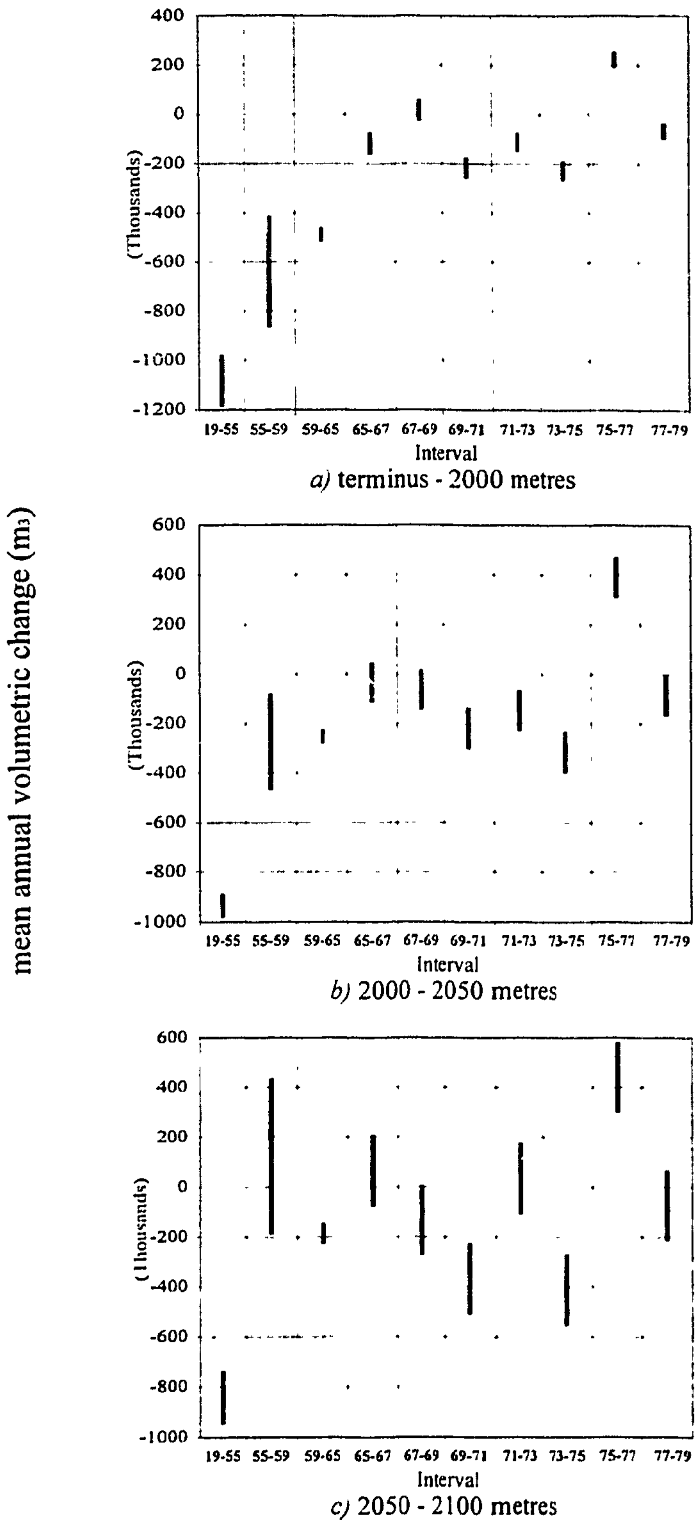
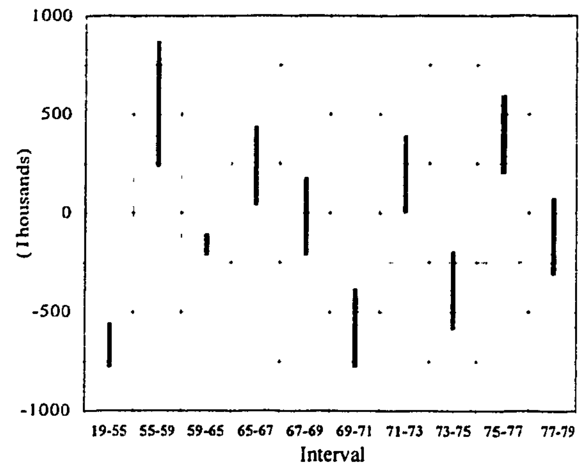
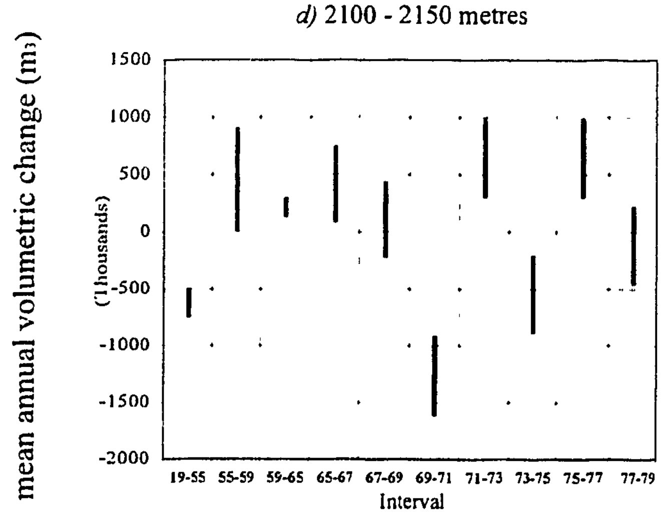


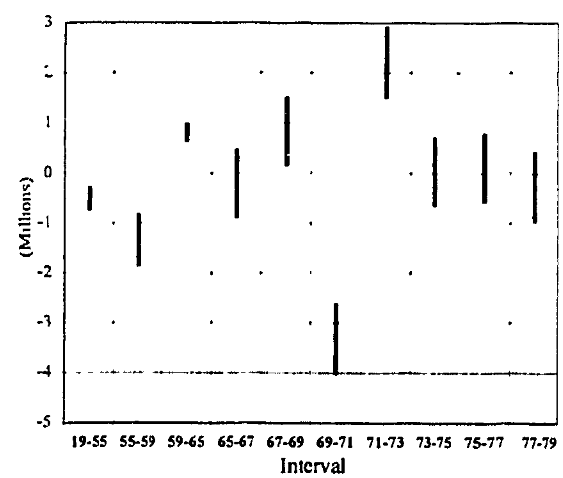
Figure 4.4: Average annual volumetric change



d) 2100 - 2150 metres

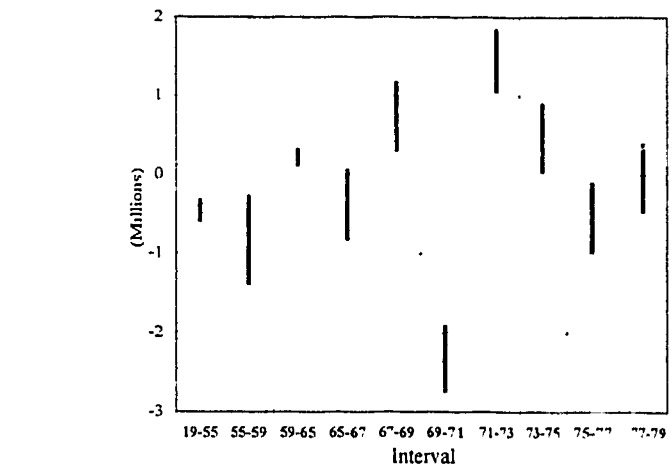


e) 2150 - 2200 metres

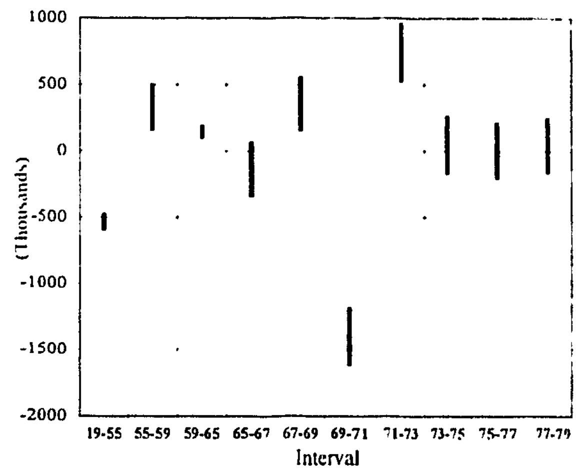


f) 2200 - 2250 metres

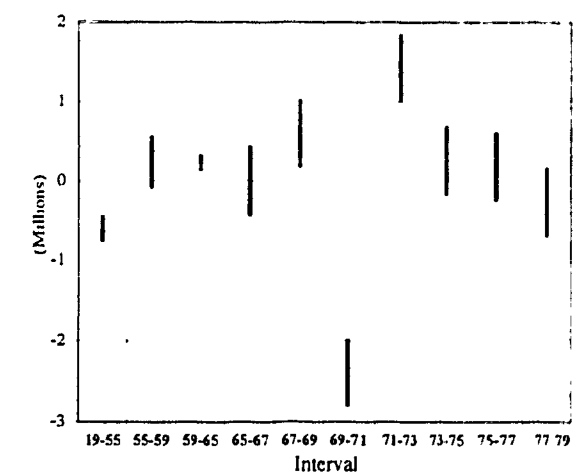
Figure 4.4: Average annual volumetric change



g) 2250 - 2300 metres



h) 2300 - 2350 metres



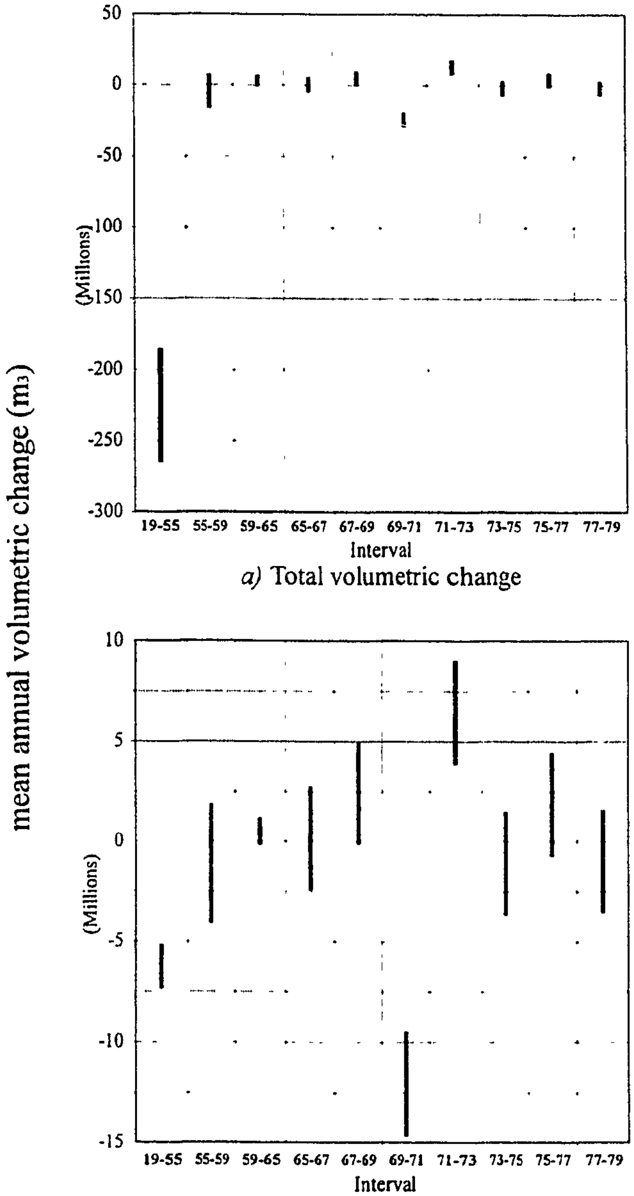
i) 2350 - 2400 metres

Figure 4.4: Average annual volumetric change

4.4*a,b,* most of the intervals show a substantial loss in volume. In both of those figures, the interval 1919-1955 shows the greatest rate of loss: more than one million cubic metres per annum in the terminus to 2000 metre elevation zone. This matches expectations since in that time the terminus of the glacier retreated substantially. Above 2050 metres, the elevation zones of the glacier show substantial annual variability both from interval to interval within a given elevation zone and across elevation zones in an interval.

When total volumetric change figures are compared (table 4.5; figure 4.5*a*), it can be seen that the interval 1919-1955 had the greatest volumetric change of any of the intervals between DEMs, and that the bulk of the total volumetric change from 1919-1979 occurred in these years. However, when mean annual volumetric change figures are compared (table 4.5; figure 4.5*b*) it becomes evident that the greatest annual rate of volumetric change occurred between 1969 and 1971.

This assumes that all of the maps are of equal accuracy and are without substantial error. It is possible that a systematic error was made in the production of the 1971 map, portraying the surface of the glacier as being lower than it in fact was. If this is the case, then the volume and elevation change figures for the intervals 1969-1971 and 1971-73 are incorrect. Difficulties with the 1971 map are discussed in section 4.1.4.1.



a) Total volumetric change

b) Average annual volumetric change

Figure 4.5: Volumetric change: Terminus - 2400 metres

Table 4.5: Total volumetric change of the complete ice surface, terminus-2400 m, 1919-1979

Interval	Total Change (m^3) (,000)	σ_v (m^3) (,000)	Annual Change (m^3) (,000)	σ_v (m^3) (,000)
1919-1955	-225,000	40,070	-6,250	1,113
1955-1959	-5,517	12,475	-1,379	3,119
1959-1965	3,185	4,208	531	701
1965-1967	-315	5,216	-158	2,608
1967-1969	4,863	5,144	2,432	2,572
1969-1971	-24,160	5,221	-12,080	2,611
1971-1973	12,940	5,192	6,470	2,596
1973-1975	-2,163	5,213	-1,081	2,606
1975-1977	3,719	5,188	1,860	2,594
1977-1979	-1,953	5,194	-976	2,597
1977(A,T)	-10,420	10,810	-	-
Sum Total: 1919-1979	-234,400	-	-3,907	-

Figures 4.5*a,b* show that only three intervals display change from the terminus to the 2400 metre contour which is of greater magnitude than the standard deviation of volumetric change for that elevation zone: 1919-1955, 1969-1971 and 1971-1973, although the latter two could be due to error in the 1971 map. This demonstrates the effects that variability of change in elevation zones in the intervals has: one part of the glacier can downwaste while a different portion upgrades. The magnitude of change over selected portions of the glacier surface can be greater than the standard deviation of volumetric change of that portion, but variability in the undivided interval causes the magnitude of change over the entire glacier surface to be less than the total standard

deviation of volumetric change.

4.1.3 Uncertainty of measurement

Kick (1966) stated in his study of glacier mapping that care was necessary to ensure that errors in mapping were less than the change measured. Since the change in elevation of a glacier is unlikely to exceed several tens of centimetres in a year except near the terminus, a frequent mapping program must be exceedingly accurate. Observing the results of elevation and volumetric change in comparison with values of uncertainty of vertical or volumetric estimation, it can be seen that often errors in mapping were greater than the change measured.

Figure 4.1 shows elevation change on the glacier surface in intervals between consecutive mappings. Figures showing elevation change were much easier to prepare than values of volumetric change, and thus more than consecutive mappings were compared. Figures 4.6 and 4.7 show changes over six- and ten-year intervals. When those diagrams are compared with figures that show change over two-year intervals (figures 4.1*d-j*) a substantial difference is noted.

Table 4.6 compares the mean area of the glacier between large-scale maps with the area over which measured change is less than one standard deviation of vertical estimation from the mean of measurement. This table shows that in the two-year interval this value is above 40%, with two intervals (1965-1967, 1977-1979) being more than 60% covered. This proportion declines to about one third of the surface for a six-year interval and 25.8% using a ten-year interval.

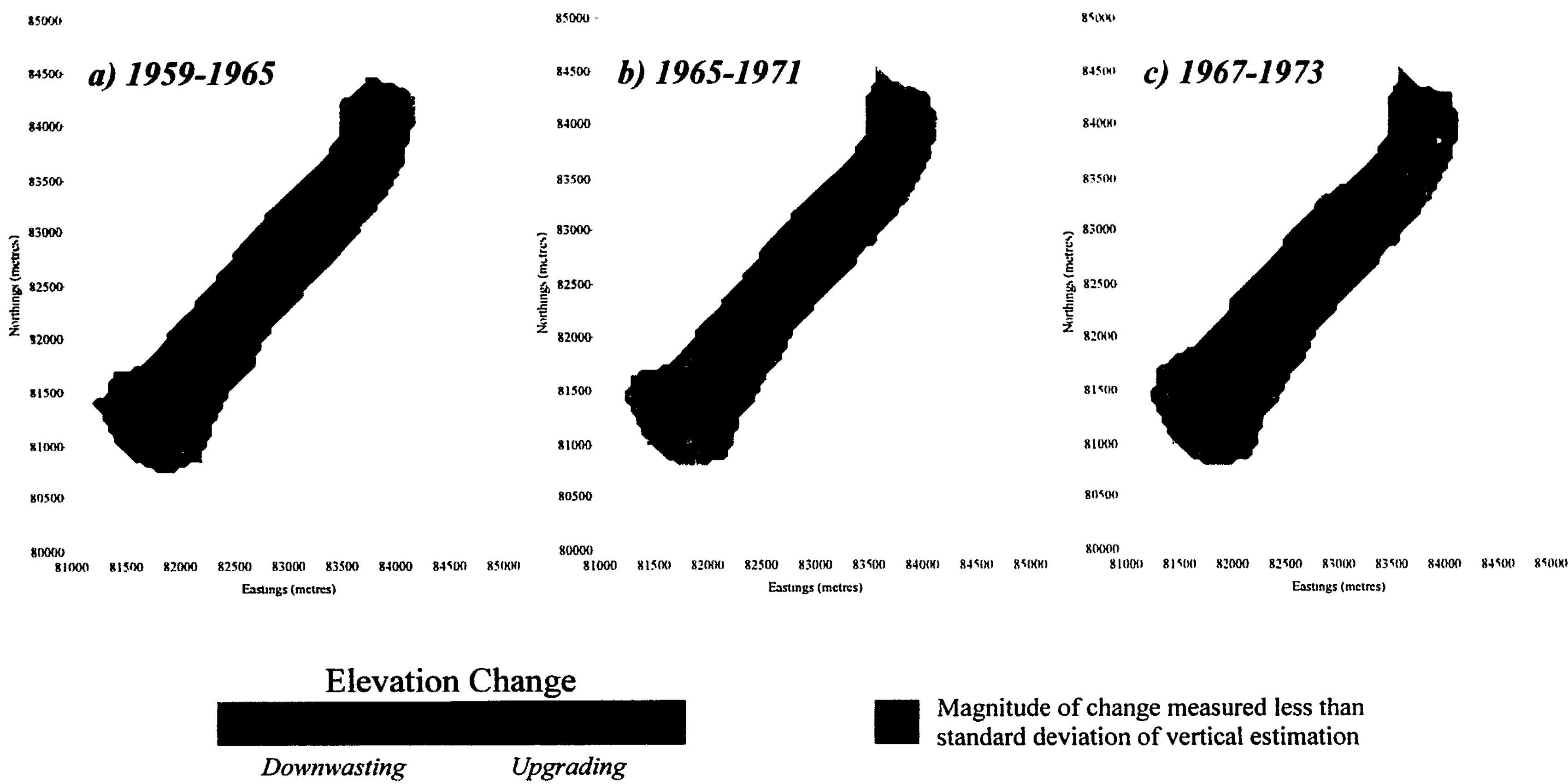


Figure 4.6: Average annual elevation change of large-scale maps over six-year interval. Contour interval 50 m. Isoline interval 1 m/yr.

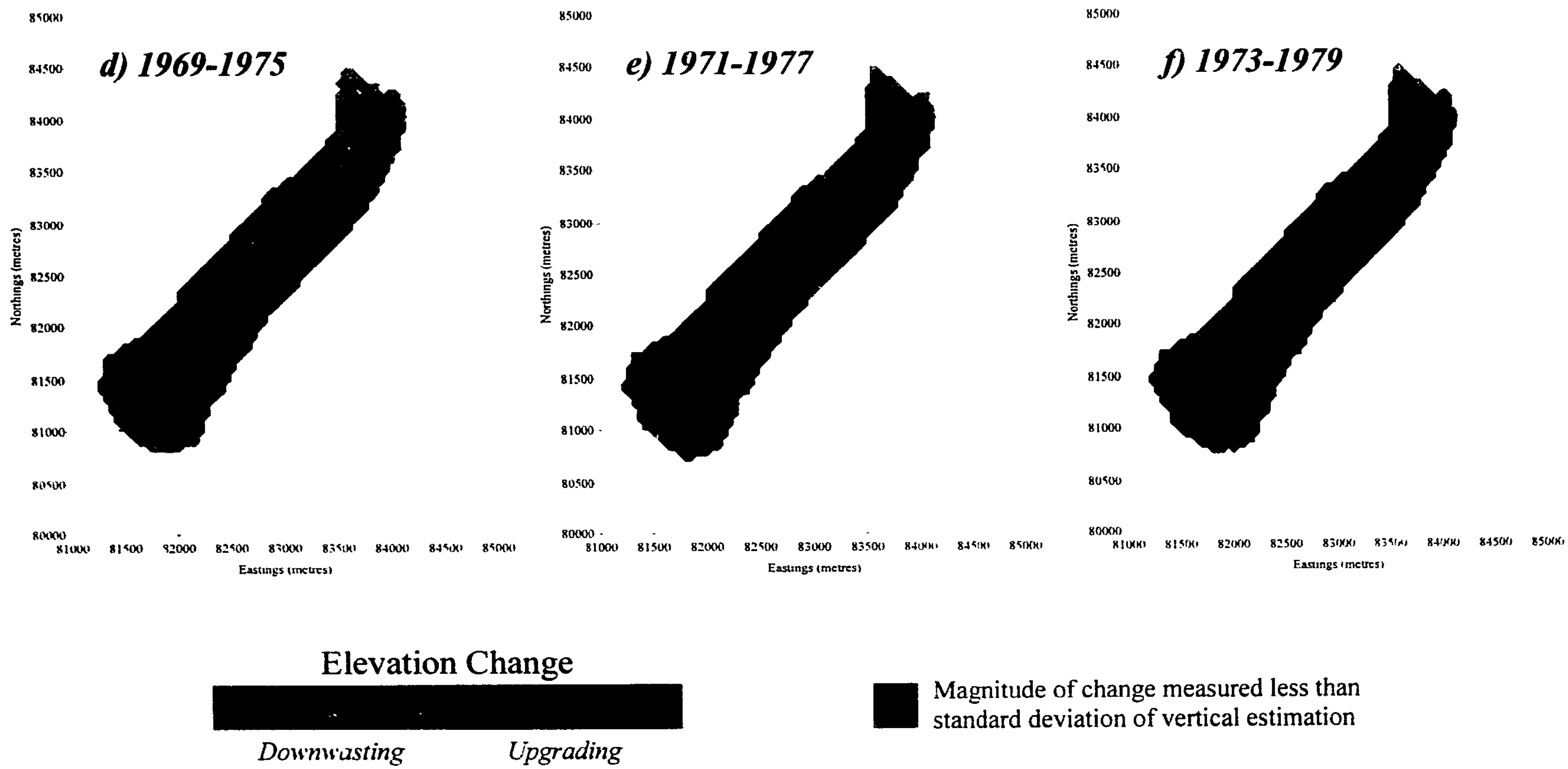


Figure 4.6: Average annual elevation change of large-scale maps over six-year period.
Contour interval 50 m. Isoline interval 1 m/yr.

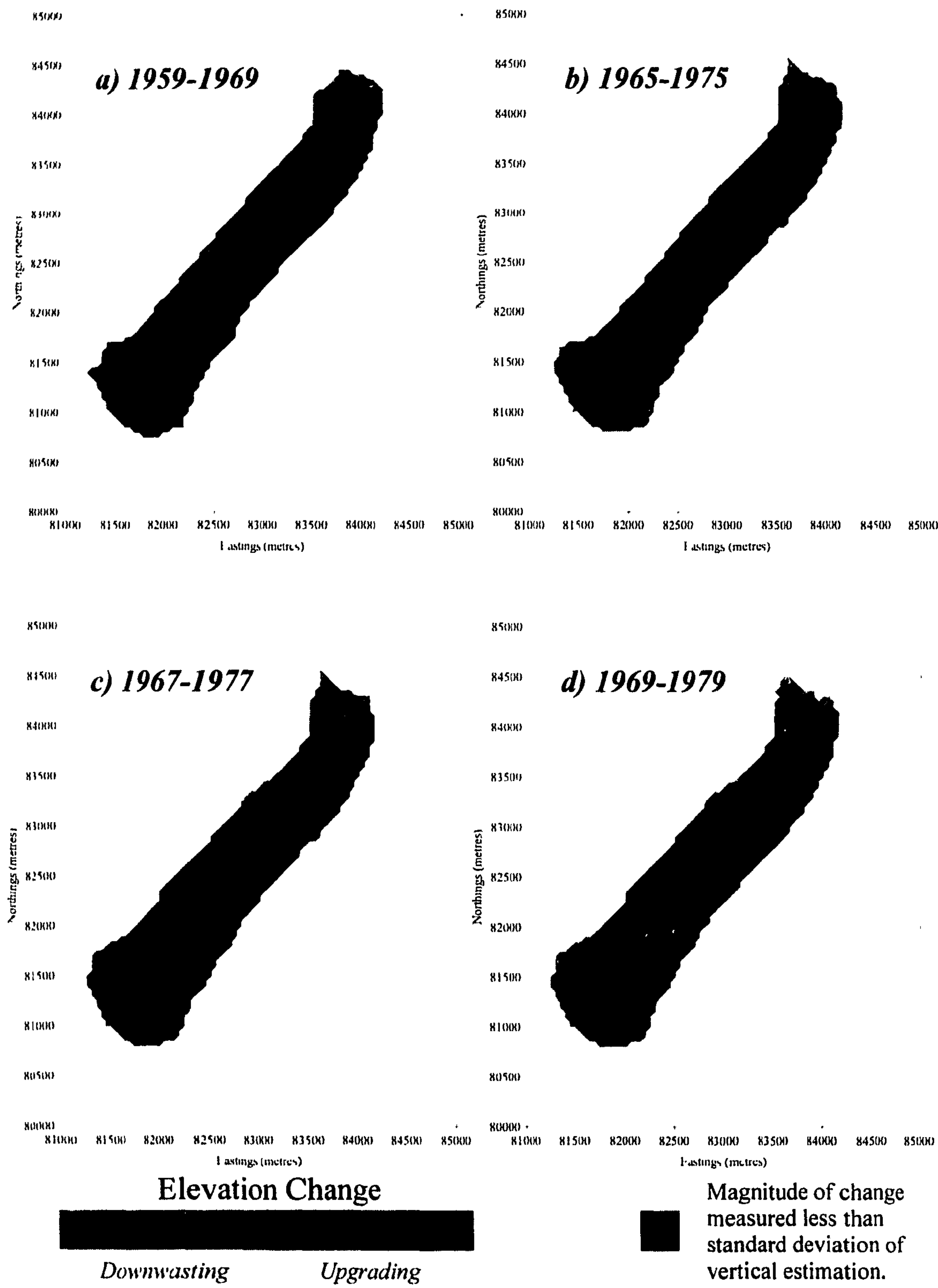


Figure 4.7: Average annual elevation change of large-scale maps over ten-year interval. Contour interval 50 m. Isoline interval 1 m/yr.

Table 4.6: Percentage of Glacial Area Where Elevation Change (Δh) is less than the uncertainty of vertical estimation (σ_e)

Duration	Years	Area of Glacier (m^2) (,000)	Percentage where $\Delta h < \sigma_e$	Average percentage
Two-year interval	1965-1967	3336	60.1	42.4
	1967-1969	3550	42.1	
	1969-1971	3558	1.0	
	1971-1973	3563	17.2	
	1973-1975	3574	47.3	
	1975-1977	3551	47.3	
	1977-1979	3559	81.9	
Six-year interval	1959-1965	3590	36.4	32.4
	1965-1971	3336	4.0	
	1967-1973	3550	41.9	
	1969-1975	3558	17.5	
	1971-1977	3563	13.2	
	1973-1979	3574	81.4	
Ten-year interval	1959-1969	3590	18.8	25.8
	1965-1975	3336	33.1	
	1967-1977	3550	43.8	
	1969-1979	3558	7.6	

Although volumetric change was only calculated for successive intervals, the results of volumetric change calculation in 10 m elevation zones for two-year intervals (table 4.1*d-j*) also suggests that an interval greater than two years between measurements is desirable. Table 4.7 shows that a substantial portion of the 10 m elevation zones in most intervals have volumetric change figures which are less than the standard deviation of volumetric change. Greater differences between the surfaces, which would occur over longer intervals, would result in fewer elevation zones where volumetric change is less than the standard deviation of volumetric estimation.

Table 4.7: Number of elevation zones where σ_v is greater than ΔV

Year	Elevation Zones		%
	Number	$\sigma_v > \Delta V$	
1919-1955	48	0	0.0
1955-1959	48	13	27.1
1959-1965	48	8	16.7
1965-1967	48	32	66.7
1967-1969	48	27	56.3
1969-1971	48	0	0.0
1971-1973	47	7	14.9
1973-1975	47	17	36.2
1975-1977	47	17	36.2
1977-1979	46	32	69.6
1977A,T	46	25	54.3

4.1.4 Analysis and Discussion: *Surfer* results

Some of the results presented above require additional analysis. Surface and volumetric change values produced using the 1971 DEM are unlike the corresponding information from other intervals in many ways; this brings the validity of the 1971 map into question. Its soundness is discussed. The comparison of the two DEMs from 1977 reveals that there are substantial differences between their surfaces. This brings the accuracy of the other small-scale maps used in this thesis into question. These matters are discussed below.

4.1.4.1 1971 DEM

Table 4.1*f,g* shows the results of calculation of elevation and volumetric change for the intervals 1969-1971 and 1971-1973. The results show that the glacier lost an enormous amount of volume and displayed a sharp lowering of its surface in 1969-1971. In the following interval, the glacier regained approximately half of the volume it lost in the previous interval: the result of the general raising of its surface above the 2050 m contour, shown in figure 4.1*g*.

The 1971 DEM is a faithful representation of the contour map it was digitized from. It was suspected that there might have been a systematic error involved in the production of the 1971 map. If the map surface was portrayed as being substantially lower than it was in reality, then results similar to those reported could be produced: great loss from 1969 to 1971, followed by the regaining of half of the loss from 1971 to 1973. Comparisons were performed to determine whether the 1971 data is anomalous. Cross sections of the terrestrial-photogrammetry based DEMs were compared to determine the magnitude of the possible anomaly. Following this, Athabasca volumetric change data was compared with similar change data from Peyto, a nearby glacier.

Longitudinal cross-sections of the 1971 DEM were compared with the DEMs from other 1:10,000 maps. A line approximating the midpoint of the glacier was drawn, extending from above the 2400 metre contour to slightly beyond the 1919 terminus (figure 4.8). Cross-sections showing glacier surfaces mapped with terrestrial photogrammetry from 1965-1979 were prepared for the representative elevation zone 2250-2300 m and appear in figure 4.9.

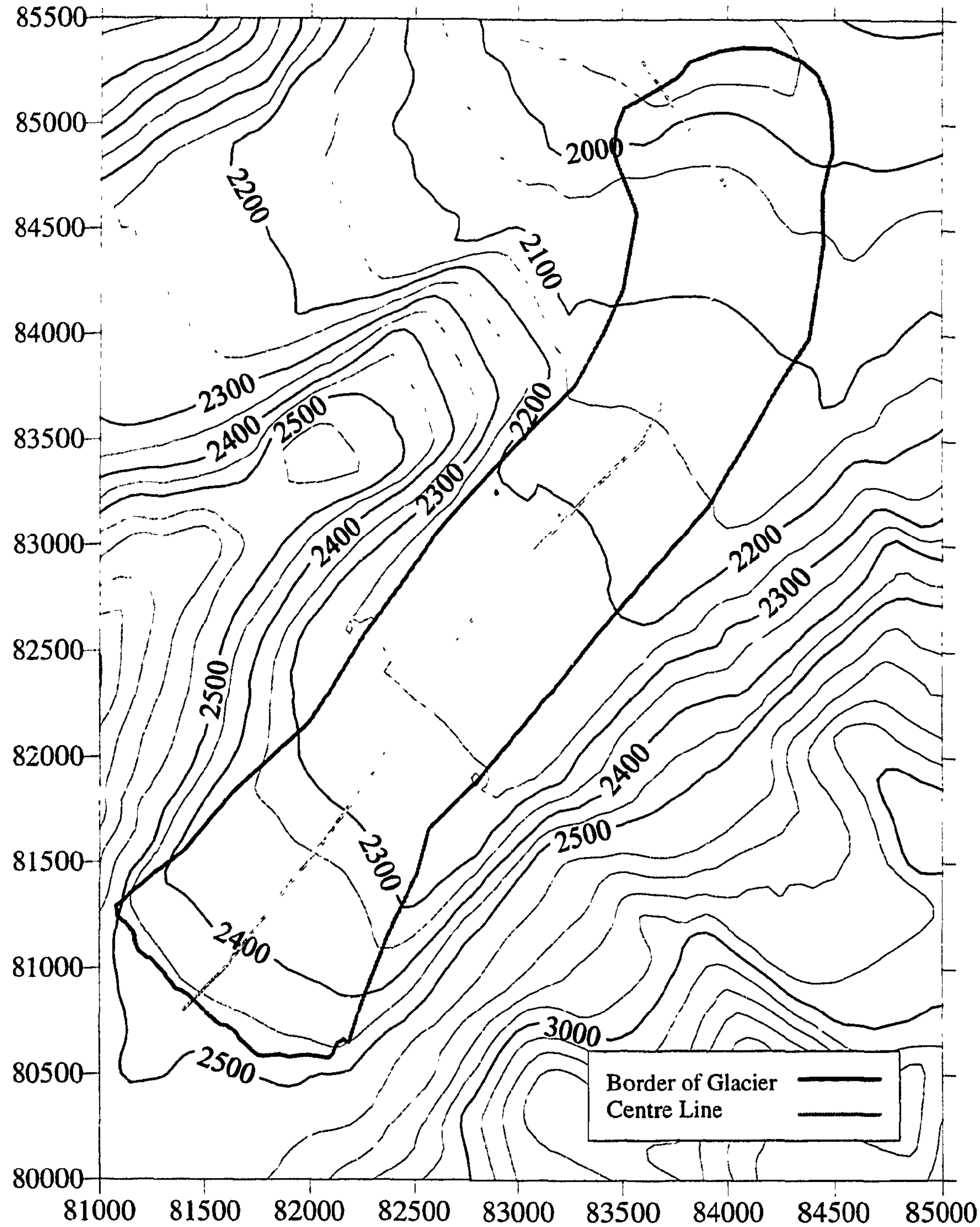


Figure 4.8: Location of longitudinal cross-section through approximate centre of glacier. Map and border are from 1919 Boundary Commission.

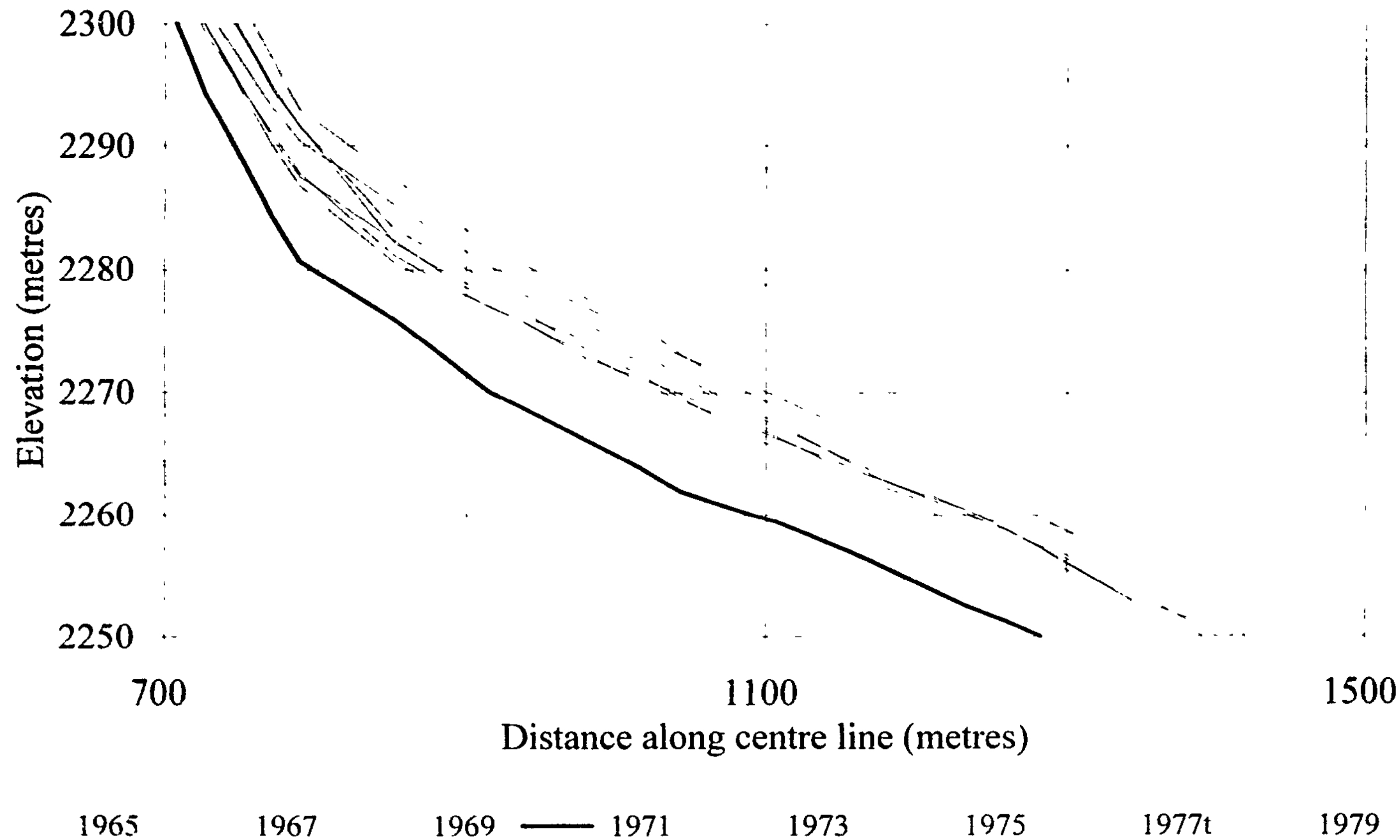


Figure 4.9: Cross-sectional profiles of 1:10,000 scale terrestrial photogrammetry maps, elevation zone 2250-2300 metres.

In that 50 m elevation zone, the glacier surface of most of the terrestrial-based DEMs lie in a range of roughly 5 m from the highest to the lowest surface. The exception is 1971, which is 5-10 m below the other values. Other 50 m elevation zones above 2050 m show similar results, with the 1971 surface much lower than the range of values from other years.

Data from the Peyto Glacier (51°40'N, 116°35'W) was compared with the Athabasca data. Peyto is a nearby glacier which is also located in the Front Range of the Canadian Rocky Mountains. It also flows off an icefield in a north-northeast direction, and has approximately the same elevation as the Athabasca. The Peyto was the subject of a series of studies in the International Hydrological Decade (Young and Stanley, 1977).

Mass balance, in metres of water equivalent, was one of the variables studied for Peyto Glacier in the time period in which the large-scale Athabasca Glacier maps were made. The Peyto data was collected by measuring annual mass balance at a number of points and extrapolating over the entire glacier (Young, 1981). It was hoped that the values would have a significant correlation with the measured volumetric change of the Athabasca Glacier. The relationship between net specific annual balance of Peyto Glacier below 2400 m and the total volumetric change of the Athabasca Glacier below 2400 metres was calculated.

Unfortunately, the correlation calculated is not significant. However, it is evident that for both Athabasca and Peyto data, the intervals surrounding 1971 displayed the same general pattern. 1969-1971 was characterized by marked decline in volume at Athabasca and a strongly negative mass balance at Peyto, while 1971-1973 was a period of volumetric and mass balance increase.

This comparison with the Peyto data suggests that the 1969-71 interval was a period of volumetric decrease at both Peyto and Athabasca. However, this does not account for the great loss in elevation from 1969-71 followed by the great gain from 1971-73, shown in figures 4.1*f,g* and in cross-section in figure 4.9. It seems unlikely that such a decline followed by a large increase could be due to processes such as kinematic waves.

It is probable that there was a systematic error in the production of the 1971 map, portraying the surface as being lower than it in fact was. This would exaggerate the loss that likely occurred in 1969-71 and would overstate the amount of volumetric and elevation gain from 1971-73.

4.1.4.2 Effects of low precision of vertical estimation of the small-scale maps

Three small scale maps (1919, 1955 and 1977A) were used in this thesis. Their precision of vertical estimation is substantially less than the large-scale maps. The results of this are discussed below, first for the 1977A map, which can be compared with the large-scale terrestrial-based DEM of 1977T, and then for the earlier maps 1955 and 1919, which were used to extend the record of change back in time. These will be discussed in reverse chronological order since it is the proximity in time between one mapping and the next that determines the ability to compare maps for their precision.

4.1.4.2.1 1977A,T

Tables 4.1*k* and 4.2*k* and figures 4.1*k* and 4.3*k* show that despite the two DEMs from 1977 representing the same surface, substantial differences exist between them. These differences are due to differing scales, methods of production, contour intervals and different interpretations of the borders of the glacier.

Table 4.1*k* shows that the average elevation difference between the two DEMs is slight in the lower portion of the study area, tending to be less than 2 m between 1970 m and 2180 m. Above 2180 m, elevation differences between the DEMs increase substantially, with average elevation differences in 10 m zones as great as 6.3 metres. When volumetric difference is computed between the two surfaces, the total difference is 1.06×10^7 m^3, a greater difference than is recorded in most of the intervals between successive DEMs. Most of this difference (8.48×10^6 m^3) is due to the surface differences above 2180 metres. In addition, in the portion of the glacier above 2150 metres, the area measured in each elevation zone in 1977A shows great differences from areas derived from the 1977T DEM (table 4.4). This is due to the differing slopes of the DEM surfaces and different definitions of the border of the glacier.

A comparison of the cross-sections of the terrestrial and aerial DEMs (figure 4.10), measured along the cross-sectional line shown in figure 4.8, is helpful in clarifying the cause of the lack of agreement of the volume and elevation results above 2180 m. This diagram shows that the two cross-sections display very similar elevations below 2180 m. Above that elevation, the differences between the surfaces become substantial. This is the cause of the elevation and volume differences noted in tables 4.1*k*, and 4.2*k*.

The aerial and terrestrial maps have been compared before, as part of Young *et al.* (1978). The results reported in that study were discussed in section 2.2.2.2.2; they were similar to the results produced in this thesis in that lower portions of the glacier displayed small map-to-map differences and that with increasing elevation differences between maps increased.

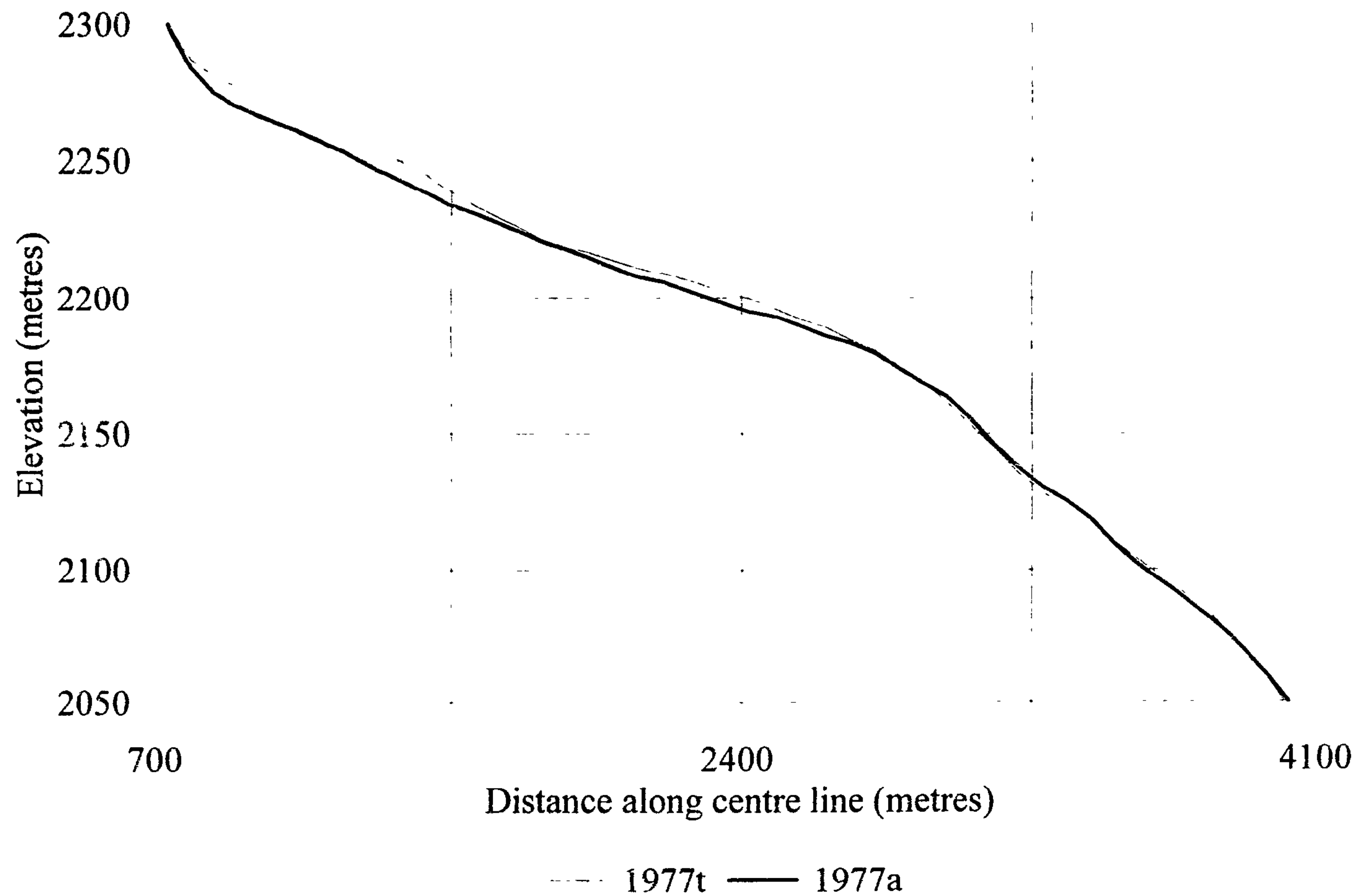


Figure 4.10: Cross-section profile along centre line of elevation zone 2050-2300 metres: 1977 DEMs.

The fact that there were great differences in maps that portrayed the same area at different scales raises questions regarding the precision of small-scale maps. The surface plot comparing 1977A and 1977T (figure 4.1*k*) shows that the elevation difference between the surfaces is often greater than one standard deviation of vertical estimation, especially in the higher elevation zones of the glacier.

The volumetric difference between the two 1977 surfaces was less than the calculated standard deviation of volumetric difference. If the other small scale maps have accuracy problems of a similar magnitude to those shown on the small-scale 1977 map, it is hoped that any errors in the measurement of volumetric change are also less than their standard deviation of volumetric change.

4.1.4.2.2 1955, 1919 small-scale maps

Figure 4.1*b* compares the small scale based 1955 DEM with the large-scale 1959 DEM. The comparison displays a pattern of elevation change unlike any of the other elevation change diagrams. Figure 4.1*b* shows downwasting at the terminus, upgrading between roughly 2100 and 2150 m, and downwasting again between the 2200 and 2400 metre contours. Above 2300 m, upgrading and downwasting are both recorded in the same 10 m elevation zones.

The diagrams of figure 4.1 that compare large-scale maps which show both downwasting and upgrading in the same interval show a different pattern of the distribution of change. In each case, zones of downwasting and upgrading extend over the width of the glacier. If the interval contains both downwasting and upgrading, it never passes from one to the other and then back to the first with increasing elevation as the 1955-1959 interval does.

Although this is an argument from analogy and is not conclusive, it suggests that the 1955 map has precision problems that are at least as great as the 1977A map. If this is the case, it would explain the unusual characteristics observed on figure 4.1*b*. If the precision of the 1955 map is less than the magnitude of change between it and the 1959 map, then ΔV and Δh values that are produced for that interval have little real meaning.

The remaining small scale map used in this study, from 1919, has an uncertainty of vertical estimation that is greater than that for the 1955 map. However, the interval between the generation of that map and the 1955 map is sufficiently large so that the majority of the surface is unquestionably downwasting.

4.2 Comparison: Results from Glacier Surveys in Alberta and from *Surfer*

Elevation and volumetric change values for the Athabasca Glacier were previously published in the Inland Waters Directorate report series Glacier Surveys in Alberta (GSAb). As was discussed in chapter two, certain problems were noted with the methodology used in that series. These included the lack of consideration for debris-covered ice, an incorrect method of calculating elevation change and occasional summation oddities.

A total of nine intervals were measured in GSAb between 1959 and 1979. Five of these intervals (1969-1979) were compared with *Surfer* results. Comparisons of the GSAb intervals from 1959 to 1969 were not made for a variety of reasons. First, the 1962 glacier map could not be located, and hence was not converted into a DEM. Thus two of the GSAb intervals (1959-1962; 1962-1965) could not be reproduced. In addition, since the maps from 1959 to 1967 were made using imperial measurements, to compare the GSAb results would require DEMs with elevations that were in imperial measurements. It was not thought worthwhile to make a lengthy and cumbersome series of changes to the various programs written to add two more intervals to the comparison figures.

Table 4.8 shows that volumetric change values calculated by *Surfer* are often of a different magnitude and sometimes of a different sign to those calculated in GSAb, while corresponding mean area values appear to be very similar. With such irregularities noted, it was thought worthwhile to perform correlation tests on the data to determine how strong the relationship was between different measures of mean area and volumetric change. In each interval the coefficient of determination (r^2) was calculated to determine the strength of the relationship between the different methods of calculating the same value.

Comparisons occurred between the two measurements of mean area and between the three methods of calculating volumetric change· using the *Surfer* surfaces (Ss), using *Surfer* to calculate the Haumann method (SH) and Glacier Surveys in Alberta (GSAb).

For each of the tests, the terminus area and each 10 m elevation zone above it in table 4.8 were part of the data set. The summation values which occur in the final row of each of the tables in 4.8 were not included in the test. Table 4.9 shows the results of these tests.

Table 4.8a: The volumetric change of the clear ice surface, 1969 to 1971: A Comparison of Surfer and Glacier Surveys in Alberta results

Elevation Zone (m)	Volumetric Change: *Surfer* Surface (m^3) (,000)	*Surfer* Haumann	GSAb Haumann (,000)	Elevation Change: *Surfer* (m)	GSAb (m)	Mean Areas: *Surfer* (m^2) (,000)	GSAb (,000)
1932-1950	(11.37)	(23.04)	(57.93)	-1.37	-7.00	10.06	8.28
1950-1960	(25.75)	(24.67)	(69.70)	-2.58	-7.91	9.63	8.81
1960-1970	(34.97)	(30.32)	(84.39)	-3.25	-8.63	10.70	9.78
1970-1980	(32.06)	(35.25)	(103.90)	-2.50	-8.06	13.02	12.89
1980-1990	(41.93)	(44.78)	(149.80)	-2.84	-9.42	15.63	15.90
1990-2000	(49.53)	(50.78)	(102.60)	-2.77	-5.62	18.11	18.26
2000-2010	(59.45)	(61.01)	(98.49)	-3.03	-4.86	19.75	20.27
2010-2020	(63.84)	(63.03)	(108.30)	-2.81	-4.69	23.73	23.10
2020-2030	(51.37)	(52.56)	(96.85)	-2.03	-3.62	26.94	26.75
2030-2040	(63.72)	(66.23)	(109.60)	-2.16	-3.75	27.94	29.22
2040-2050	(82.50)	(80.15)	(121.60)	-2.61	-3.64	33.15	33.42
2050-2060	(70.42)	(77.88)	(118.10)	-1.84	-3.08	38.84	38.36
2060-2070	(102.80)	(107.70)	(163.80)	-2.30	-3.91	42.46	41.89
2070-2080	(150.60)	(144.32)	(222.60)	-2.90	-4.16	52.79	53.50
2080-2090	(152.10)	(154.66)	(251.60)	-2.58	-3.95	62.95	63.71
2090-2100	(205.70)	(199.75)	(309.60)	-2.90	-4.56	67.33	67.89
2100-2110	(220.40)	(230.73)	(341.90)	-3.11	-4.75	71.90	71.98
2110-2120	(190.90)	(209.94)	(322.20)	-2.99	-4.90	66.54	65.75
2120-2130	(213.00)	(219.63)	(345.70)	-3.49	-6.05	56.36	57.15
2130-2140	(238.50)	(236.42)	(371.60)	-4.26	-6.37	57.00	58.34
2140-2150	(193.10)	(190.92)	(345.60)	-3.77	-6.37	54.65	54.26
2150-2160	(214.00)	(230.76)	(394.50)	-3.56	-6.93	55.65	56.92
2160-2170	(317.00)	(321.30)	(499.70)	-4.62	-7.08	70.78	70.58
2170-2180	(422.50)	(433.34)	(627.20)	-3.88	-5.97	106.20	105.10
2180-2190	(604.80)	(596.66)	(778.80)	-5.39	-7.42	105.50	105.00
2190-2200	(672.00)	(729.39)	(917.50)	-5.75	-7.66	120.80	119.80
2200-2210	(1,114.00)	(1,051.20)	(1,274.00)	-5.98	-6.98	186.10	182.60
2210-2220	(1,216.00)	(1,167.65)	(1,452.00)	-7.09	-9.27	156.40	156.70
2220-2230	(938.00)	(964.87)	(1,264.00)	-7.61	-10.06	127.80	125.60
2230-2240	(928.60)	(979.45)	(1,281.00)	-7.74	-10.76	119.50	119.10
2240-2250	(1,130.00)	(1,106.61)	(1,444.00)	-8.86	-12.73	117.30	113.40
2250-2260	(1,207.00)	(1,148.36)	(1,508.00)	-9.60	-12.17	125.50	123.90
2260-2270	(1,051.00)	(1,045.04)	(1,428.00)	-9.23	-12.36	115.10	115.50
Totals	**(12,070.00)**	**(12,080.00)**	**(16,760.00)**	**-5.48**	**-7.71**	**2186.00**	**2174.00**

Table 4.8b: The volumetric change of the clear ice surface, 1971 to 1973: A Comparison of Surfer and Glacier Surveys in Alberta results							
	Volumetric Change			Elevation Change		Mean Areas	
Elevation Zone (m)	*Surfer* Surface (m^3) (,000)	*Surfer* Haumann (m^3) (,000)	GSAb Haumann (,000)	*Surfer* (m)	GSAb (m)	*Surfer* (m^2) (,000)	GSAb (,000)
1936-1950	(30.57)	(33.16)	(9.15)	-4.30	-1.55	7.07	5.90
1950-1960	(32.66)	(33.09)	(6.78)	-4.70	-0.89	7.79	7.62
1960-1970	(35.33)	(37.56)	(10.15)	-4.19	-1.10	8.48	9.23
1970-1980	(45.21)	(50.27)	(19.22)	-3.68	-1.48	13.26	12.98
1980-1990	(48.64)	(50.28)	(15.04)	-3.32	-0.98	15.60	15.34
1990-2000	(58.94)	(52.46)	(12.00)	-3.15	-0.65	18.69	18.46
2000-2010	(60.46)	(59.65)	(26.28)	-3.09	-1.31	20.19	20.06
2010-2020	(54.76)	(54.93)	(28.96)	-2.67	-1.37	21.43	21.14
2020-2030	(46.40)	(47.78)	(23.79)	-1.84	-0.92	26.11	25.86
2030-2040	(49.24)	(47.31)	(15.81)	-1.70	-0.53	29.80	29.83
2040-2050	(44.68)	(42.80)	(5.40)	-1.36	-0.17	32.11	31.76
2050-2060	(28.12)	(23.51)	10.10	-0.83	0.28	36.72	36.05
2060-2070	2.68	0.35	36.60	0.04	0.83	44.52	44.10
2070-2080	9.35	18.26	60.95	0.15	1.15	52.74	53.00
2080-2090	34.68	29.39	78.46	0.56	1.25	63.77	62.80
2090-2100	40.85	33.81	90.34	0.58	1.27	71.28	71.14
2100-2110	49.69	53.71	105.20	0.71	1.53	69.22	68.78
2110-2120	74.95	77.96	109.80	1.30	1.74	62.49	63.09
2120-2130	80.88	82.16	49.71	1.39	0.82	60.12	60.62
2130-2140	81.17	75.55	93.36	1.44	1.67	56.42	55.90
2140-2150	59.50	59.36	76.18	1.14	1.44	52.04	52.90
2150-2160	59.17	66.09	84.25	0.99	1.38	62.61	61.05
2160-2170	85.79	98.39	117.50	1.22	1.64	71.92	71.67
2170-2180	187.70	222.95	297.10	1.93	2.87	104.40	103.50
2180-2190	373.60	375.52	347.00	3.66	3.28	105.80	105.80
2190-2200	438.60	465.07	434.60	3.65	3.61	120.50	120.40
2200-2210	762.40	676.06	671.70	4.33	3.59	191.30	187.10
2210-2220	795.30	733.66	755.10	5.43	5.03	150.30	150.10
2220-2230	630.20	633.61	653.90	5.26	5.51	120.10	118.70
2230-2240	663.30	700.24	687.80	6.03	5.77	120.90	119.20
2240-2250	765.70	754.54	713.40	6.56	6.10	117.90	117.00
2250-2260	772.40	780.28	639.50	6.80	5.31	117.00	120.40
2260-2270	735.60	746.65	546.90	7.07	5.09	107.70	107.50
2270-2280	542.20	552.44	324.00	6.89	4.03	78.34	80.41
2280-2290	292.00	290.26	33.09	6.04	0.75	44.01	44.12
2290-2300	170.40	182.83	(111.00)	5.71	-3.47	32.06	31.99
2300-2310	159.00	167.75	(142.40)	5.43	-4.79	29.05	29.73
2310-2320	180.40	179.31	(157.00)	6.44	-5.52	30.61	28.44
2320-2330	185.50	190.03	(144.00)	6.22	-5.08	27.33	28.34
2330-2340	212.30	232.32	(128.00)	6.01	-3.68	37.00	34.78
2340-2350	335.10	364.88	24.49	6.08	0.46	52.95	53.24
Totals	**8,245.00**	**8,311.00**	**6,186.00**	**3.36**	**2.49**	**2492.00**	**2480.00**

Table 4.8c: The volumetric change of the clear ice surface, 1973 to 1975: A Comparison of Surfer and Glacier Surveys in Alberta results

Elevation Zone (m)	Volumetric Change			Elevation Change		Mean Areas	
	Surfer Surface (m^3) (,000)	*Surfer* Haumann (m^3) (,000)	GSAb Haumann (,000)	*Surfer* (m)	GSAb (m)	*Surfer* (m^2) (,000)	GSAb (,000)
1938-1950	(22.05)	(25.20)	(10.40)	-3.61	-2.54	5.39	4.09
1950-1960	(35.83)	(35.42)	(14.87)	-5.41	-2.30	6.65	6.47
1960-1970	(43.77)	(43.77)	(4.24)	-5.58	-0.48	8.57	8.84
1970-1980	(61.35)	(67.75)	(9.19)	-5.07	-0.71	13.22	12.94
1980-1990	(90.62)	(86.89)	(22.19)	-5.78	-1.54	15.27	14.41
1990-2000	(78.79)	(85.04)	(15.80)	-4.90	-0.88	17.82	17.96
2000-2010	(85.22)	(86.92)	(16.20)	-4.27	-0.81	20.50	20.00
2010-2020	(95.79)	(96.59)	(33.38)	-4.53	-1.60	20.70	20.86
2020-2030	(98.34)	(97.75)	(40.01)	-4.01	-1.57	25.67	25.48
2030-2040	(95.22)	(97.82)	(40.80)	-3.32	-1.39	29.45	29.36
2040-2050	(103.60)	(106.44)	(47.96)	-3.25	-1.51	32.00	31.76
2050-2060	(117.40)	(126.75)	(63.49)	-3.44	-1.82	35.32	34.89
2060-2070	(140.50)	(141.23)	(88.41)	-3.30	-2.03	42.99	43.55
2070-2080	(148.80)	(156.80)	(121.70)	-2.98	-2.33	52.39	52.22
2080-2090	(170.20)	(170.61)	(139.70)	-2.86	-2.26	62.02	61.80
2090-2100	(158.60)	(160.39)	(174.10)	-2.41	-2.53	68.61	68.80
2100-2110	(136.40)	(145.20)	(124.10)	-2.06	-1.89	66.57	65.67
2110-2120	(140.90)	(139.54)	(120.30)	-2.28	-1.94	61.81	62.01
2120-2130	(134.00)	(141.53)	(121.40)	-2.21	-2.05	60.52	59.21
2130-2140	(133.50)	(133.14)	(119.80)	-2.44	-2.20	54.56	54.48
2140-2150	(148.90)	(147.58)	(141.40)	-2.64	-2.53	55.22	55.88
2150-2160	(175.70)	(180.37)	(186.90)	-2.92	-3.10	60.86	60.30
2160-2170	(198.20)	(205.86)	(233.10)	-2.80	-3.25	71.93	71.70
2170-2180	(184.20)	(207.32)	(253.20)	-2.14	-2.75	91.67	92.06
2180-2190	(166.00)	(178.79)	(231.60)	-1.69	-2.30	100.50	100.70
2190-2200	(201.60)	(174.36)	(233.50)	-1.78	-2.02	115.80	115.60
2200-2210	(139.60)	(144.48)	(224.90)	-0.92	-1.37	164.00	164.20
2210-2220	(36.34)	(42.44)	(144.50)	-0.23	-0.95	151.20	152.10
2220-2230	45.13	54.11	(65.56)	0.41	-0.56	117.00	117.10
2230-2240	111.30	109.45	(32.39)	1.05	-0.29	110.20	111.70
2240-2250	115.50	142.82	(35.59)	1.05	-0.32	112.00	111.20
2250-2260	186.30	187.05	(6.78)	1.78	-0.06	113.30	113.00
2260-2270	260.80	255.39	2.24	2.55	0.02	108.50	112.00
2270-2280	224.20	235.39	(80.39)	2.26	-0.80	101.10	100.50
2280-2290	157.90	154.38	(185.80)	2.53	-3.16	60.17	58.80
2290-2300	86.58	91.21	(276.20)	2.08	-7.50	37.86	36.83
2300-2310	31.44	40.35	(354.20)	1.08	-11.42	34.65	31.01
2310-2320	15.44	12.30	(405.30)	0.31	-14.31	30.70	28.32
2320-2330	2.71	(2.85)	(417.30)	0.03	-14.01	33.07	29.79
2330-2340	(17.69)	(12.45)	(408.40)	-0.42	-13.86	35.02	29.46
2340-2350	7.82	9.35	(407.40)	0.19	-9.31	49.52	43.76
2350-2360	74.75	52.50	(479.00)	1.05	-6.64	71.53	72.15
Totals	**(2,039.00)**	**(2,097.00)**	**(6,129.00)**	**-0.81**	**-2.45**	**2526.00**	**2503.00**

Table 4.8d: The volumetric change of the clear ice surface, 1975 to 1977: A Comparison of Surfer and Glacier Surveys in Alberta results							
	Volumetric Change			Elevation Change		Mean Areas	
Elevation Zone (m)	*Surfer* Surface (m^3) (,000)	*Surfer* Haumann	GSAb Haumann (,000)	*Surfer* (m)	GSAb (m)	*Surfer* (m^2) (,000)	GSAb (,000)
1938-1950	24.24	14.76	4.89	5.84	2.15	2.30	2.27
1950-1960	25.15	36.10	29.29	6.47	6.02	5.49	4.87
1960-1970	45.35	47.63	40.94	6.52	5.49	6.84	7.46
1970-1980	56.87	71.62	45.95	6.48	3.73	10.74	12.32
1980-1990	91.79	95.00	52.65	6.19	3.70	14.63	14.23
1990-2000	91.40	95.31	49.81	5.19	2.80	17.43	17.79
2000-2010	99.84	101.52	52.81	5.12	2.63	20.00	20.08
2010-2020	109.10	110.62	61.47	5.07	2.92	20.65	21.05
2020-2030	108.50	111.86	63.18	4.20	2.47	25.42	25.58
2030-2040	106.90	107.69	54.79	3.48	1.80	30.11	30.44
2040-2050	105.60	109.08	51.42	3.45	1.62	31.41	31.74
2050-2060	119.60	123.48	56.28	3.28	1.58	35.56	35.62
2060-2070	120.00	128.91	51.46	2.78	1.17	43.51	43.98
2070-2080	146.80	146.84	58.85	2.86	1.15	51.14	51.17
2080-2090	184.70	172.84	73.28	3.06	1.21	60.97	60.56
2090-2100	166.80	175.54	74.40	2.60	1.11	66.38	67.03
2100-2110	164.90	159.64	53.25	2.55	0.81	65.21	65.74
2110-2120	134.50	135.01	21.92	2.27	0.36	60.94	60.88
2120-2130	132.30	123.20	(4.72)	2.23	-0.08	57.90	58.94
2130-2140	114.30	127.56	(33.58)	2.23	-0.61	53.65	55.05
2140-2150	142.10	139.94	(40.57)	2.70	-0.72	54.75	56.35
2150-2160	185.00	193.01	(4.74)	3.24	-0.08	58.45	59.26
2160-2170	269.00	274.31	63.54	3.94	0.92	67.34	69.07
2170-2180	314.40	317.22	89.64	3.44	0.97	89.94	92.41
2180-2190	241.20	251.45	8.46	2.18	0.08	104.50	105.70
2190-2200	196.10	203.88	(46.68)	1.73	-0.40	115.80	116.70
2200-2210	232.70	232.94	(25.38)	1.48	-0.16	157.60	158.60
2210-2220	114.70	119.79	(131.60)	0.69	-0.83	157.50	158.50
2220-2230	(3.71)	(10.88)	(271.00)	-0.05	-2.34	114.30	115.80
2230-2240	(68.43)	(47.88)	(309.10)	-0.63	-2.74	111.60	112.80
2240-2250	(37.73)	(82.11)	(386.00)	-0.32	-3.49	105.80	110.60
2250-2260	(141.20)	(141.24)	(471.10)	-1.23	-4.31	111.10	109.30
2260-2270	(292.80)	(277.48)	(656.40)	-2.39	-5.53	113.00	118.70
2270-2280	(287.30)	(306.20)	(689.30)	-3.03	-6.97	98.27	98.90
2280-2290	(190.50)	(186.40)	(586.80)	-3.31	-10.18	57.65	57.64
2290-2300	(105.10)	(107.44)	(515.70)	-2.86	-13.73	37.25	37.56
2300-2310	(48.57)	(52.99)	(483.70)	-1.63	-15.26	33.16	31.70
2310-2320	(22.26)	(17.84)	(467.30)	-0.64	-16.23	29.35	28.79
2320-2330	9.80	7.93	(437.90)	0.25	-14.90	29.82	29.39
2330-2340	20.41	20.31	(448.50)	0.71	-14.93	31.75	30.04
2340-2350	2.27	(0.59)	(512.60)	0.01	-12.40	44.80	41.34
2350-2360	(3.07)	23.60	(482.10)	-0.01	-7.94	63.91	60.72
Totals	**2,676.00**	**2,748.00**	**(5,946.00)**	**1.10**	**-2.39**	**2468.00**	**2487.00**

Table 4.8e: The volumetric change of the clear ice surface, 1977 to 1979: A Comparison of Surfer and Glacier Surveys in Alberta results

Elevation Zone (m)	Volumetric Change *Surfer* Surface (m^3) (,000)	Volumetric Change *Surfer* Haumann	Volumetric Change GSAb Haumann (,000)	Elevation Change *Surfer* (m)	Elevation Change GSAb (m)	Mean Areas *Surfer* (m^2) (,000)	Mean Areas GSAb (,000)
1944-1950	(7.41)	(7.44)	(1.60)	(1.65)	(0.99)	1.82	1.61
1950-1960	(9.86)	(13.65)	(5.56)	-2.72	(1.23)	5.61	4.52
1960-1970	(17.77)	(16.13)	(9.88)	-2.27	(1.39)	7.02	7.11
1970-1980	(17.72)	(21.21)	(9.44)	-2.02	(0.86)	10.03	10.98
1980-1990	(22.38)	(24.14)	(2.42)	-1.76	(0.17)	14.31	14.21
1990-2000	(18.45)	(20.98)	(1.99)	-1.08	(0.11)	17.53	18.09
2000-2010	(25.21)	(23.39)	(0.95)	-1.25	(0.05)	19.03	19.08
2010-2020	(23.34)	(25.83)	(2.73)	-1.25	(0.13)	20.64	21.02
2020-2030	(29.37)	(25.05)	(17.24)	-1.11	(0.65)	25.98	26.52
2030-2040	(21.06)	(17.36)	(17.94)	-0.68	(0.59)	30.27	30.40
2040-2050	(15.85)	(16.81)	(19.18)	-0.50	(0.60)	31.89	31.97
2050-2060	(21.89)	(18.84)	(20.93)	-0.61	(0.60)	35.16	34.89
2060-2070	(5.35)	(8.80)	(9.02)	-0.15	(0.21)	42.65	42.96
2070-2080	(15.91)	(14.71)	(16.52)	-0.33	(0.33)	49.98	50.06
2080-2090	(29.10)	(29.62)	(36.25)	-0.52	(0.61)	59.13	59.43
2090-2100	(30.94)	(46.16)	(56.02)	-0.43	(0.83)	67.52	67.50
2100-2110	(69.95)	(62.74)	(71.56)	-1.01	(1.06)	67.44	67.51
2110-2120	(32.32)	(60.79)	(82.80)	-0.60	(1.44)	59.35	57.50
2120-2130	(41.54)	(35.74)	(73.89)	-0.72	(1.25)	58.09	59.11
2130-2140	(26.44)	(25.99)	(73.90)	-0.49	(1.30)	55.74	56.85
2140-2150	(27.64)	(24.44)	(76.97)	-0.51	(1.41)	53.80	54.59
2150-2160	(15.21)	(21.60)	(67.61)	-0.27	(1.21)	56.83	55.88
2160-2170	(39.13)	(36.48)	(77.47)	-0.61	(1.17)	65.10	66.21
2170-2180	(71.37)	(63.41)	(106.40)	-0.76	(1.12)	95.11	94.96
2180-2190	(58.26)	(55.38)	(101.00)	-0.53	(0.92)	109.90	109.80
2190-2200	(23.08)	(36.26)	(78.40)	-0.18	(0.68)	115.90	115.30
2200-2210	(134.30)	(132.82)	(175.40)	-0.84	(1.05)	166.40	167.00
2210-2220	(132.70)	(153.31)	(208.00)	-0.84	(1.27)	163.00	163.80
2220-2230	(68.02)	(93.72)	(149.30)	-0.56	(1.26)	118.40	118.50
2230-2240	(71.15)	(102.39)	(146.30)	-0.59	(1.29)	113.50	113.40
2240-2250	(127.50)	(99.37)	(156.80)	-1.15	(1.33)	114.40	117.90
2250-2260	(77.02)	(80.34)	(158.50)	-0.70	(1.31)	114.70	121.00
2260-2270	(16.45)	(23.00)	(106.60)	-0.11	(0.83)	120.50	128.40
2270-2280	5.68	10.12	(68.52)	0.05	(0.72)	90.79	95.16
2280-2290	1.90	8.91	(66.26)	0.08	(1.23)	51.79	53.87
2290-2300	(0.20)	2.37	(78.00)	0.06	(2.14)	34.28	36.45
2300-2310	10.60	7.80	(73.98)	0.25	(2.44)	29.14	30.32
2310-2320	36.98	32.36	(47.32)	1.31	(1.63)	27.25	29.03
2320-2330	31.04	29.41	(53.11)	1.18	(1.47)	30.10	36.13
2330-2340	18.04	16.64	(77.91)	0.39	(1.98)	32.58	39.35
2340-2350	22.00	17.20	(83.26)	0.53	(1.45)	47.93	57.42
2350-2360	15.34	3.35	(104.30)	0.28	(1.45)	61.23	71.93
Totals	**(1,202.00)**	**(1,310.00)**	**(2,791.00)**	**(0.49)**	**(1.09)**	**2492.00**	**2558.00**

Table 4.9: Correlation (r^2) of Mean area and Volume Calculations: Glacier Surveys in Alberta and *Surfer*

Years	n	Mean Areas	Volume		
			SH - GSAb	Ss - GSAb	SH - Ss
1969-1971	33	0.9995	**0.9953**	0.9906	*0.9969*
1971-1973	41	0.9993	**0.7648**	**0.7788**	*0.9948*
1973-1975	42	0.9986	**0.0012**	**0.0006**	0.9944
1975-1977	42	0.9987	**0.7078**	**0.7078**	*0.9933*
1977-1979	42	**0.9955**	**0.4756**	**0.4192**	*0.9343*

Values which fail the runs test at $\alpha = 0.05$ in **bold**.
Values that show autocorrelation at $\alpha = 0.01$ in *italics*.

Producing correlation coefficients by comparing differing measures of area and volume runs a strong risk of violating the assumptions of independence made for the test of correlation. When the values were tested for autocorrelation by comparing the magnitude of their residuals with the independent variable and tested using the runs test, it was clear that the relationship between paired values was often not independent.

4.2.1 Mean Area

The areas within elevation zones were calculated using planimetry in the GSAb reports and then averaged with the corresponding elevation zone in the preceding year. This was evidently a highly accurate method of calculation, because the area results produced using *Surfer* were very similar. Table 4.9 shows that the five correlation coefficients are extremely high. One of the five values (1977-1979) failed the runs test. No autocorrelation at the $\alpha = 0.01$ level was reported. This suggests that measurements of area made using *Surfer* and GSAb display a definite relationship. This was known to be the case before the test was made, however.

4.2.2 Volume Comparison

The raw values produced in *Surfer* were read into the program DVOL.BAS which calculated the volumetric change of the clear ice surface using the Haumann method, which was the method used to calculate volumetric change in GSAb. In addition, the volumetric change of the clear ice surface was calculated using the surfaces of the DEMs as described in section 3.3.3.4.1. These *Surfer*-generated values of volumetric change were compared with the corresponding values published in GSAb. Table 4.8 displays the values as produced. Table 4.9 shows the statistical comparison between them. Table 4.9 shows that when *Surfer*-generated values of Ss or SH are compared with GSAb values, the results fail the runs test nine out of ten times. When Ss and SH values are compared, the runs test is passed, but four of the five values display autocorrelation at the $\alpha = 0.01$ level.

4.2.3 Analysis of Comparison of Glacier Surveys in Alberta and *Surfer* Results

With relationships between the results that showed high correlation and high autocorrelation, it was decided to produce a series of scattergrams of the comparisons of the measurements in the five intervals. Figure 4.11*a-e* consists of scattergrams based on table 4.8*a-e*. The different scattergrams show the relationship between volumetric change values produced using Surfer to calculate volumetric change using the Haumann method and as reported in GSAb. Values in adjacent elevations are connected by lines. This allows the relationship between the paired values to be traced from the terminus to the upper limit of measurement.

If the DEMs faithfully represent the maps, and if there were no problems calculating volumetric change in the GSAb reports, then the volumetric change values

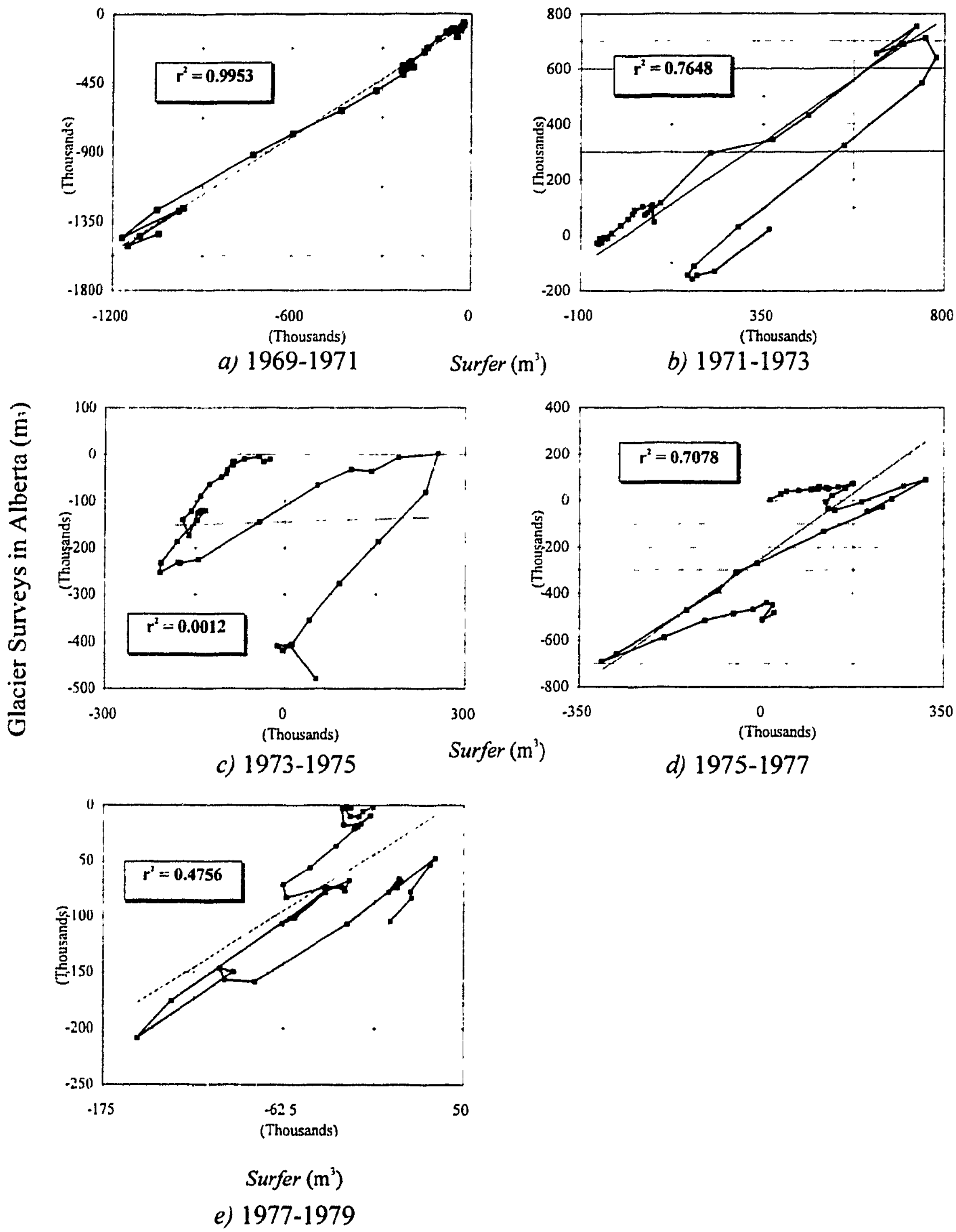


Figure 4.11: Scattergrams showing the relationship between Haumann calculations of volumetric change using DEMs and from Glacier Surveys in Alberta. Measurements from successive elevation zones connected by lines. This shows that often several different linear relationships exist in the data set of each interval.

calculated by GSAb and *Surfer* would be very similar. The correlation (r^2) between them would be high, and the Scattergram plot of the values would be linear and would pass through the origin.

Upon examination of the scattergrams which make up figure 4.11, only figure 4.11*a* shows these characteristics. Figures 4.11*b-e* vary markedly from this ideal. In each of these figures, many series of adjacent elevation zones display marked linear correlation. However, this does not extend over all of the elevation zones. Sudden breaks occur, which are followed by a series of successive elevation zones which display a different slope coefficient. It is necessary to establish why this nonlinear relationship exists between values calculated using different methods.

4.2.3.1 The Sources of Inaccuracy of Volumetric Change Calculation

It is unlikely that problems caused due to differences in width of borders, as discussed in appendix 4, are a contributing factor to the lack of agreement between the GSAb measures of volumetric change and those produced with *Surfer*. If this was a factor, then the *Surfer* Haumann (SH) volume figures would not agree with the *Surfer* surface (Ss) volumetric figures as well as they do. It is only in the terminus region of the glacier that substantial differences exist between SH and Ss values (*cf* table 4.8*d,e*), suggesting that in all other areas the width of the glacier is constant or near-constant between mappings.

The Haumann method required both the mean areas of 10 m or 25 foot contour elevation zones on the two maps between which change was measured and the area between identical contour lines on different maps. In the GSAb report series, the area between two contour lines on the same map was simple to calculate and was done using planimetry. Comparison between maps was more difficult to accomplish. Identically placed grids were overlaid on both maps. The area on the glacier between a given contour line and the grid is calculated for both maps; the area between corresponding contour lines on different maps is the difference between the corresponding areas. It was mentioned that it was desirable to place the grids parallel to the lateral limits of the glacier. This suggests, although it was not explicitly stated, that different grids may have been used for different series of elevation zones to more closely approximate parallel sides (Reid and Charbonneau, 1975).

This method of comparison would produce consistent results if each of the grids used were placed in precisely the same position on each map. The slightest imprecision of placement of the grids, either in translation or rotation, would be sufficient to have substantial effects on the results of volumetric change calculations. The slope of the line of best fit between volumetric change calculated with grid placement inconsistencies and more accurate measures would not necessarily have a slope of 1. However, volumetric change calculations made using the same pair of grids would produce values that showed a consistent relationship with the more accurate values. When a series of volumetric change calculations for this same interval that were based on a different pair of grids are compared however, the different inconsistency in measurement would result in a relationship for the next few elevation zones that had a different slope than the previous pair of comparison grids.

The scattergrams of figures 4.11*b-e* suggest that this was the source of the inaccuracy: the grids used in GSAb did not precisely match from map to map and from elevation zone to elevation zone.

CHAPTER FIVE: DISCUSSION, CONCLUSIONS AND FURTHER RESEARCH

5.0 Introduction

This chapter will present conclusions that can be made from the results presented in the previous chapter. The conclusions will be discussed first for the *Surfer* results, and then for the results of the comparison of Glacier Surveys in Alberta and *Surfer*. The chapter will conclude with ideas for further research that were suggested by this thesis.

5.1 Discussion of *Surfer* Results

Area, elevation and volumetric change results presented in chapter four (figures 4.1-4.4) demonstrate that the Athabasca Glacier changed substantially in the time of record. Between 1919 and 1979, the glacier lost a great amount of volume, shrunk in surface area, and downwasted significantly. Change was most pronounced and unidirectional in the lower elevation zones (terminus-2050 m; figures 4.4*a,b*). That portion of the glacier recorded loss of elevation, reduced areas and a general reduction of volume consistent with the recession of the terminus (figure 4.2). In higher elevations, change was more variable from year to year in a given elevation zone.

Over the entire portion of the glacier studied, figures 4.5*b* and 4.9 suggest that there may be a systematic error involved in the production of the 1971 map, distorting the values of volumetric and elevation change produced for the 1969-71 and 1971-73 intervals.

The results that compare the two maps made at different scales in 1977 (section 4.1.3.2.1) suggest that small-scale mapping is insufficiently precise for determining change over short intervals or where change is slight. The 1977A map was shown to have substantial differences from the 1977T map above the 2180 m contour line. Also discussed was the strong possibility that the 1955 map had precision problems that were at least as serious as the 1977A map. However, when the 1919 and 1955 maps were compared, most of the surface showed that the magnitude of downwasting recorded in those years was greater than the uncertainty of vertical estimation.

This leads to the restating of an important point. When a series of maps is created to measure change, the interval between them should be determined by a comparison of the expected magnitude of elevation change and the precision of vertical estimation of the method of mapping used. Despite the problem of high imprecision in both the 1919 and 1955 maps, their comparison (figure 4.1*a*) reveals that very little of the elevation change recorded between those two DEMs was less than the uncertainty of vertical estimation. This is in contrast with many of the higher-precision DEMs that show substantial portions of the glacier having elevation change that is less than the uncertainty of vertical estimation. With the large-scale maps, increasing the interval between mapping decreased the proportion of the glacier over which Δh was less than σ_c (figures 4.1, 4.6, 4.7; table 4.6). This suggests that in this period two years was too short an interval between maps.

In the early 1960s, when it was decided to produce the Inland Waters map series, two years between maps did not seem to be an unreasonably short length of time between measurements. Figure 2.2*a-c* shows three air photos of the Athabasca Glacier in 1938, 1948 and 1959. These photos demonstrate that in those years the glacier was changing rapidly. Terminus recession studies that the WSC performed on the Athabasca Glacier (Davis *et al.*, 1962) and archival information also showed that the glacier was changing rapidly.

There was no way of knowing that the twenty years following 1959 would see the recession of the terminus slow substantially (figure 2.2*c-e*; WSC, 1982), presumably caused by an alteration in the rate of change of volume and elevation change. It is unfortunate that only after the project was cancelled did the Athabasca Glacier resume a rapid recession (figure 2.2*f*).

5.2 Discussion of Comparison Results

Glacier Surveys in Alberta (GSAb) results were compared with results produced using *Surfer*. Tests were performed and presented in the previous chapter which demonstrated that of the five intervals compared, most were heavily autocorrelated, a result to be expected when the same source was used to produce the data being compared. In addition, it was shown in chapter two that the values of elevation change that were produced in GSAb used an incorrect method of calculation and were hence not comparable to values of elevation change produced using *Surfer*. The result produced from GSAb that compares best with *Surfer*-calculated values is mean area in elevation zones. When a correlation is performed on these values, the worst r^2 of the five intervals is 0.9955. This shows marked linearity.

It is reasonable to expect that the difficulties observed in the five compared intervals of the Athabasca Glacier are not confined to the compared intervals. It is likely that the non-compared intervals have similar calculation problems.

5.3 Conclusions

This thesis has demonstrated that raster DEMs can be used effectively to calculate the elevation and volumetric change of the ablation area of the Athabasca Glacier. It has been shown that in the period 1919-1979 the glacier lost 2.34×10^8 m^3 of volume and downwasted significantly. Additionally, it is apparent that there are substantial precision problems associated with small-scale portrayals of the glacier in both the horizontal and vertical directions (tables 3.3, 3.7), although only vertical uncertainty values were dealt with in detail. Only in the case where there is a considerable interval between maps, such as exists between 1919 and 1955, is it advantageous to use small scale maps in a study of this nature. In addition, the calculations of elevation change made in GSAb were demonstrated to be misleading, since they attempted to calculate a vertical measure from horizontal volumetric change information.

The package *Surfer for Windows* was used extensively to prepare DEMs and to generate raw data for elevation and volumetric change, as well as area results. Although it is excellent at calculating and displaying surfaces of change (*cf* figures 4.1, 4.6, 4.7), the process of producing quantitative volumetric and elevation change values for individual elevation zones was a laborious and time-consuming task. Most of the effort involved concerned the writing of GSMAC and QBASIC programs (especially *SURELEV.BAS* and *DVOL.BAS*; appendix 7). Once these programs were written, the production of values was swift. It is hoped that later versions of *Surfer* will make these calculations easier.

5.4 Further Research

There are many potential avenues of investigation suggested by this thesis. The most obvious is incorporating the effects of the uncertainty of horizontal estimation into the uncertainty figure results. It would be possible to do this using *Surfer*, however it would be a very difficult task requiring lengthy GSMAC and QBASIC programming, copious amounts of hard disk space, and long periods of time to run the programs. It was the combination of these factors that kept the quantification of horizontal uncertainty out of this thesis.

It has been stated that differences in albedo cause substantial differences in the precision of measurement (Young and Arnold, 1977). It would be interesting to investigate the effects that different surfaces (ice, debris, bedrock and snow) had on the relationship between the quantity of measured elevation and volumetric change compared with the increased quantities of uncertainty due to higher imprecision. Since the uncertainty of vertical estimation on snow-covered areas, above 2450-2500 m on the Athabasca, can be as great as ± 10 or 20 metres (Young *et al.*, 1978), it is likely that any volume and elevation change measures from snow-covered areas would be of little use; nevertheless, it would be of interest to quantify the variation in precision of the other land covers on the maps of that area.

In addition, the maps of the glacier which could not be found for this thesis could be located and incorporated into the database. These include the 1962 map and the 1977 DEM prepared from aerial orthophotography (1977D). If the 1962 map was added to the database, the entire Inland Waters volumetric change record could be compared to the *Surfer*-calculated values. If the 1977D DEM was acquired, then all of the results of Young *et al.* (1978) could be reproduced and compared using current techniques.

Unfortunately, the 1962 map proved difficult to locate. No Canadian university has a copy. Neither the Geological Survey of Canada nor the National Hydrology Research Centre (NHRI) has a copy, and the World Data Centre 'A' for Glaciology in Boulder does not have a copy either. Despite its elusiveness, it would be surprising if all copies of this map had been lost. The 1977D DEM may prove easier to locate.

The 1959 and 1965 maps did not contour the entire surface of the glacier. The photos from which the maps were produced still exist, however. It would be worthwhile to produce new maps that showed the entire ablation area of the glacier from those photos. This would not only allow the production of quantitative information for the entire ablation area in those years, but would also allow the direct comparison of different maps made from the same information. The empirical generation of values of mapping precision for those years would be the result.

Flights of aerial photography of the Athabasca Glacier were flown in 1938 and 1948, before the first large-scale map was made (see appendix 3). If the ground control points on the 1959 photographs could be used to georeference the earlier images, maps and DEMs could be generated for each of those years. Efforts in this direction should concentrate on the 1948 photography. It depicts the ice surface to the first icefall and is only slightly overexposed on the glacier surface. The earliest air photos, from 1938, are not well suited to mapping. These photos do not go as far up the ice surface as the 1948 images, and in addition are very overexposed on the glacier surface. Despite the fact that the 1938 photography is intermediate in age between the 1919 and 1955 maps, it is unlikely that any worthwhile measurements of the glacier surface can be made from these photos.

It has been discovered that at least three maps or DEMs of the Athabasca Glacier made from information postdating 1979 have been created or are in the process of being created. This is taking place at NHRI with 1986 and 1992 data and at the Canadian Centre for Remote Sensing with 1995 data. Obviously, it would be beneficial to acquire any or all of these DEMs to compare with the DEMs produced for this thesis. With the

1986 and 1995 DEMs, only formatting difficulties stand in the way of comparisons. The 1992 map is in the process of being made (Brugman and Demuth, in preparation; P. Vachon, personal communication).

This thesis did not compute the volume or depth of the Athabasca Glacier. To make such measures, a DEM portraying the base of the glacier is required. Although studies are being undertaken at NHRI to determine this (Brugman and Demuth, in preparation), the basal DEM of the Athabasca Glacier does not cover the entire base of the glacier that is studied in this thesis; most depth measures are restricted to the relatively flat portions of the glacier surface. In addition, the depth studies which have been undertaken reveal a substantial precision problem in some of the measurements (Kite and Reid, 1977). When the precision problems have been dealt with and more of the base of the glacier is known, depth and volume measurements could be made by comparing the existing DEMs with a DEM representing the base of the glacier.

Kite and Reid (1977) estimated the volumetric change of the Athabasca Glacier between the glacial maximum (which they took to be in 1870) and 1971. The 1870 surface of the glacier was based on the position of terminal moraines, the crests of lateral moraines, and estimated contour lines. Wallace (1995) performed a similar series of calculations for Peyto Glacier, using a terrestrial photograph of the glacier taken in 1896 to measure the position of the glacial terminus and estimate its surface in that year. Luckman (1988) produced a detailed study of the recession of the Athabasca Glacier; using the terminal positions reported in that article, it would be possible to produce a DEM that estimated the contours of the glacier at its position of maximum advance and in other years in which the terminus position is recorded but no map exists. These estimates could be compared with DEMs representing empirical measurements to quantify volumetric and surface change over the time periods in question.

The Athabasca was one of seven glaciers studied in the Inland Waters program, the others being Sentinel, Sphinx, Nadahini, Kokanee, Bugaboo and Saskatchewan. It would be interesting to produce a time-series of DEMs using the procedure given in this thesis for each of the other six studied glaciers. If this is done, it is suggested that the Saskatchewan Glacier receive high priority. Its proximity to the Athabasca Glacier and the fact that it was mapped from photos taken only a few days before or after the photos of the Athabasca would lend itself to useful comparisons. Other Canadian glaciers studied in the International Hydrological Decade that also have large-scale maps suitable for comparison using the methodology of this thesis would merit a study similar to this one. Clearly however, many of the problems experienced in this study will be encountered using other maps.

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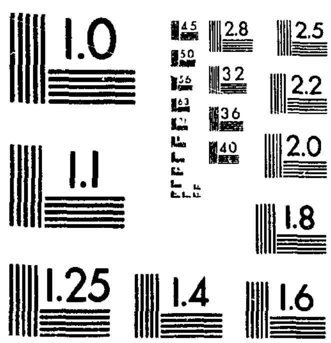
3 of/de 3

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT



PRECISION℠ RESOLUTION TARGETS

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APPENDIX 1: EARLY PHOTOGRAPHY OF THE ATHABASCA GLACIER

The Whyte Museum of the Canadian Rockies contains the records of the Alpine Club of Canada. It also contains many early photographs taken in the contiguous mountain parks, as well as diaries, travel records, and other archival information concerning the early explorers of the parks.

The photographic record of the glaciers observed begins in depth with the work of Bill Gibbons, Byron Harmon and George Noble. This work is contained in the Visual Reference File of the museum. A list of the catalogue numbers of the relevant photography follows. The pictures are arranged in order of subject headings in the cataloguing system that the Whyte Museum employs: Mount Athabasca, Athabasca Glacier and Columbia Icefield are the subjects listed.

At the end of the table the catalogue numbers of the two earliest photographs of the Athabasca are reproduced. These are not part of the visual reference file of the museum. They are part of the Mary Schaffer photographic collection that the museum possesses.

Table A1.1: Early photography of the Athabasca Glacier in the Whyte Museum of the Canadian Rockies

Photographic Target	Catalogue		Photographer	Date	Comments
	Placement	Number			
Mount Athabasca	V227	3542-3547 4355-4368	Bill Gibbons	1945-1958	
	V263 NA71	1834	Byron Harmon	1948	Mt. Athabasca and Sunwapta River from Banff-Jasper Highway
		1896		1914	Mt. Athabasca
		2142		1924	from Mt. Bryce showing Mts. Athabasca, Andromeda and Castleguard
		2214, 2215		1924	Mt. Athabasca and Hilda Peak
		2236, 2253, 2364, 2393	Byron Harmon and Lewis Freeman	1924	series of photos taken of Mount Athabasca during Columbia Icefield 1924 expedition (see Freeman, 1925)
		5713, 5714, 5716	Byron Harmon	ca. 1940	Mt. Athabasca
		5911		1924	Lewis Freeman on Wilcox Pass looking at Mt. Athabasca
		6163		1924	Mt. Athabasca and Mt. Andromeda

Table A1.1: Early photography of the Athabasca Glacier in the Whyte Museum of the Canadian Rockies

Photographic Target	Catalogue		Photographer	Date	Comments
	Placement	Number			
		6170		1924	Lewis Freeman looking at Mt. Athabasca
	V469	567-582	George Noble	unknown	
Athabasca Glacier	V227	3013, 3015, 3525	Bill Gibbons	?	Athabasca Glacier
	V263 NA71	1812, 1813	Byron Harmon	1914-1917	Athabasca Glacier
		1815, 1816 1819-1821		1924, 1948	
		1833, 1835, 1846, 1847		1948	
		1958, 1960, 1962		1914	men on horses at ice cave in Athabasca Glacier
		2039		1914	Athabasca Glacier
		2170,2173		1924	Athabasca Glacier from Wilcox Pass
		2364		1924	Mt. Athabasca, toe of Athabasca Glacier and headwaters of Sunwapta River
		5718, 5722		ca. 1940	Athabasca Glacier

Table A1.1: Early photography of the Athabasca Glacier in the Whyte Museum of the Canadian Rockies

Photographic Target	Catalogue		Photographer	Date	Comments
	Placement	Number			
		5735	?	?	snowmobile on Athabasca Glacier
		5859	Byron Harmon	ca.1923	Athabasca Glacier
		5912		1923?	Athabasca Glacier from Wilcox Pass
		5916		1924	Ulysses LaCasse and Soapy Smith on Athabasca Glacier
	V469	567-584	George Noble	?	Athabasca Glacier
Columbia Icefield	V227	764, 1217-1235, 1238, 3312-3321, 3460-3478, 3701, 3702, 3704-3707, 3794, 4171, 4174	Bill Gibbons	?	people and busses in front of Athabasca Glacier
	V263 NA71	2156, 2158, 2163	Byron Harmon	1924	taken on the Columbia Icefields expedition of 1924
	V469	567-582	George Noble	?	Columbia Icefield

Table A1.1: Early photography of the Athabasca Glacier in the Whyte Museum of the Canadian Rockies

Photographic Target	Catalogue		Photographer	Date	Comments
	Placement	Number			
Schaffer pictures	V527	PS-131	Mary Schaffer	1906?	Pictures Mt. Athabasca and Athabasca Glacier from a fairly high elevation. Shows most of the toe of the glacier.
	V527	NG#4		1908?	Glass negative view across the toe of the glacier looking up the valley of Dome glacier (which is also visible)

APPENDIX 2: WATER RESOURCES BRANCH REPORTS

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APPENDIX 3: AERIAL PHOTOGRAPHY OF THE ATHABASCA GLACIER AT THE NATIONAL AIR PHOTO LIBRARY

Table A3.1: Air photos of the Athabasca Glacier

Date (y/m/d)	Photo		AASL (m)	Lens (mm)	Scale at 1950 metres	Comments
	Roll	Numbers				
1938/7/10	A6619	42-44	5,700	152.4	1:24,600	Only lower part of glacier visible. Glacier overexposed.
1948/9/19	A11729	5-7	6,100	152.4	1:27,200	Visible to lowest icefall. Glacier somewhat overexposed.
1950/9/11	A13068	225	5,200	152.4	1:21,300	No stereo. Lowest part of glacier only.
1951?	A13013	110,111			1:	
1955/8/28	A14895	73,74	10,700	152.4	1:57,400	Picture used for 1955 mapping.
1958/7/16	A16085	184-186	2,900	152.4	1:6,200	Terminus of glacier only. Plane turning for new line of photos.
1959	A16703	16-24	3,410	152.4	1:9,600	Excellent contrast on glacier. Ground control visible.
		3-9	4,110	152.4	1:14,200	Excellent contrast on glacier. Ground control visible. Used for 1959 mapping.
1962/7/31	A17597	7-17	4,110	152.4	1:14,200	Excellent contrast on glacier. Ground control visible.
1966/8/22	A19684	29,30	9,100	152.4	1:46,900	Good overview of area.
1968/7/7	A20888	3,4	4,420	152.4	1:16,200	Some snow on glacier. Otherwise a good shot.
1970/9/13	A21540	188,189	11,600	152.307	1:63,400	Good overview shot. Glacier not well defined.

APPENDIX 3: AERIAL PHOTOGRAPHY OF THE ATHABASCA GLACIER AT THE NATIONAL AIR PHOTO LIBRARY

Table A3.1: Air photos of the Athabasca Glacier

Date (y/m/d)	Photo		AASL (m)	Lens (mm)	Scale at 1950 metres	Comments
	Roll	Numbers				
1970/9/28	A21547	8-11	9,100	152.307	1:46,900	
1972/8/27	A23015	38-40	12,200	152.51	1:67,200	Very small scale. Heavy shadows on glacier.
1977/8/16	A40065	1-73	10,000	153.12		Air photos used to create the Parks Canada (1977) map.
1977/8/17	A40066	1-118	10,000	153.12		
1979/7/9	A25165	99,100	10,400	153.37	1:55,100	Good overview shot.
1979/8/7	A25266	89-92	2,700	151.95	1:4,900	Large scale. Only terminus of glacier shown.
1981/8/4	A25818	10,11	3,410	152.618	1:9,600	Large scale. Only terminus of glacier shown.
1981	A25818	6-8	3,570	152.618	1:10,600	Large scale. Only terminus of glacier shown.
1992/8/01	A31610	1-128	3120-4160	304.969	varies	Colour photography. Across the glacier at various elevations.
1992/8/13	A31609	34,35	10,060	152.844	1:53,100	Good overview shot.

APPENDIX 4: COMPARISON OF HAUMANN METHOD AND GEOMETRIC METHOD OF CALCULATING VOLUMETRIC DIFFERENCES

The Haumann method is a means of calculating the volumetric change between two corresponding contour lines on different maps. This appendix presents in detail how the Haumann method compares with a geometric method of calculating volumetric difference. To do this, the two nested polyhedrons portrayed in figure 2.3 are compared.

In figure 2.3,

Δh = height of the elevation zone
L_{A1} = lower length of polyhedron *A*
L_{A2} = upper length of polyhedron *A*
L_{B1} = lower length of polyhedron *B*
L_{B2} = upper length of polyhedron *B*
W_A = width of polyhedron *A*
W_B = width of polyhedron *B*
V_A = volume of polyhedron *A*
V_B = volume of polyhedron *B*
ΔV = volumetric difference between polyhedrons *A* and *B*, $V_A - V_B$

The geometric difference in volume is V_A subtract V_B:

$$V_A = (L_{A_2} - L_{B_2})\Delta h \cdot W_A + \frac{(L_{A_1} - L_{A_2})\Delta h \cdot W_B}{2}$$

$$V_B = \frac{(L_{B_1} - L_{B_2})\Delta h \cdot W_B}{2}$$

Thus,

$$\Delta V = (L_{A_2} - L_{B_2})\Delta h \cdot W_A + \frac{(L_{A_1} - L_{A_2})\Delta h \cdot W_A}{2} - \frac{(L_{B_1} - L_{B_2})\Delta h \cdot W_B}{2}$$

$$\Delta V = \Delta h\left(W_A\left(L_{A_2} - L_{B_2} + \frac{L_{A_1} - L_{A_2}}{2}\right) - \frac{W_B(L_{b_1} - L_{B_2})}{2}\right)$$

The Haumann method of calculation is not as direct. It is produced as follows:

$$F_1 = W_B(L_{A_1} - L_{A_2})$$

$$F_2 = W_B(L_{B_1} - L_{B_2})$$

$$\Delta F_1 = W_B(L_{A_1} - L_{B_1})$$

$$\Delta F_2 = W_B(L_{A_2} - L_{B_2})$$

$$F_1'' = W_A(L_{A_1} - L_{A_2})$$

$$F_m = \frac{F_2 + F_1''}{2} = \frac{W_B(L_{B_1} - L_{B_2}) + W_A(L_{A_1} - L_{A_2})}{2}$$

$$dh = \frac{(\Delta F_1 + \Delta F_2)\Delta h}{F_1 + F_2} = \frac{(L_{A_1} - L_{B_1} + L_{A_2} - L_{B_2})\Delta h}{L_{A_1} - L_{A_2} + L_{B_1} - L_{B_2}}$$

When combined, ΔV is calculated from the above equations as follows:

$$\Delta V = F_m \cdot dh$$

$$\Delta V = \Delta h\left(\frac{W_B(L_{B_1} - L_{B_2}) + W_A(L_{A_1} - L_{A_2})}{2}\right)\left(\frac{L_{A_1} - L_{B_1} + L_{A_2} - L_{B_2}}{L_{A_1} - L_{A_2} + L_{B_1} - L_{B_2}}\right)$$

This is not equal to the previous ΔV equation. However, if it is assumed that $W_A = W_B$, then both equations simplify to

$$\Delta V = \frac{\Delta h \cdot W_A}{2}(L_{A_1} - L_{B_1} + L_{A_2} - L_{B_2})$$

Q.E.D.

Only when $W_A = W_B$ does the Haumann method produce a result identical to the geometric method. Additionally, it can be seen that the calculations of the Haumann method assume that the receding face of the glacier is a flat-sided prism. As $\Delta h \rightarrow 0$, this assumption becomes more accurate. However, for the calculations in Glacier Surveys in Alberta, Δh was 25 feet (1959-1969) or 10 metres (1969-1979). These values are large enough to allow substantial error in the calculations in those reports.

APPENDIX 5: REFERENCES OF GLACIER SURVEYS IN ALBERTA, INLAND WATER REPORTS

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APPENDIX 6: BIBLIOGRAPHIC LIST OF SOURCE MAPS

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Athabasca Glacier, Alberta Canada, scale 1:10,000, July 25, 1969. Inland Waters Branch, Glacier Map Series No. 3, Sheet Number 6, produced by the Inland Waters Branch 1971, printed by the Surveys and Mapping Branch, Department of Energy, Mines and Resources, Ottawa, Canada.

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Athabasca Glacier, Alberta Canada, scale 1:10,000, August 3, 1973. Inland Waters Directorate, Glacier Map Series No. 5, Sheet Number 6, produced by the Inland Waters Directorate 1976, Environmental Management Service, Department of the Environment, printed by the Surveys and Mapping Branch, Department of Energy, Mines and Resources, Ottawa, Canada.

Athabasca Glacier, Alberta Canada, scale 1:10,000, August 27, 1975. Inland Waters Directorate, Glacier Map Series No. 6, Sheet Number 6, produced by the Inland Waters Directorate, Fisheries and Environment Canada 1977, printed by the Surveys and Mapping Branch, Department of Energy, Mines and Resources, Ottawa, Canada.

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APPENDIX 7: PROGRAM CODES

A7.1: DVOL.BAS

```
DECLARE SUB elsom (ye$(), es'(), fp$(), nh'(), es$(),
nv'())
DECLARE SUB elsomi (ye$(), es'(), fp$(), nh'(), es$(),
nv!())
DECLARE SUB elsomo (ye$(), es'(), fp$(), nh!(), es$(),
nv'())
DECLARE SUB head5 (es$, el', bft!, a!)
DECLARE SUB calc3 (clt!(), be!, en!, st!, a', mxl'(),
yr$(), bft$(), co', oa!, vavg'(), year', hn!, h'(), l'(),
tav'(), planar'(), sm!(), yd!)
DECLARE SUB tots (sm!(), hn', sm$(), yd!)
DECLARE SUB finpr (bft!, cl', co', vavg!, pl!, sig', yd')
DECLARE SUB areap (hcl'(), yr$(), u1'(), u2'())
DECLARE SUB areafiles (be'(), en'(), st!(), ne'(),
elk'(), yc$(), hcl'(), yr$(), u1'(), u2'())
DECLARE SUB arear (be!(), en!(), st'(), ne'(), elk'(),
yc$(), hcl'())
DECLARE SUB head4 (yr$())
DECLARE SUB commer (y!, l', c$)
DECLARE SUB wipe (nv', o'())
DECLARE SUB nvv (nv', o'())
DECLARE SUB smmy (be'(), en'(), st'(), fp$(), yr$(),
nh'(), smy!(), nv'(), s'(), t'())
DECLARE SUB innp (a$, smy'(), fp', nv!(), s'(), t'())
DECLARE SUB head3 (yr$(), year', pest!, n!)
DECLARE SUB head2 (year', pest', yr$(), bft', n')
DECLARE SUB sp (a$, a1$, a2$, a2')
DECLARE SUB haupr (hn', h', l', vavg', planar', co',
shv!(), ef!, sm'())
DECLARE SUB penn (ne'(), elk!(), be'(), en'(), st'(),
yc$(), cho$(), clt'(), mxl!(), yr$(), bft$(), vavg'(),
elz$(), li!(), l'(), h'(), tav'(), planar'(), shv'(),
sm'(), sm$())
DECLARE SUB fly (fl!(), a', b!)
DECLARE SUB summe (sm'(), bft!, vss', vsh', vgs', sig',
sa!, ag!)
DECLARE SUB limits (hn!, year', yc$(), li!(), l'(), a',
st!)
DECLARE SUB avt1 (h!, tav!(), vavg!, planar', sig!, cl!,
co!, l!, bft')
DECLARE SUB head1 (bft$(), yr$(), st', year', a', hn')
DECLARE SUB hck (l!, cl!, h!)
DECLARE SUB avt2 (tav!(), vavg!, planar!, sig', bft!)
DECLARE SUB hncs (hn!, cs!)
DECLARE SUB tv (tav!(), shv!())
DECLARE SUB hp (c1!, c2!, s1!, s2!, s3!, gv!, ga!)
DECLARE SUB svh (shv!(), vavg!, shv!, planar')
DECLARE SUB sf (year!, s!, f!)
DECLARE SUB hop (hn!, y1$, y2$)
DECLARE SUB haumann (hau!(), elk!(), yr$(), ne!)
DECLARE SUB rh (hau!(), elk'(), yr$(), ne!)
DECLARE SUB haucalc (hau!(), yr$(), elk!(), ne')
DECLARE SUB hv (hau'(), year', elf', cl!)
DECLARE SUB original2 (be'(), en'(), st'(), yc$(),
cho$(), ne!(), elk'(), aclv'(), fl'(), yr$())
DECLARE SUB acquire (a!, year!, aclv'(), p', hn')
DECLARE SUB ho (hn', sur!, bft!, av', year!, bfy', aclv',
c$)
DECLARE SUB cm (bfy!, b$, c$)
DECLARE SUB h5 (hn!)
DECLARE SUB o2op (hn!, yc$)
DECLARE SUB cn (hn!, cs!)
DECLARE SUB hes (h1$, h2$, h3$, h4$, hn')
DECLARE SUB sigfig (a!, b!, n!)
DECLARE SUB haunec (a!, year!, hn', bft!)
DECLARE SUB op (aa$, yc$, cho$(), s!, f')
DECLARE SUB finis1 (a!, year', cl !(), ane')
DECLARE SUB area (clt!(), year!, bft!, planar', pest')
DECLARE SUB vmatrix (clt'(), year', pest!, mxl'(), bft',
mean!, sig!)
DECLARE SUB mexl (mxl'(), clt'(), year', pest', bft')
DECLARE SUB sigma (mxl'(), mean', sig')
DECLARE SUB closer (a', b!)
DECLARE SUB ny (yc$())
DECLARE SUB egg (a!, eg!)
DECLARE SUB original1 (be!(), en'(), st'(), ne'(),
elk!(), yc$(), elz$(), sur$(), bio$(), bft$(), bfy$(),
cho$(), a!(), r!, rw!(), fl!(), av'())
DECLARE SUB cvt (aa$, yc$, cho$())
DECLARE SUB calc2 (a!, year!, aclv!(), l!, h', fl!(), c$)
DECLARE SUB lh (l!, h!)
DECLARE SUB refine (a!, year!, av!(), rw!(), c$, fl'())
DECLARE SUB char (aa$, yc$, cho$(), r!)
DECLARE SUB choi (elf!, cl!, bio!, bfy!, e!)
DECLARE SUB calc1 (rw!(), bfy', bft', sur', av'(), c$,
fl!())
DECLARE SUB nom1 (yc$, elz$, cl!, sur$, bio$, bft$, bfy$,
a!(), c$, e!, fl!())
DECLARE SUB reader1 (yc$, tfy$, a!(), c$, e!, fl!())
DECLARE SUB nom2 (e!, a!())
DECLARE SUB ch (a!, a$)
DECLARE SUB chf (a!, a$)
DECLARE SUB lire1 (a'(), e')
DECLARE SUB np (a!, b!)
DECLARE SUB minz (v')
DECLARE SUB stre (v!)
DECLARE SUB fc (a$, b!)
DECLARE SUB blanked (a', year', sur!, b!, f', s!)

DIM bfy$(-1 TO 4), cho$(4), es$(11), fp$(8), ofy$(4)
DIM sm$(3, 8), sur$(-1 TO 1), yc$(13), ye$(11), yr$(13)

DIM a(4), aclv(2, 2, -1 TO 4, 2, -1 TO 1), av(2)
DIM clt(2, 13, -1 TO 4, 2, -1 TO 1), es(10, 2, 11, 5),
fl(4, 4), flg(13)
DIM h(2), hau(5 TO 10, 9, 9, -1 TO 1), hcl(13, 0 TO 50,
2), l(2), li(2, 2)
DIM mxl(2, 3), nh(5), nv(5), planar(2), rw(2, 0 TO 2, -1
TO 4, 3, -1 TO 1)
DIM s(5), shv(3), sm(3, 8), smy(5), t(5), tav(2, 3),
u1(12, 2), u2(12, 2)
DIM vavg(2)

DIM SHARED jr(13), ss1, ee1

DATA 1,10,1
DATA 9,11,2
DATA 12,13,1
DATA 9,5
DATA 1910,2000, 2000,2050, 2050,2100
DATA 2100,2150, 2150,2200, 2200,2250
DATA 2250,2300, 2300,2350, 2350,2400
DATA 1900,2000, 2000,2100, 2100,2200, 2200,2300,
2300,2400
DATA "35", "59", "65", "67", "69", "71", "73"
DATA "75", "77", "79", "78", "19", "55"
DATA "h", "j", "k", "l", "n", "o", "p", "q", "r"
DATA "v", "w", "x", "y", "z"
DATA "s", "t", "u"
DATA "o", "i"
DATA "m", "n"
DATA "a", "b", "c", "d", "e", "f"
DATA "z", "v", "a"
DATA "fm-", "fn-", "fh-", "sm-", "sn-", "sh ", "em", "en"
DATA 5, 5, 5
DATA 28, 28, 24
DATA "19", "55", "59", "65", "67", "69", "71", "73",
"75", "77", "79"

FOR a = 1 TO 3. READ be(a)  READ en(a)  READ st(a)  NEXT
a
FOR a = 1 TO 2. READ ne(a)  NEXT a
FOR a = 1 TO 2  FOR b = 1 TO ne(a)  FOR c = 1 TO 2  READ
elk(a, b, c)· NEXT c  NEXT b  NEXT a
FOR a = 1 TO 13  READ yr$(a)  yc$(a) = CHR$(64 + a)  NEXT
a
FOR a = 1 TO 2  FOR b = 1 TO ne(a)  READ elz$(b, a)  NEXT
b: NEXT a
FOR a = -1 TO 1  READ sur$(a)  NEXT a
FOR a = 0 TO 1  READ bio$(a)  NEXT a
FOR a = 1 TO 2. READ bft$(a)  NEXT a
FOR a = -1 TO 4  READ bfy$(a). NEXT a
FOR a = 2 TO 4· READ cho$(a)  NEXT a
FOR a = 1 TO 8  READ fp$(a)  NEXT a
FOR a = 1 TO 3  READ nv(a)  NEXT a
FOR a = 1 TO 3  READ nh(a)  NEXT a
FOR a = 1 TO 11  READ ye$(a)  NEXT a

b$(0) = ","

ss1 = 1  ee1 = 3

CLS

' If the raw data has been read into the clv files
properly, r1 = 2.
' If not, r1 = 1  (r1 = 2 is MUCH faster)
r1 = 2

' If the CLV files have been produced, then r2 = 2  This
will read directly
' from them to produce the HCL files
r2 = 2

' If the HCL files have been converted into FM, FN, and
FH files, then r3 = 2
' This will allow the program to proceed directly to
producing summary values
r3 = 1

' If the summary files have already been created, then r4
= 2.  If they need
' to be produced, then r4 = 1
r4 = 1

' If areafiles need to be produced, r5=1  Otherwise,
r5=2.
r5 = 1

r6 = 1

IF r1 = 1 THEN
   CALL original1(be(), en(), st(), ne(), elk(), yc$(),
elz$(), sur$(), bio$(), bft$(), bfy$(), cho$(), a(), r1,
rw(), fl(), av())
END IF

IF r2 = 1 THEN
   CALL original2(be(), en(), st(), yc$(), cho$(), ne(),
elk(), aclv(), fl(), yr$())
```

```
      CALL haumann(hau(), elk(), yr$(), ne(1))
   END IF

   IF r3 = 1 THEN
      CALL penn(ne(), elk(), be(), en(), st(), yc$(),
cho$(), clt(), mxl(), yr$(), bft$(), vavg(), elz$(),
li(), l(), h(), tav(), planar(), shv(), sm(), sm$())
   END IF

   IF r4 = 1 THEN
      CALL smmy(be(), en(), st(), fp$(), yr$(), nh(), smy(),
nv(), s(), t())
   END IF

   IF r5 = 1 THEN
      CALL areafiles(be(), en(), st(), ne(), elk(), yc$(),
hcl(), yr$(), u1(), u2())
   END IF

   IF r6 = 1 THEN
      CALL elsom(ye$(), es(), fp$(), nh(), es$(), nv())
   END IF

SUB acquire (a, year, aclv(), p, hn)
' inputs all values from current contour line of the
"clv-" file of the
' appropriate year
   c$ = CHR$(44)
   FOR sur = -1 TO 1
      CALL blanked(a, year, sur, b, f, pest)
      FOR bft = 1 TO 2
         FOR bfy = b TO f STEP pest
            FOR av = 1 TO 2
               INPUT #av, aclv(p, av, bfy, bft, sur)
               CALL ho(hn, sur, bft, av, year, bfy,
aclv(p, av, bfy, bft, sur), c$)
            NEXT av
         NEXT bfy
      NEXT bft
   NEXT sur
END SUB

SUB area (clt(), year, bft, planar, pest)
   pl = (clt(2, year, 0, bft, 0) + clt(2, year + pest,
-1, bft, 0)) / 2
   CALL sigfig(pl, planar, 4)
END SUB

SUB areafiles (be(), en(), st(), ne(), elk(), yc$(),
hcl(), yr$(), u1(), u2())
   PRINT TAB(0), "areafiles"
   CALL arear(be(), en(), st(), ne(), elk(), yc$(),
hcl())
   CALL areap(hcl(), yr$(), u1(), u2())
END SUB

SUB areap (hcl(), yr$(), u1(), u2())
   PRINT TAB(0), "areap"
   fo1$ = "am-hist txt"
   fo2$ = "an-hist txt"
   OPEN fo1$ FOR OUTPUT AS #1
   OPEN fo2$ FOR OUTPUT AS #2
   CALL head4(yr$())
   FOR co = 1910 TO 2400 STEP 10
      cl = (co - 1900) / 10
      IF co / 50 = INT(co / 50) THEN
         f1 = 1
         IF co = 1950 THEN
            c1$ = "Terminus"
         ELSE
            CALL ch(co - 50, c1$)
         END IF
         CALL ch(co, c2$)
      ELSE
         f1 = 0
      END IF
      FOR bft = 1 TO 2
         IF f1 = 1 THEN
            PRINT #bft, c1$, "-"; c2$, ",",
         END IF
         FOR year = 1 TO 12
            IF year = 1 THEN
               y1 = 12
            ELSE
               y1 = year - 1
            END IF
            u1(year, bft) = u1(year, bft) + hcl(y1, cl,
bft)
            u2(year, bft) = u2(year, bft) + hcl(y1, cl,
bft)
            IF f1 = 1 THEN
               CALL commer(year, 12, c$)
               CALL sigfig(u1(year, bft), h, 4)
               CALL ch(h, h$)
               PRINT #bft, h$, c$,
               u1(year, bft) = 0
            END IF
         NEXT year
         IF f1 = 1 THEN
            PRINT #bft,
         END IF
      NEXT bft
   NEXT co
   FOR bft = 1 TO 2
      PRINT #bft, "Totals,",
      FOR year = 1 TO 12
         IF year = 1 THEN
            y1 = 12
         ELSE
            y1 = y - 1
         END IF
         CALL commer(year, 12, c$)
         CALL sigfig(u2(year, bft), h, 4)
         CALL ch(h, h$)
         PRINT #bft, h$; c$;
      NEXT year
   NEXT bft
   CALL closer(1, 2)
END SUB

SUB arear (be(), en(), st(), ne(), elk(), yc$(), hcl())
   PRINT TAB(0), "arear ",
   FOR a = ss1 TO ee1
      CALL egg(a, eg)
      CALL ch(a, aa$)
      FOR year = be(a) TO en(a) STEP st(a)
         PRINT yc$(year);
         fi$ = "hcl-" + aa$ + yc$(year) + "a.txt"
         OPEN fi$ FOR INPUT AS #1
         FOR elf = 1 TO ne(eg)
            FOR co = (elk(eg, elf, 1) + 10) TO elk(eg,
elf, 2) STEP 10
               cl = (co - 1900) / 10
               FOR sur = -1 TO 1
                  CALL blanked(a, year, sur, b, f, p)
                  FOR bft = 1 TO 2
                     FOR bfy = b TO f STEP p
                        INPUT #1, ar
                        IF bfy = 0 AND sur = 0 AND NOT
(year = 9 AND a = 2) THEN
                           ar = ar / 1000
                           hcl(year, cl, bft) = ar
                        END IF
                     NEXT bfy
                  NEXT bft
               NEXT sur
            NEXT co
         NEXT elf
         CLOSE #1
      NEXT year
   NEXT a
END SUB

SUB avt1 (h, tav(), vavg, planar, sig, cl, co, l, bft)
   IF h = 1 THEN
      CALL avt2(tav(), vavg, planar, sig, bft)
      cl = 0
   ELSEIF h = 2 THEN
      CALL avt2(tav(), vavg, planar, sig, bft)
      cl = INT(l)
   ELSE
      cl = co
   END IF
END SUB

SUB avt2 (tav(), vavg, planar, sig, bft)
   tav(bft, 1) = tav(bft, 1) + vavg
   tav(bft, 2) = tav(bft, 2) + planar
   tav(bft, 3) = tav(bft, 3) + sig
END SUB

SUB blanked (a, year, sur, b, f, s)
' determines which blanking files need to be read
   IF sur = 0 THEN
      b = -1. f = 1: s = 1
      IF a = 1 THEN
         IF year = 1 THEN
            b = 0
         ELSEIF year = 10 THEN
            f = 0
         END IF
      ELSEIF a = 2 THEN
         IF year = 9 THEN
            b = 0: f = 3: s = 3
         ELSEIF year = 11 THEN
            b = 0. f = 4: s = 4
         END IF
      ELSEIF a = 3 THEN
         IF year = 12 THEN
            b = 0
         ELSEIF year = 13 THEN
            f = 0
         END IF
      END IF
   ELSE
      b = -1: f = 0: s = 1
      IF a = 1 THEN
         IF year = 1 THEN
            b = 0
         END IF
      ELSEIF a = 2 THEN
         IF year = 9 THEN
            b = 0: f = 3: s = 3
         ELSEIF year = 11 THEN
            b = 0: f = 4: s = 4
         END IF
      ELSEIF a = 3 THEN
         IF year = 12 THEN
            b = 0: f = 1: s = 1
         END IF
      END IF
   END IF
END SUB
```

```
SUB calc1 (rw(), bfy, bft, sur, av(), c$, fl())
' calculates areas and volumes within blanking files from
within and without
' files for given surface, blanking file type and
blanking file year
   ' Area
   av(2) = (rw(2, 0, bfy, bft, sur) + (rw(2, 2, 2, 3,
sur) - rw(2, 1, bfy, bft, sur))) / 2
   ' Area multiplier
   IF rw(2, 0, bfy, bft, sur) <> 0 THEN
      s = av(2) / rw(2, 0, bfy, bft, sur)
   ELSE
      s = 1
   END IF
   ' Volume
   av(1) = rw(1, 0, bfy, bft, sur) / s

   FOR n = 1 TO 2
      CALL chf(av(n), av$)
      IF fl(2, n) = 0 THEN
         PRINT #n + 4, av$,
         fl(2, n) = 1
      ELSE
         PRINT #n + 4, c$, av$,
      END IF
   NEXT n
END SUB

SUB calc2 (a, year, aclv(), l, h, fl(), c$)
' produces "hcl-" files from "clv-" files  HCL files
represent elevation
' zones-- the volume defined by the surface gven and two
elevation zones.
' The area is the planimetric area between the two
contour lines and within
' the blanking polygons
   FOR sur = -1 TO 1
      CALL blanked(a, year, sur, b, f, p)
      FOR bft = 1 TO 2
         FOR bfy = b TO f STEP p
            FOR av = 1 TO 2
               hcl = aclv(l, av, bfy, bft, sur) - aclv(h,
av, bfy, bft, sur)
               CALL chf(hcl, a$)
               IF fl(3, av) = 0 THEN
                  PRINT #(av + 2), a$;
                  fl(3, av) = 1
               ELSE
                  PRINT #(av + 2), c$, a$,
               END IF
            NEXT av
         NEXT bfy
      NEXT bft
   NEXT sur
END SUB

SUB calc3 (clt(), be, en, st, a, mxl(), yr$(), bft$(),
co, oa, vavg(), year, hn, h(), l(), tav(), planar(),
sm(), yd)
' outputs area and volume figures for final .txt output
   FOR bft = 1 TO 2
      CALL hck(l(bft), co, h(bft))
      CALL vmatrix(clt(), year, st, mxl(), bft,
vavg(bft), sig)
      CALL area(clt(), year, bft, planar(bft), st)
      CALL summe(sm(), bft, vavg(bft), 0, 0, sig,
planar(bft), 0)
      CALL avtl(h(bft), tav(), vavg(bft), planar(bft),
sig, cl, co, l(bft), bft)
      IF h(bft) = 2 THEN
         FOR y = 1 TO 3
            x = tav(bft, y)
            CALL sigfig(x, tav(bft, y), 4)
         NEXT y
         CALL finpr(bft, cl, co, tav(bft, 1), tav(bft,
2), tav(bft, 3), yd)
      ELSEIF h(bft) = 0 THEN
         CALL finpr(bft, cl, co, vavg(bft), planar(bft),
sig, yd)
      END IF
   NEXT bft
END SUB

SUB ch (a, a$)
   a$ = MID$(STR$(a), 2)
END SUB

SUB char (aa$, yc$, cho$(), r)
' opens "RAW-" file for input or output, depending on the
value of "r".
   FOR cha = 2 TO 4
      ofy$ = "raw-" + aa$ + yc$ + cho$(cha) + ".txt"
      IF r = 1 THEN
         OPEN ofy$ FOR OUTPUT AS #cha
      ELSEIF r = 2 THEN
         OPEN ofy$ FOR INPUT AS #cha
      END IF
   NEXT cha
   FOR cha = 5 TO 6
      ofy$ = "clv-" + aa$ + yc$ + cho$(cha - 2) + ".txt"
      OPEN ofy$ FOR OUTPUT AS #cha
   NEXT cha
END SUB

SUB chf (a, a$)
   a$ = STR$(a)
END SUB

SUB choi (elf, cl, bio, bfy, e)
' determines whether z-min needs to be read
   IF elf = 1 AND cl = 0 AND bio = 0 AND bfy = 0 THEN
      e = 1
   ELSE
      e = 0
   END IF
END SUB

SUB closer (a  b)
   FOR c = a TO b
      CLOSE #c
   NEXT c
END SUB

SUB cm (bfy, b$, c$)
   IF bfy <> -1 THEN
      b$ = c$
   ELSE
      b$ = ""
   END IF
END SUB

SUB cn (hn, cs)
   IF hn = 1 THEN
      cs = 5
   ELSE
      cs = 4
   END IF
END SUB

SUB commer (y, l, c$)
   IF y = 1 THEN
      c$ = ""
   ELSE
      c$ = ","
   END IF
END SUB

SUB cvt (aa$, yc$, cho$())
' opens (modified) input file (CLV) and opens (HCL)
output file
   FOR a = 1 TO 2
      fi$ = "clv-" + aa$ + yc$ + cho$(a + 2) + " txt"
      fo$ = "hcl-" + aa$ + yc$ + cho$(a + 2) + " txt"
      OPEN fi$ FOR INPUT AS #a
      OPEN fo$ FOR OUTPUT AS #(a + 2)
   NEXT a
END SUB

SUB egg (a, eg)
' determines which elevation groups to use
   IF a = 3 THEN
      eg = 2
   ELSE
      eg = 1
   END IF
END SUB

SUB elsom (ye$(), es(), fp$(), nh(), es$(), nv())
   PRINT "Elsom"
   CALL elsomi(ye$(), es(), fp$(), nh(), es$(), nv())
   CALL elsomo(ye$(), es(), fp$(), nh(), es$(), nv())
END SUB

SUB elsomi (ye$(), es(), fp$(), nh(), es$(), nv())
   PRINT TAB(5), "elsomi"
   FOR year = 1 TO 10
      f$ = ye$(year) + ye$(year + 1) + " txt"
      FOR bft = 1 TO 2
         fi$ = fp$(bft + 3) + f$
         OPEN fi$ FOR INPUT AS #1
         FOR a = 1 TO nh(bft)
            INPUT #1, a$
         NEXT a
         FOR el = 1 TO 11
            INPUT #1, es$(el)
            FOR vl = 1 TO nv(bft)
               INPUT #1, es(year, bfr, el, vl)
            NEXT vl
            IF es$(el) = "Totals" THEN el = 11
         NEXT el
         FOR vl = 1 TO nv(bft)
            es(year, bft, 2, vl) = es(year, bft, 2, vl) +
es(year, bft, 1, vl)
         NEXT vl
         CLOSE #1
      NEXT bft
   NEXT year
   es$(2) = "Termini-2000"
END SUB

SUB elsomo (ye$(), es(), fp$(), nh(), es$(), nv())
   PRINT TAB(5); "elsomo"
   c$ = ","
   OPEN "el-m.txt" FOR OUTPUT AS #1
   OPEN "el-n txt" FOR OUTPUT AS #2
   FOR el = 2 TO 11
      FOR bft = 1 TO 2
         fo$ = fp$(bft + 6) + f$
         CALL head5(es$(el), el, bft, bft)
         FOR year = 1 TO 10
            y$ = ye$(year) + "-" + ye$(year + 1)
            CALL chf((es(year, bft, el, 1)   es(year,
bft, el, 2)) * 1000, v1$)
            CALL chf((es(year, bft, el, 1) + es(year,
bft, el, 2)) * 1000, v2$)
```

```
            CALL chf((es(year, bft, el, 3) - es(year,
bft, el, 4)) * 1000, v3$)
            CALL chf((es(year, bft, el, 3) + es(year,
bft, el, 4)) * 1000, v4$)
            PRINT #bft, y$, c$, v1$, c$, v2$, c$, v3$;
c$, v4$
         NEXT year
         FOR s = 1 TO 2
            PRINT #bft,
         NEXT s
      NEXT bft
   NEXT el
END SUB

SUB fc (a$, b)
' finds out where colon is in a Surfer-file line
   FOR a = 1 TO LEN(a$)
      b$ = MID$(a$, a, 1)
      IF b$ = " " THEN
         b = a + 1
      END IF
   NEXT a
END SUB

SUB finisl (a, year, clt(), ane)
' inputs area and volume values from identical contour
zones from two
' adjacent years
   IF year = ane THEN
      s1 = 1  s2 = 2
   ELSE
      s1 = 3  s2 = 4
   END IF
   IF a = 2 THEN
      IF ane = 9 THEN
         b = 0  f = 1  p = 1
      ELSEIF ane = 11 THEN
         b = 0  f = -1  p = -1
      ELSE
         BEEP  STOP
      END IF
   END IF
   FOR sur = -1 TO 1
      IF a <> 2 THEN
         CALL blanked(a, ane, sur, b, f, p)
      END IF
      FOR bft = 1 TO 2
         FOR bfy = b TO f STEP p
            FOR av = s1 TO s2
               a1 = av + 1 - s1
               INPUT #av, clt(a1, ane, bfy, bft, sur)
            NEXT av
         NEXT bfy
      NEXT bft
   NEXT sur
END SUB

SUB finpr (bft, c1, co, vavg  pl, sig, yd)
   c$ = CHR$(44)
   CALL ch(c1, c1$)
   CALL ch(co + 10, c2$)
   CALL chf((vavg / 1000), va$)
   CALL ch((sig / 1000), sig$)
   CALL ch((pl / 1000), pl$)
   CALL sigfig(vavg / (yd * 1000), vy, 4)
   CALL sigfig(sig / (yd * 1000), sy, 4)
   CALL chf(vy, vy$)
   CALL ch(sy, sy$)
   PRINT #(bft + 4), c1$, "-", c2$, c$; va$, c$; sig$,
c$, vy$, c$, sy$, c$, pl$
END SUB

SUB fly (fl(), a, b)
' sets all flag values to zero
   FOR ch1 = 1 TO a
      FOR ch2 = 1 TO b
         fl(ch1, ch2) = 0
      NEXT ch2
   NEXT ch1
END SUB

SUB h5 (hn)
   IF hn = 1 THEN
      PRINT #5,
   END IF
END SUB

SUB haucalc (hau(), yr$(), elk(), ne)
   FOR year = 5 TO 9
      f$ = "hv-" + yr$(year) + yr$(year + 1) + ".txt"
      OPEN f$ FOR OUTPUT AS #1
      FOR elf = 1 TO ne
         FOR co = elk(1, elf, 1) TO (elk(1, elf, 2) - 10)
STEP 10
            c1 = (co - elk(1, elf, 1))   10
            CALL hv(hau(), year, elf, c1)
         NEXT co
      NEXT elf
   CLOSE #1
   NEXT year
END SUB

SUB haumann (hau(), elk(), yr$(), ne)
   PRINT TAB(0), "Haumann"
   CALL rh(hau(), elk(), yr$(), ne)
   CALL haucalc(hau(), yr$(), elk(), ne)
END SUB

SUB haunec (a, year, hn, bft)
' determines whether Haumann value of volumetric change
should be output
   hn = 0
   IF a = 1 THEN
      IF bft = 2 THEN
         IF year >= 5 THEN
            hn = 1
         END IF
      END IF
   END IF
END SUB

SUB haupr (hn, h, l, vavg, planar, co, shv(), ef, sm())
   IF hn = 1 AND ef = 0 THEN
      c2 = co + 10
      IF c2 > 1950 THEN
         f1 = 2
      ELSEIF c2 = 1950 THEN
         f1 = 1
      ELSE
         f1 = 0
      END IF
      IF f1 >= 1 THEN
         INPUT #9, gv, ga
         ef = EOF(9)
      ELSE
         gv = 0  ga = 0
      END IF
      INPUT #7, shv
      IF f1 = 1 THEN
         CALL svh(shv(), vavg, shv, planar)
         c1 = INT(l)
      ELSEIF f1 = 0 THEN
         CALL svh(shv(), vavg, shv, planar)
         c1 = 0
      ELSEIF f1 = 2 THEN
         c1 = co
      END IF
      CALL summe(sm(), 3, vavg, shv, gv, 0, planar, ga)
      IF f1 = 1 THEN
         FOR n = 1 TO 3
            box = shv(n)
            CALL sigfig(box, shv(n), 4)
         NEXT n
         CALL hp(c1, c2, shv(1), shv(2), shv(3), gv, ga)
      ELSEIF f1 = 2 THEN
         CALL hp(c1, c2, vavg, shv, planar, gv, ga)
      END IF
   END IF
END SUB

SUB hck (l, c1, h)
   c2 = c1 + 10
   FOR i = 1 TO 2
      IF l > c1 AND l <= c2 THEN
         h = 2
      ELSEIF l > c2 THEN
         h = 1
      ELSE
         h = 0
      END IF
   NEXT i
END SUB

SUB head1 (bft$(), yr$(), st, year, a, hn)
' opens final files for output ("F-M" for total ice,
"F-N" for clear ice)
   FOR bft = 1 TO 2
      otf$ = "f" + bft$(bft) + "-" + yr$(year) + yr$(year
+ st) + ".txt"
      OPEN otf$ FOR OUTPUT AS #(bft + 4)
      CALL head2(year, st, yr$(), bft, 4)
   NEXT bft
   IF hn = 1 THEN
      f$ = "fh-" + yr$(year) + yr$(year + st) + ".txt"
      OPEN f$ FOR OUTPUT AS #8
      CALL head3(yr$(), year, st, 8)
   END IF
END SUB

SUB head2 (year, pest, yr$(), bft, n)
' places header information on each "F-" file opened
   IF bft = 1 THEN
      bf$ = "completeice"
   ELSEIF bft = 2 THEN
      bf$ = "clearice"
   ELSE
      bf$ = ""
   END IF
   o = bft + n
   PRINT #o, "Thevolumetricchangeofthe"
   PRINT #o, bf$, "surface·19"; yr$(year), "to19",
yr$(year + pest)
   PRINT #o, ",Total,,PerYear,,Mean"
   PRINT #o, "Elevation,dV,Standard,dV,Standard,Area"
   PRINT #o, "Zone,Surfer,Deviation,Surfer,Deviation,",
yr$(year), "-", yr$(year + pest)
   PRINT #o, "(m),(m^3),(m^3),(m^3/yr),(m^3/yr),(m^2)"
   PRINT #o, ",(000)"
END SUB

SUB head3 (yr$(), year, pest, n)
   PRINT #n,
"Thevolumetricchangeoftheclearicesurface.19";
   PRINT #n, yr$(year), "to19", yr$(year + pest)
   PRINT #n,
```

```
"AComparisonofSurferandGlacierSurveysinAlbertaresults"
   PRINT #n, ",VolumetricChange, ,MeanAreas"
   PRINT #n, "Elevation,Surfer,,GSAb,Surfer GSAb"
   PRINT #n  "Zone,Surface,Haumann,Haumann"
   PRINT #n, ",(m), m^3),,,(m^2)"
   PRINT #n, ",(000)"
END SUB

SUB head4 (yr$())
' header information for area istogram material
   FOR bft = 1 TO 2
      IF bft = 1 THEN
         d$ = "Complete"
      ELSE
         d$ = "Clear"
      END IF
      PRINT #bft, ",AreaineachElevationZonefor", d$,
"IceSurface"
      PRINT #bft, ",(thousandsofm^2)"
      PRINT #bft, ",",
      FOR y = 1 TO 12
         IF y = 1 THEN
            y1 = 12
         ELSE
            y1 = y - 1
         END IF
         CALL commer(y, 12, c$)
         PRINT #bft, "19", yr$(y1), c$,
      NEXT y
      PRINT #bft,
   NEXT bft
END SUB

SUB head5 (es$, el, bft, a)
   IF bft = 1 THEN
      i$ = "complete"
   ELSE
      i$ = "clear"
   END IF
   PRINT #a, "TheVolumetricChangeof"
   PRINT #a, "elevationzone ", es$
   PRINT #a, ",TotalChange,,AnnualChange"
   PRINT #a, ",Estimates,,Estimates"
   PRINT #a, "Years,Lower,Upper,Lower,Upper"
   PRINT #a, ",(m^3),(m^3),(m^3/yr),(m^3/yr)"
END SUB

SUB hes (h1$, h2$, h3$, h4$, hn)
' determines Haumann header information
   IF hn = 1 THEN
      h1$ = ","
      h2$ = "Haumann,"
      h3$ = "(m^3),"
      h4$ = ","
   ELSE
      h1$ = ""
      h2$ = ""
      h3$ = ""
      h4$ = ""
   END IF
END SUB

SUB hncs (hn, cs)
   IF hn = 1 THEN
      cs = 9
   ELSE
      cs = 6
   END IF
END SUB

SUB ho (hn, sur, bft, av, year, bfy, aclv, c$)
   IF hn = 1 AND sur = 0 AND bft = 2 AND av = 2 THEN
      IF (year = 5 AND bfy = -1) OR bfy = 1 OR (year = 10
AND bfy = 0) THEN
         d$ = ""
      ELSE
         d$ = c$
      END IF
      IF year = 5 AND bfy = -1 THEN
         ac$ = ""
      ELSE
         CALL ch(aclv, ac$)
      END IF
      PRINT #5, ac$, d$;
   END IF
END SUB

SUB hop (hn, y1$, y2$)
   IF hn = 1 THEN
      f$ = "hv-" + y1$ + y2$ + ".txt"
      g$ = "gsa-" + y1$ + y2$ + ".txt"
      OPEN f$ FOR INPUT AS #7
      OPEN g$ FOR INPUT AS #9
   END IF
END SUB

SUB hp (c1, c2, s1, s2, s3, gv, ga)
   c$ = CHR$(44)
   CALL ch(c1, c1$)
   CALL ch(c2, c2$)
   CALL chf(INT(s1 * 1000 + .5) / 1000000, s1$)
   CALL chf(INT(s2 * 1000 + .5) / 1000000, s2$)
   CALL ch(INT(s3 * 1000 + .5) / 1000000, s3$)
   CALL chf(gv, gv$)
   CALL ch(ga, ga$)
   PRINT #8, c1$, "-", c2$, c$, s1$, c$, s2$, c$, gv$,
c$; s3$; c$, ga$
END SUB

SUB hv (hau(), year, elf, c1)
   F1 = hau(year, elf, c1, 1)   hau(year, elf, c1 + 1, 1)
   F2   hau(year + 1, elf, c1, 0)   hau(year + 1, elf, c1
+ 1, 0)
   dF1 = hau(year, elf, c1, 1)   hau(year + 1, elf, c1,
0)
   dF2 = hau(year, elf, c1 + 1, 1)   hau(year + 1, elf,
c1 + 1, 0)
   F1PP = hau(year, elf, c1, 0) - hau(year, elf, c1 + 1,
0)
   F1P - F1PP - F1
   Fm = (F2 + F1PP) / 2
   IF F1 + F2 = 0 THEN
      dh = 0
   ELSE
      dh = -10 * (dF1 + dF2)   (F1 + F2)
   END IF
   vc = Fm * dh
   CALL sigfig(vc, dv, 4)
   CALL chf(vc, dv$)
   PRINT #2, dv$
END SUB

SUB innp (a$, smy(), fp, nv(), s(), t())
   INPUT #1, a$
   IF a$ <> "Totals" THEN
      FOR a = 1 TO nv(fp)
         INPUT #1, s(a)
         smy(a) = smy(a) + s(a)
         t(a) = t(a) + s(a)
      NEXT a
   ELSE
      FOR a = 1 TO nv(fp)
         INPUT #1, z
      NEXT a
   END IF
END SUB

SUB lh (l, h)
   IF l = 1 THEN
      l = 2  h - 1
   ELSE
      l = 1  h - 2
   END IF
END SUB

SUB limits (hn, year, yc$(), li(), l(), a, st)
   CALL ch(a, aa$)
   IF hn = 1 THEN
      INPUT #9, li 'reads GSAB z limit
   END IF
   FOR y = year TO (year + st) STEP st
      y1 = (y - year) / st + 1
      f$ = "raw-" + aa$ + yc$(y) + "z.txt"
      OPEN f$ FOR INPUT AS #10
      INPUT #10, s1, s2, li(y1, 1), li(y1, 2)
      CLOSE #10
   NEXT y

   FOR y = 1 TO 2
      IF li(1, y) < li(2, y) THEN
         l(y) = li(1, y)
      ELSE
         l(y) = li(2, y)
      END IF
   NEXT y
END SUB

SUB lirel (a(), e)
' reads values from Surfer text file
   IF e = 1 THEN
      CALL np(1, 11)
      CALL minz(a(2))
      CALL np(13, 23)
   ELSE
      a(2) = 0
      CALL np(1, 23)
   END IF
   CALL stre(a(3))
   CALL np(25, 29)
   CALL stre(a(4))
END SUB

SUB mex1 (mx1(), clt(), year, pest, bft)
' places divers total volume figures in matrix to be
compared in SIGMA
' subroutine
   FOR s = 1 TO 2
      IF s = 1 THEN
         yp = year
         bfy = 0
      ELSE
         yp = year + pest
         bfy = 1
      END IF
      FOR t = 1 TO 3
         mx1(s, t) = clt(1, yp, bfy, bft, t   2)
      NEXT t
   NEXT s
END SUB

SUB minz (v)
' inputs minimum z-value from Surfer text file
   INPUT #1, a$
   CALL fc(a$, b)
```

```
   FOR a = b TO LEN(a$)
      b$ = MID$(a$, a, 1)
      IF b$ = "t" THEN
         c = a - b
         a = LEN(a$)
      END IF
   NEXT a
   v = VAL(MID$(a$, b, c))
END SUB

SUB nom1 (yc$, elz$, cl, sur$, bio$, bft$, bfy$, a(), c$,
e, fl())
' determines filename of Surfer file to be read next,
passes info to READER1
' subroutine
   cl$ = CHR$(65 + cl)
   tfy$ = yc$ + cl$ + elz$ + bio$ + bfy$ + bft$ + sur$ +
".txt"
   CALL reader1(yc$, tfy$, a(), c$, e, fl())
END SUB

SUB nom2 (e, a())
' reads value from "RAW-" file
   f = 4
   IF e = 1 THEN
      s = 2
   ELSE
      s = 3: a(2) = 0
   END IF
   FOR r = s TO f
      INPUT #r, a(r)
   NEXT r
END SUB

SUB np (a, b)
' inputs a line from Surfer text file (nothing is done
with the inputted
' information)
   FOR c = a TO b
      INPUT #1, a$
   NEXT c
END SUB

SUB nvv (nv, o())
' prints in a row the nv values in o()
   FOR b = 1 TO nv
      IF b = nv THEN
         c$ = ""
      ELSE
         c$ = CHR$(44)
      END IF
      CALL sigfig(o(b), s, 4)
      CALL chf(s, s$)
      PRINT #2, s$, c$;
   NEXT b
   PRINT #2,
   CALL wipe(nv, o())
END SUB

SUB ny (yc$())
   FOR year = 1 TO 13
      PRINT "Number of ", yc$(year), " files ", TAB(25);
jr(year)
      js = js + jr(year)
   NEXT year
   PRINT "Total number of files.", TAB(25), js
END SUB

SUB o2op (hn, yc$)
   IF hn = 1 THEN
      f$ = "hau-" + yc$ + ".txt"
      OPEN f$ FOR OUTPUT AS #5
   END IF
END SUB

SUB op (aa$, yc$, cho$(), s, f)
' opens HCL file for input
   FOR a = s TO f
      fi$ = "hcl-" + aa$ + yc$ + cho$(a + 3 - s) + ".txt"
      OPEN fi$ FOR INPUT AS #a
   NEXT a
END SUB

SUB opie (flg(), year, yc$(), pest, cho$(), a)
   CALL ch(a, aa$)
   FOR n = 1 TO 2
      fi$ = "pen-" + aa$ + yc$(year) + yc$(year + pest) +
cho$(n + 2) + ".txt"
      IF flg(year) = 0 THEN
         OPEN fi$ FOR OUTPUT AS #n
         flg(year) = 1
      ELSE
         OPEN fi$ FOR APPEND AS #n
      END IF
   NEXT n
END SUB

SUB original1 (be(), en(), st(), ne(), elk(), yc$(),
elz$(), sur$(), bio$(), bft$(), bfy$(), cho$(), a(), r,
rw(), fl(), av())
' Reads in data. If this has not been done before, it
reads the original
' Surfer-produced files and produces RAW files from them.
If it has been
' done before, it reads the data from the series of RAW
files already
' produced. The second option is much faster.
' Modified area and volume values are calculated and
output as CLV files.
   b1 = 2: b2 = 2: b3 = 3
   PRINT "Original1"
   FOR a = ss1 TO ee1
      CALL egg(a, eg)
      CALL ch(a, aa$)
      FOR year = be(a) TO en(a) STEP st(a)
         CALL char(aa$, yc$(year), cho$(), r)
         PRINT TAB(0), yc$(year), TAB(3),
         FOR elf = 1 TO ne(eg)
            PRINT elz$(elf, eg),
            FOR co = elk(eg, elf, 1) TO elk(eg, elf, 2)
STEP 10
               CALL fly(fl(), 3, 4)
               c$ = ""
               cl = (co - elk(eg, elf, 1)) / 10
               FOR sur = -1 TO 1
                  CALL blanked(a, year, sur, bfyb, bfyf,
pest)
                  IF r = 1 THEN
                     CALL nom1(yc$(year), elz$(elf, eg),
cl, sur$(sur), "u", "u", "d", a(), c$, e, fl())
                     c$ = CHR$(44)
                  ELSE
                     CALL nom2(e, a())
                  END IF
                  jr(year) = jr(year) + 1
                  FOR cha = 3 TO 4
                     rw(cha - 2, b1, b2, b3, sur) =
a(cha)
                  NEXT cha
                  c$ = CHR$(44)
                  FOR bio = 0 TO 1
                     FOR bft = 1 TO 2
                        FOR bfy = bfyb TO bfyf STEP pest

                           CALL choi(elf, cl, bio, bfy,
e)
                           IF r = 1 THEN
                              CALL nom1(yc$(year),
elz$(elf, eg), cl, sur$(sur), bio$(bio), bft$(bft),
bfy$(bfy), a(), c$, e, fl())
                           ELSE
                              CALL nom2(e, a())
                           END IF
                           FOR cha = 3 TO 4
                              rw(cha - 2, bio, bfy, bft,
sur) = a(cha)
                           NEXT cha
                           jr(year) = jr(year) + 1
                        NEXT bfy
                     NEXT bft
                  NEXT bio
               NEXT sur
               IF r = 1 THEN
                  FOR cha = 3 TO 4
                     PRINT #cha,
                  NEXT cha
               END IF
               CALL refine(a, year, av(), rw(), c$, fl())
               FOR cha = 5 TO 6
                  PRINT #cha,
               NEXT cha
            NEXT co
         NEXT elf
         CALL closer(2, 6)
         BEEP
      NEXT year
      BEEP
   NEXT a
   BEEP
   PRINT
   CALL ny(yc$())
END SUB

SUB original2 (be(), en(), st(), yc$(), cho$(), ne(),
elk(), aclv(), fl(), yr$())
' Converts CLV files to HCL files.  CLV are contour line
values.  HCL are
' contour zone values.
   PRINT "Original2"
   c$ = CHR$(44)
   FOR a = ss1 TO ee1
      CALL egg(a, eg)
      CALL ch(a, aa$)
      FOR year = be(a) TO en(a) STEP st(a)
         PRINT TAB(0); yc$(year),
         CALL haunec(a, year, hn, 2)
         CALL cn(hn, cs)
         CALL cvt(aa$, yc$(year), cho$())
         CALL o2op(hn, yr$(year))
         FOR elf = 1 TO ne(eg)
            PRINT elf;
            l = 1: h = 2
            CALL acquire(a, year, aclv(), l, hn)
            CALL h5(hn)
            FOR co = (elk(eg, elf, 1) + 10) TO elk(eg,
elf, 2) STEP 10
               CALL fly(fl(), 3, 4)
               CALL acquire(a, year, aclv(), h, hn)
               CALL h5(hn)
               CALL calc2(a, year, aclv(), l, h, fl(),
c$)
               CALL lh(l, h)
               FOR cha = 3 TO 4
                  PRINT #cha,
```

```
            NEXT cha
         NEXT co
      NEXT elf
      CALL closer(1, cs)
   NEXT year
 NEXT a
END SUB

SUB penn (ne(), elk(), be(), en(), st(), yc$(), cho$(),
clt(), mx1(), yr$(), bft$(), vavg(), elz$(), li(), l(),
h(), tav(), planar(), shv(), sm(), sm$())
' takes volume and area elevation zone figures from HCL
files and converts
' them to final values ("F-" files)
   c$ = CHR$(44)
   PRINT TAB(0); "Penn"
   FOR a = ss1 TO ee1
      CALL ch(a, aa$)
      CALL egg(a, eg)
      FOR year = be(a) TO en(a) - 1 STEP st(a)
         yd = VAL(yr$(year + st(a))) - VAL(yr$(year))
         ef = 0
         CALL tv(tav(), shv())
         CALL op(aa$, yc$(year), cho$(), 1, 2)
         CALL op(aa$, yc$(year + st(a)), cho$(), 3, 4)
         CALL haunec(a, year, hn, 2)
         CALL head1(bft$(), yr$(), st(a), year, a, hn)
         CALL hncs(hn, cs)
         CALL hop(hn, yr$(year), yr$(year + 1))
         CALL limits(hn, year, yc$(), li(), l(), a,
st(a))
         PRINT TAB(4), "19", yr$(year), "-19", yr$(year +
st(a)), " ";
         oa = 1
         FOR elf = 1 TO ne(eg)
            PRINT elz$(elf, eg),
            FOR co = elk(eg, elf, 1) TO (elk(eg, elf, 2)
- 10) STEP 10
               FOR ane = year TO year + st(a) STEP st(a)
                  CALL finisl(a, year, clt(), ane)
               NEXT ane
               CALL calc3(clt(), be(a), en(a), st(a), a,
mx1(), yr$(), bft$(), co, oa, vavg(), year, hn, h(), l(),
tav(), planar(), sm(), yd)
               oa = 2
               CALL haupr(hn, h(2), l(2), vavg(2),
planar(2), co, shv(), ef, sm())
            NEXT co
         NEXT elf
         CALL tots(sm(), hn, sm$(), yd)
         CALL fly(sm(), 3, 6)
         CALL closer(1, cs)
      NEXT year
   NEXT a
END SUB

SUB reader1 (yc$, tfy$, a(), c$, e, fl())
' opens and reads values for each Surfer-based text file
   path$ = "c \temp\jrr\" + yc$ + "\"
   OPEN path$ + tfy$ FOR INPUT AS #1
   CALL lire1(a(), e)
   IF e = 1 THEN
      s = 2
   ELSE
      s = 3
   END IF
   FOR n = s TO 4
      CALL chf(a(n), a$)
      IF fl(1, n) = 0 THEN
         PRINT #n, a$,
         fl(1, n) = 1
      ELSE
         PRINT #n, c$; a$,
      END IF
   NEXT n
   CLOSE #1
END SUB

SUB refine (a, year, av(), rw(), c$, fl())
' Sends correct order of surface, blanking file type and
blanking file year
' to subroutine CALC1.
   FOR sur = -1 TO 1
      CALL blanked(a, year, sur, bfyb, bfyf, pest)
      FOR bft = 1 TO 2
         FOR bfy = bfyb TO bfyf STEP pest
            CALL calc1(rw(), bfy, bft, sur, av(), c$,
fl())
         NEXT bfy
      NEXT bft
   NEXT sur
END SUB

SUB rh (hau(), elk(), yr$(), ne)
   FOR year = 5 TO 10
      CALL sf(year, s, f)
      f$ = "hau-" + yr$(year) + " txt"
      OPEN f$ FOR INPUT AS #1
      FOR elf = 1 TO ne
         FOR co = elk(1, elf, 1) TO elk(1, elf, 2) STEP
10
            cl = (co - elk(1, elf, 1)) / 10
            FOR bfy = s TO f
               INPUT #1, hau(year, elf, cl, bfy)
            NEXT bfy
         NEXT co
      NEXT elf
      CLOSE #1
   NEXT year
END SUB

SUB sf (year, s, f)
   s = -1  f = 1
   IF year = 5 THEN
      s = 0
   ELSEIF year = 10 THEN
      f = 0
   END IF
END SUB

SUB sigfig (a, b, n)
   IF a = 0 THEN
      b = 0
   ELSE
      lg = INT(LOG(ABS(a)) / LOG(10) - (n - 1))
      pw = 10 ^ lg
      b = INT(a / pw + 5) * pw
   END IF
END SUB

SUB sigma (mx1(), mean, sig)
' calculates average and standard deviation of divers
surfaces when compared
   FOR s = 1 TO 3
      FOR t = 1 TO 3
         u = u + 1
         mx2 = mx1(1, s) - mx1(2, t)
         sm = sm + mx2
         ssq = ssq + mx2 ^ 2
      NEXT t
   NEXT s
   mn = (sm / u)
   CALL sigfig(mn, mean, 4)
   mean = mean * -1
   sg = (ssq / u - mean ^ 2) ^  5
   CALL sigfig(sg, sig, 4)
END SUB

SUB smmy (be(), en(), st(), fp$(), yr$(), nh(), smy(),
nv(), s(), t())
   CLS
   PRINT TAB(0), "smmy"
   FOR a = ss1 TO ee1
      CALL egg(a, eg)
      FOR year = be(a) TO en(a) - 1 STEP st(a)
         PRINT TAB(0), "19", yr$(year), "-19", yr$(year +
st(a)),
         CALL haunec(a, year, hn, 2)
         FOR fp = 1 TO (hn + 2)
            f$ = yr$(year) + yr$(year + st(a)) + " txt"
            fi$ = fp$(fp) + f$
            fo$ = fp$(fp + 3) + f$
            OPEN fi$ FOR INPUT AS #1
            OPEN fo$ FOR OUTPUT AS #2
            IF fp = 3 THEN
               CALL head3(yr$(), year, st(a), 2)
            ELSE
               CALL head2(year, st(a), yr$(), fp, 2   fp)
            END IF
            FOR b = 1 TO nh(fp)
               INPUT #1, a$
            NEXT b
            fl = 0
            DO
               CALL innp(a$, smy(), fp, nv(), s(), t())
               CALL sp(a$, a1$, a2$, a2)
               IF fl = 0 THEN
                  st$ = a1$
                  fl = 1
               END IF
               IF a$ <> "Totals" THEN
                  IF a2 / 50 = INT(a2 / 50) THEN
                     PRINT #2, st$ + "-" + a2$, CHR$(44),
                     fl = 0
                     CALL nvv(nv(fp), smy())
                  END IF
               ELSE
                  PRINT #2, a$, CHR$(44),
                  CALL nvv(nv(fp), t())
                  CALL wipe(nv(fp), smy())
               END IF
            LOOP UNTIL EOF(1)
            CALL closer(1, 2)
         NEXT fp
      NEXT year
   NEXT a
END SUB

SUB sp (a$, a1$, a2$, a2)
   a1$ = LEFT$(a$, 4)
   a2$ = RIGHT$(a$, 4)
   a2 = VAL(a2$)
END SUB

SUB stre (v)
' produces a numeric value of a text line following a
colon
   INPUT #1, a$
   CALL fc(a$, b)
   v = VAL(MID$(a$, b))
END SUB

SUB summe (sm(), bft, vss, vsh, vgs, sig, sa, ag)
   sm(bft, 1) = sm(bft, 1) + vss
```

```
   sm(bft, 2) = sm(bft, 2) + vsh
   sm(bft, 3) = sm(bft, 3) + vgs
   sm(bft, 4) = sm(bft, 4) + sig
   sm(bft, 5) = sm(bft, 5) + sa
   sm(bft, 6) = sm(bft, 6) + ag
END SUB

SUB svh (shv(), vavg, shv, planar)
   shv(1) = shv(1) + vavg
   shv(2) = shv(2) + shv
   shv(3) = shv(3) + planar
END SUB

SUB tots (sm(), hn, sm$(), yd)
   c$ = CHR$(44)
   t = hn + 2
   FOR bft = 1 TO t
      FOR j = 1 TO 6
         IF j = 1 OR j = 2 OR j = 4 OR j = 5 THEN
            d = 1000
         ELSE
            d = 1
         END IF
         x = sm(bft, j)
         CALL sigfig(x, sm(bft, j), 4)
         CALL chf(sm(bft, j) / d, sm$(bft, j))
      NEXT j
      CALL sigfig(sm(bft, 1) / (yd * 1000), sm(bft, 7),
4)
      CALL sigfig(sm(bft, 4) / (yd * 1000), sm(bft, 8),
4)
      CALL chf(sm(bft, 7), sm$(bft, 7))
      CALL chf(sm(bft, 8), sm$(bft, 8))
   NEXT bft

   FOR bft = 1 TO 2
      PRINT #(bft + 4), "Totals", c$, sm$(bft, 1), c$;
sm$(bft, 4); c$, sm$(bft, 7), c$; sm$(bft, 8); c$,
sm$(bft, 5)
   NEXT bft
   IF hn = 1 THEN
      PRINT #8, "Totals", c$,
      FOR n = 1 TO 6
         IF n <> 6 THEN
            c1$ = c$
         ELSE
            c1$ = ""
         END IF
         IF n <> 4 THEN
            PRINT #8, sm$(3, n); c1$,
         END IF
      NEXT n
      PRINT #8,
   END IF
END SUB

SUB ttl (el, es$, f$)
   IF el = 2 THEN
      b = 9
   ELSE
      b = 6
   END IF
   f$ = LEFT$(es$, 3) + MID$(es$, b, 3)
END SUB

SUB tv (tav(), shv())
   FOR b = 1 TO 3
      shv(b) = 0
      FOR a = 1 TO 2
         tav(a, b) = 0
      NEXT a
   NEXT b
END SUB

SUB vmatrix (clt(), year, pest, mx1(), bft, mean, sig)
' sends values for volume-related calculation
   CALL mex1(mx1(), clt(), year, pest, bft)
   CALL sigma(mx1(), mean, sig)
END SUB

SUB wipe (nv, o())
   FOR b = 1 TO nv
      o(b) = 0
   NEXT b
END SUB
```

A7.2: ELCH.BAS

```
' This program can only be run after FACE BAS (a
GSMAC-program) is run.
' It takes as input a series of ASCII  GRD files (saved
with .DAT extensions)
' and compares the surface file with the corresponding
surface difference
' file.  The result is a series of .TXT files which
summarize surface change.

DECLARE SUB ezb (ez'())
DECLARE SUB sigfig (a', b', n')
DECLARE SUB ch (a', a$)
DECLARE SUB chf (a!, a$)
DECLARE SUB reader2 (e', elzo'(), p!, vl'(), ez'(),
year')
DECLARE SUB vad (ez'(), a', vl', el', year')
DECLARE SUB writer (s'(), p', pr$(), e', sx$(), elzo'(),
ez'(), elzo$(), bft$())
DECLARE SUB cm (a', b', c$)
DECLARE SUB reader1 (s'(), p', e', elzo$(), pr$(), yr$(),
sx$(), bft$(), elzo'(), ez'(), vl'())
DECLARE SUB h (n')
DECLARE SUB closer (a', b')

DIM SHARED null, path$
DIM yr$(13), pr$(4), elzo$(2, 9), sx$(2), bft$(2)
DIM ez(12, 2, 0 TO 10, 2), vl(3), elzo(2, 9, 2)

DATA "19", "55", "59", "65", "67", "69", "71", "73",
"75", "77", "79", "77", "78"
DATA "a", "b", "d", "z"
DATA 9,5
DATA "h", "j", "k", "l", "n", "o", "p", "q", "r"
DATA "v", "w", "x", "y", "z"
DATA ".dat", " txt"
DATA "m", "n"
DATA 1920,2000, 2000,2050, 2050,2100
DATA 2100,2150, 2150,2200, 2200,2250
DATA 2250,2300, 2300,2350, 2350,2400
DATA 1920,2000, 2000,2100, 2100,2200, 2200,2300,
2300,2400
DATA 2,12, 1,1

FOR a = 1 TO 13: READ yr$(a): NEXT a
FOR a = 1 TO 4  READ pr$(a): NEXT a
FOR a = 1 TO 2  READ ne(a): NEXT a
FOR a = 1 TO 2  FOR b = 1 TO ne(a): READ elzo$(a, b)
NEXT b  NEXT a
FOR a = 1 TO 2: READ sx$(a): NEXT a
FOR a = 1 TO 2  READ bft$(a): NEXT a
FOR a = 1 TO 2  FOR b = 1 TO ne(a): FOR c = 1 TO 2  READ
elzo(a, b, c): NEXT c  NEXT b. NEXT a
FOR a = 1 TO 2  FOR b = 1 TO 2  READ s(a, b). NEXT b
NEXT a

null = 1.70141E+38
path$ = "c \students\james\txt\bas\"
p = 2

FOR e = 1 TO 5
 CALL reader1(s(), p, e, elzo$(), pr$(), yr$(), sx$(),
bft$(), elzo(), ez(), vl())
 CALL writer(s(), p, pr$(), e, sx$(), elzo(), ez(),
elzo$(), bft$())
 CALL ezb(ez())
NEXT e

SUB ch (a, a$)
 a$ = MID$(STR$(a), 2)
END SUB

SUB chf (a, a$)
 a$ = STR$(a)
END SUB

SUB closer (a, b)
 FOR c = a TO b
  CLOSE #c
 NEXT c
END SUB

SUB cm (a, b, c$)
 IF a = b THEN
  c$ = ""
 ELSE
  c$ = ","
 END IF
END SUB

SUB ezb (ez())
 FOR a = 1 TO 12
  FOR b = 1 TO 2
   FOR c = 0 TO 8
    FOR d = 1 TO 2
      ez(a, b, c, d) = 0
    NEXT d
   NEXT c
  NEXT b
 NEXT a
END SUB
```

```
SUB h (n)
 FOR a = 1 TO 9
  INPUT #n, b$
 NEXT a
END SUB

SUB reader1 (s(), p, e, elzo$(), pr$(), yr$(), sx$(),
bft$(), elzo(), ez(), vl())
 CLS
 PRINT "Reader1 in elevation zone ", elzo$(p, e)
 FOR year = s(p, 1) TO s(p, 2)
  IF year <> 11 THEN
   PRINT "19", yr$(year), "-19", yr$(year + 1), " "
   f1$ = pr$(p) + elzo$(p, e) + "_" + yr$(year) + "u" +
sx$(1)
   OPEN f1$ FOR INPUT AS #1
   CALL h(1)
   FOR bft = 1 TO 2
    f2$ = pr$(3) + elzo$(p, e) + bft$(bft) + yr$(year) +
yr$(year + 1) + sx$(1)
    OPEN f2$ FOR INPUT AS #(bft + 1)
    CALL h(bft + 1)
   NEXT bft
   CALL reader2(e, elzo(), p, vl(), ez(), year)
   CALL closer(1, 3)
  END IF
  PRINT
 NEXT year
END SUB

SUB reader2 (e, elzo(), p, vl(), ez(), year)
 elow = elzo(p, e, 1)
 ehigh = elzo(p, e, 2)
 DO
  j = j + 1
  IF j / 2000 = INT(j / 2000) THEN PRINT " ",
  FOR a = 1 TO 3  INPUT #a, vl(a)  NEXT a
  IF vl(1) >= elow AND vl(1) < ehigh THEN
   FOR a = 2 TO 3
    IF vl(a) <> null THEN
     CALL vad(ez(), a, vl(a), 0, year)
     el = INT((vl(1) - elow) / 10) + 1
     CALL vad(ez(), a, vl(a), el, year)
    END IF
   NEXT a
   END IF
 LOOP UNTIL EOF(1)
END SUB

SUB sigfig (a, b, n)
 IF a = 0 THEN
  b = 0
 ELSE
  lg = INT(LOG(ABS(a)) / LOG(10) - (n - 1))
  pw = 10 ^ lg
  b = INT(a / pw + 5) * pw
 END IF
END SUB

SUB vad (ez(), a, vl, el, year)
 bft = a - 1
 ez(year, bft, el, 1) = ez(year, bft, el, 1) + vl
 ez(year, bft, el, 2) = ez(year, bft, el, 2) + 1
END SUB

SUB writer (s(), p, pr$(), e, sx$(), elzo(), ez(),
elzo$(), bft$())
 FOR bft = 1 TO 2
  f$ = path$ + pr$(4) + elzo$(p, e) + bft$(bft) + sx$(2)
  OPEN f$ FOR OUTPUT AS #1
  em = (elzo(p, e, 2) - elzo(p, e, 1)) / 10
  FOR el = 0 TO em
   IF el = 0 THEN
    PRINT #1, "Total,",
   ELSE
    CALL ch((el - 1) * 10 + elzo(p, e, 1), e1$)
    CALL ch((el) * 10 + elzo(p, e, 1), e2$)
    PRINT #1, e1$, "-", e2$, ",",
   END IF
   FOR year = s(p, 1) TO s(p, 2)
    IF year <> 11 THEN
     CALL cm(year, s(p, 2), c$)
     IF ez(year, bft, el, 2) > 0 THEN
      CALL sigfig(ez(year, bft, el, 1) / ez(year, bft,
el, 2), ez, 3)
     ELSE
      ez = 0
     END IF
     CALL chf(ez, ez1$)
     CALL ch(ez(year, bft, el, 2), ez2$)
     PRINT #1, ez1$, ",", ez2$, c$,
    END IF
   NEXT year
   PRINT #1,
  NEXT el
  CLOSE #1
 NEXT bft
END SUB
```

A7.3: FACE.BAS

```
' This program works in the GSMAC language
' It takes a series of elevation zone DEMs, saves the
binary  GRD file into
' an ASCII  GRD file (saved with a  DAT extension,
however)   It also does a
' GRID-MATH function on successive years and saves the
result as an ASCII
' .GRD file (once again saved with a  DAT extension)
This program must be
' run before ELCH BAS can be run

DIM yc$(2, 12) AS STRING
DIM pr$(2, 2) AS STRING
DIM el$(2, 9) AS STRING
DIM b$(2) AS STRING
DIM ny(2) AS INTEGER
DIM nel(2) AS INTEGER

yc$(1, 1) = "55"  yc$(1, 2) = "59"  yc$(1, 3) = "65"
yc$(1, 4) = "67"
yc$(1, 5) = "69"  yc$(1, 6) = "71"  yc$(1, 7) = "73"
yc$(1, 8) = "75"
yc$(1, 9) = "77"  yc$(1, 10) = "79"  yc$(1, 11) = "77"
yc$(1, 12) = "78"

yc$(2, 1) = "19"  yc$(2, 2) = "55"

el$(1, 1) = "h"  el$(1, 2) = "j": el$(1, 3) = "k"
el$(1, 4) = "l". el$(1, 5) = "n"  el$(1, 6) = "o"
el$(1, 7) = "p"· el$(1, 8) = "q". el$(1, 9) = "r"

el$(2, 1) = "v"  el$(2, 2) = "w"  el$(2, 3) = "x"
el$(2, 4) = "y"  el$(2, 5) = "z"

b$(1) = "m"  b$(2) = "n"

pr$(1, 1) = "a"  pr$(1, 2) = "d"
pr$(2, 1) = "b"  pr$(2, 2) = "d"

ny(1) = 11  ny(2) = 1
nel(1) = 9  nel(2) = 5
p = 2

path$ = "c \students\james\txt\bas\"
s$ = " "
o$ = "out grd"
yy = 4

Set Surf = CreateObject("Surfer.App")
FOR e = 1 TO nel(p)
 FOR yr = 1 TO ny(p)
  IF yr <> 10 THEN
   y$ = pr$(p, 1) + el$(p, e) + "_" + yc$(p, yr)
   y1$ = y$ + "u.grd"
   y3$ = y$ + "u.dat"
   y2$ = pr$(p, 1) + el$(p, e) + "_" + yc$(p, yr + 1) +
"u grd"
   Surf.GridConvert(y1$, OutGrid=y3$, OutFmt=2)
   Surf GridMath(InGridA=y1$, InGridB=y2$, OutGridC=o$,
OutFmt=1, Function="C=B-A")
   FOR bft = 1 TO 2
    bfy$ = "b" + b$(bft) + "o" + yc$(p, yr) + ".bln"
    y4$ = pr$(p, 2) + el$(p, e) + b$(bft) + yc$(p, yr) +
yc$(p, yr + 1) + ".dat"
    Surf GridBlank(InGrid=o$, BlankFile=bfy$,
OutGrid=y4$, OutFmt=2)
   NEXT
  END IF
 NEXT
NEXT
```

A7.4: ID.BAS

```
' This program takes a corrected vector file in TOSCA
format and converts
' it into an ASCII (x,y,z) data file readable by SURFER.
In SURFER, the
' file will become a .GRD file

DECLARE SUB ch (a', a$)
CLS

x$ = "eastings"
y$ = "northings"
z$ = "elevations"
c$ = CHR$(44)
q$ = CHR$(34)
xx$ = q$ + x$ + q$ + c$
yy$ = q$ + y$ + q$ + c$
zz$ = q$ + z$ + q$

mult = 1
add = 0
metric = .3048
INPUT "Filename"; f$
IF RIGHT$(f$, 4) <> " vec" THEN
 f1$ = f$ + ".vec"
ELSE
 f1$ = f$
 f$ = LEFT$(f1$, (LEN(f1$) - 4))
END IF

OPEN f1$ FOR INPUT AS #1
'OPEN f$ + "i.txt" FOR OUTPUT AS #2
'OPEN f$ + "ic.txt" FOR OUTPUT AS #3
'OPEN f$ + "m.txt" FOR OUTPUT AS #4
OPEN f$ + "a txt" FOR OUTPUT AS #2

PRINT #2, xx$, yy$, zz$
'PRINT #3, xx$, yy$, zz$
'PRINT #4, xx$; yy$; zz$
INPUT #1, z, n

jr = jr + n
'z2 = INT(z * metric + .5)
z1 = z '/10

PRINT z1 ', z2
WHILE n <> 0
 FOR a = 1 TO n
  INPUT #1, x, y
  x1 = x * 1000 - 99.46156
  y1 = y * 1000 + 83.23
  CALL ch(x1, x1$)
  CALL ch(y1, y1$)
  CALL ch(z1, z1$)
'  CALL ch(z2, z2$)
  PRINT #2, x1$; c$, y1$; c$; z1$
'  IF z1 / 100 = INT(z1 / 100) THEN
'   PRINT #3, x1$; c$, y1$; c$; z1$
'  END IF
'  PRINT #4, x1$, c$, y1$, c$; z2$
 NEXT a
 INPUT #1, z, n
 jr = jr + n
' z2 = INT(z * metric + .5)
 z1 = z ' / 10
 PRINT z1 ', z2
WEND
CLOSE #2
CLOSE #1
PRINT . PRINT jr

SUB ch (a, a$)
 a$ = MID$(STR$(a), 2)
END SUB
```

A7.5: SURELEV.BAS

```
DIM bva(4), elz(2, 9, 2), ez(4), n(2), size(2)
DIM xn(4, 9), xx(4, 9), yn(4, 9), yx(4, 9)
DIM tot(2, 11)
DIM aasl$(3) AS STRING
DIM bav$(4) AS STRING
DIM bft$(2) AS STRING
DIM bfy$(6) AS STRING
DIM bio$(2) AS STRING
DIM op$(4, 9) AS STRING
DIM pave$(3) AS STRING
DIM ptve$(2) AS STRING
DIM px$(2) AS STRING
DIM pxi$(2) AS STRING
DIM sf$(3) AS STRING
DIM sur$(3) AS STRING
DIM tx$(2) AS STRING
DIM ty$(2) AS STRING
DIM u$(2) AS STRING
DIM yc$(2, 11) AS STRING
DIM yr$(2, 11) AS STRING

Set Surf = CreateObject("Surfer.App")

yr$(1, 1) = "19"  yr$(1, 2) = "78"

yr$(2, 1) = "59"  yr$(2, 2) = "69"  yr$(2, 3) = "79"
yr$(2, 4) = "65". yr$(2, 5) = "75"  yr$(2, 6) = "67"
yr$(2, 7) = "77": yr$(2, 8) = "59"  yr$(2, 9) = "79"
yr$(2, 10) = "79"  yr$(2, 11) = "78"

yc$(1, 1) = "L"  yc$(1, 2) = "M"

yc$(2, 1) = "A"  yc$(2, 2) = "B". yc$(2, 3) = "C"
yc$(2, 4) = "D"  yc$(2, 5) = "E"  yc$(2, 6) = "F"
yc$(2, 7) = "G". yc$(2, 8) = "H"  yc$(2, 9) = "I"
yc$(2, 10) = "J": yc$(2, 11) = "K"

xn(1, 1) = 83450: xx(1, 1) = 84500: yn(1, 1) = 84050
yx(1, 1) = 85380
xn(1, 2) = 83300: xx(1, 2) = 84480: yn(1, 2) = 83220.
yx(1, 2) = 85080
xn(1, 3) = 82490: xx(1, 3) = 84330: yn(1, 3) = 82350:
yx(1, 3) = 84200
xn(1, 4) = 81590  xx(1, 4) = 83450: yn(1, 4) = 81230:
yx(1, 4) = 83420
xn(1, 5) = 81200: xx(1, 5) = 82450: yn(1, 5) = 80630:
yx(1, 5) = 82130

xn(2, 1) = 83490. xx(2, 1) = 84300  yn(2, 1) = 83900
yx(2, 1) = 84800
xn(2, 2) = 83350. xx(2, 2) = 84300: yn(2, 2) = 83650
yx(2, 2) = 84500
xn(2, 3) = 83300  xx(2, 3) = 84150: yn(2, 3) = 83200
yx(2, 3) = 84150
xn(2, 4) = 82930  xx(2, 4) = 83950  yn(2, 4) = 82820
yx(2, 4) = 83830
xn(2, 5) = 82450: xx(2, 5) = 83650  yn(2, 5) = 82250
yx(2, 5) = 83450
xn(2, 6) = 81690. xx(2, 6) = 83200  yn(2, 6) = 81420
yx(2, 6) = 83050
xn(2, 7) = 81500: xx(2, 7) = 82630: yn(2, 7) = 81220
yx(2, 7) = 82280
xn(2, 8) = 81300: xx(2, 8) = 82450: yn(2, 8) = 80950
yx(2, 8) = 82000
xn(2, 9) = 81100: xx(2, 9) = 82300. yn(2, 9) = 80620
yx(2, 9) = 81900

xn(3, 1) = 81000: xx(3, 1) = 85000  yn(3, 1) = 80000
yx(3, 1) = 85500

xn(4, 1) = 81000: xx(4, 1) = 85000  yn(4, 1) = 80000
yx(4, 1) = 85000

op$(1, 1) = "v": op$(1, 2) = "w": op$(1, 3) = "x"
op$(1, 4) = "y". op$(1, 5) = "z"

op$(2, 1) = "h": op$(2, 2) = "j": op$(2, 3) = "k"
op$(2, 4) = "l": op$(2, 5) = "n". op$(2, 6) = "o"
op$(2, 7) = "p": op$(2, 8) = "q": op$(2, 9) = "r"

op$(3, 1) = "t"

op$(4, 1) = "t"

px$(1) = "b": px$(2) = "a"
pxi$(1) = "bd_": pxi$(2) = "at_"

sf$(1) = ".txt": sf$(2) = ".grd"  sf$(3) = ".bln"

n(1) = 2: n(2) = 9
ez(1) = 5: ez(2) = 9: ez(3) = 1: ez(4) = 1

elz(1, 1, 1) = 1900. elz(1, 1, 2) = 2000
elz(1, 2, 1) = 2000: elz(1, 2, 2) = 2100
elz(1, 3, 1) = 2100. elz(1, 3, 2) = 2200
elz(1, 4, 1) = 2200: elz(1, 4, 2) = 2300
elz(1, 5, 1) = 2300. elz(1, 5, 2) = 2400

elz(2, 1, 1) = 1910: elz(2, 1, 2) = 2000
elz(2, 2, 1) = 2000: elz(2, 2, 2) = 2050
elz(2, 3, 1) = 2050  elz(2, 3, 2) = 2100
elz(2, 4, 1) = 2100  elz(2, 4, 2) = 2150
elz(2, 5, 1) = 2150  elz(2, 5, 2) = 2200
elz(2, 6, 1) = 2200  elz(2, 6, 2) = 2250
elz(2, 7, 1) = 2250  elz(2, 7, 2) = 2300
elz(2, 8, 1) = 2300  elz(2, 8, 2) = 2350
elz(2, 9, 1) = 2350  elz(2, 9, 2) = 2400

bft$(1) = "m"  bft$(2) = "n"
bio$(0) = "i"  bio$(1) = "o"
bfy$(1) = "a"  bfy$(2) = "b"  bfy$(3) = "c"  bfy$(4) =
"d"  bfy$(5) = "e". bfy$(6) = "f"
sur$(1) = "s"  sur$(2) = "t"  sur$(3) = "u"
aasl$(1) = "10700"  aasl$(2) = "4110"  aasl$(3) = "10000"
pave$(1) = "0 00045"  pave$(2) = "0 000247"  pave$(3) =
"0.00045"
tx$(1) = "88337.29"  tx$(2) = "84185.69"
ty$(1) = "87447.11"  ty$(2) = "86548 33"
ptve$(1) = "0.084631813": ptve$(2) = "1/60"
tg1$ = "z=pow(pow("  tg2$ = "-x,2)+pow("  tg3$ =
"-y,2),0.5)*tan(("  tg4$ = ")*3 1415926/180)"
ag1$ = "C=("  ag2$ = "-A)*"
size(1) = 5  size(2) = 500
path$ = "c \temp\jrr\"
rad = 1500
o$ = "out"
ot$ = "out.grd"
oun$ = "out-un grd"
ig$ = "oi grd"
u$(1) = "u"  u$(2) = "y"

Surf FileNew()

' Produces grid files for all areas, both entire glacier
and subarea zones
FOR i = 1 TO 1
   IF (i = 1 OR i = 3) THEN j = 1
   IF (i = 2 OR i = 4) THEN j = 2
   IF i <= 2 THEN j1 = 1
   IF i >= 3 THEN j1 = 2
   FOR year = 2 TO n(j)
      p = 2
      IF (i = 1 OR i = 3) THEN p = 1
      IF ((i = 2 OR i = 4) AND (year = 1 OR year = 11))
THEN p = 1
      tf$ = pxi$(p) + yr$(j, year) + sf$(1)
      FOR elzo = 4 TO ez(i)
        gf$ = px$(j) + op$(i, elzo) + "_" + yr$(j, year)
+ u$(j1) + sf$(2)
         Surf.GridData(tf$, xMin=xn(i,elzo),
xMax=xx(i,elzo), yMin=yn(i,elzo), yMax=yx(i,elzo),
xsize=size(j1), ysize=size(j1), GridMethod=1,
OutGrid=gf$, OutFmT=1, SearchMethod=3, SearchRad1=rad,
SearchRad2=rad)
      NEXT
   NEXT
NEXT

' Produces  TXT files from the GRID VOLUME command for
all grid files having
' 5m grid spacing
FOR i = 1 TO 2
   FOR year = 1 TO n(i)
      pa$ = path$ + yc$(i, year) + "\"
      fl = 0
      IF yr$(i, year) = "19" THEN t1 = 1  t2 = 1  fl = 1
      IF yr$(i, year) = "55" THEN t1 = 2: t2 = 1  fl = 1
      IF yr$(i, year) = "59" THEN t1 = 2  t2 = 2  fl = 1
      IF yr$(i, year) = "78" THEN t1 = 2  t2 = 3  fl = 1
      IF fl = 0 THEN t1 = 1  t2 = 1

      IF t1 = 1 THEN tpg$ = tg1$ + tx$(t2) + tg2$ +
ty$(t2) + tg3$ + ptve$(t2) + tg4$
      IF t1 = 2 THEN apg$ = ag1$ + aasl$(t2) + ag2$ +
pave$(t2)

      FOR elzo = 1 TO ez(i)
         gf$ = px$(i) + op$(i, elzo) + "_" + yr$(i, year)
+ u$(1) + sf$(2)
         if t1=1 then Surf.GridFunction(tpg$,
xMin=xn(i,elzo), xMax=xx(i,elzo), yMin=yn(i,elzo),
yMax=yx(i,elzo), xInc=size(1), yInc=size(1),
OutGrid=oun$, OutFmt=1)
         if t1=2 then Surf.GridMath(gf$, OutGridC=oun$,
Function=apg$)
         FOR sir = -1 TO 1
            IF year = 1 AND sir = 0 THEN bv = 2  bva(1) =
year: bva(2) = year + 1  bav$(1) = "b"  bav$(2) = "c"
            IF i = 1 AND year = 2 AND sir = 0 THEN bv =
2  bva(1) = year - 1  bva(2) = year  bav$(1) = "a"
bav$(2) = "b"
            IF i = 2 AND year >= 2 AND year <= 10 AND sir
= 0 THEN bv = 3  bva(1) = year - 1: bva(2) = year  bva(3)
= year + 1. bav$(1) = "a"  bav$(2) = "b"  bav$(3) = "c"
            IF i = 2 AND year = 9 AND sir = 0 THEN bv =
4  bva(4) = 11  bav$(4) = "e"
            IF i = 2 AND year = 10 AND sir = 0 THEN bv =
2
            IF i = 2 AND year = 11 AND sir = 0 THEN bv =
2  bva(1) = 9  bva(2) = 11  bav$(1) = "f"  bav$(2) = "b"

            IF i = 1 AND year = 1 AND sir <> 0 THEN bv =
2. bva(1) = year: bva(2) = year + 1  bav$(1) = "b"
bav$(2) = "c"
```

```
            IF ((i = 1 AND year = 2) OR (i = 2 AND year
<= 10)) AND sir <> 0 THEN bv = 2  bva(1) = year - 1
bva(2) = year  bav$(1) = "a"  bav$(2) = "b"
            IF i = 2 AND year = 1 AND sir <> 0 THEN bv =
1  bva(1) = year  bav$(1) = "b"
            IF i = 2 AND year = 9 AND sir <> 0 THEN bv =
3  bva(3) = 11  bav$(3) = "e"
            IF i = 2 AND year = 11 AND sir <> 0 THEN bv =
2  bva(1) = 9  bva(2) = 11  bav$(1) = "f"  bav$(2) = "b"

            IF sir = -1 THEN f$ = "C=A-B"
            IF sir = 1 THEN f$ = "C=A+B"
            if sir<>0 then Surf GridMath(gf$,
InGridB=oun$, OutGridC="o grd", Function=f$)
            IF sir = 0 THEN p$ = gf$
            IF sir <> 0 THEN p$ = "o grd"
            FOR l = elz(i, elzo, 1) TO elz(i, elzo, 2)
STEP 10
               el$ = CHR$((l - elz(i, elzo, 1)) / 10 +
65)
               tfy$ = pa$ + yc$(i, year) + el$ + op$(i,
elzo) + "udu" + sur$(sir + 2) + sf$(1)
               tot(i, year) = tot(i, year) + 1
               Surf GridVolume(UpGrid=p$, LowConstant=1)
               Surf FileSaveAs (tfy$)
               Surf FileClose()
            NEXT                'l

            FOR blft = 1 TO 2
               FOR blfy = 1 TO bv
                  FOR blio = 0 TO 1
                     bf$ = "b" + bft$(blft) + bio$(blio)
+ yr$(i, bva(blfy)) + sf$(3)
                     Surf.GridBlank(p$, BlankFile=bf$,
OutGrid=ot$, OutFmt=1)
                     FOR l = elz(i, elzo, 1) TO elz(i,
elzo, 2) STEP 10
                        el$ = CHR$((l - elz(i, elzo, 1))
/ 10 + 65)
                        tfy$ = pa$ + yc$(i, year) + el$ +
op$(i, elzo) + bio$(blio) + bav$(blfy) + bft$(blft) +
sur$(sir + 2) + sf$(1)
                        tot(i, year) = tot(i, year) + 1
                        Surf.GridVolume(UpGrid=ot$,
LowConstant=1)
                        Surf.FileSaveAs (tfy$)
                        Surf.FileClose()
                     NEXT         'l
                  NEXT            'blio
               NEXT               'blfy
            NEXT                  'blft
         NEXT                     'sir
      NEXT                        'elzo
   NEXT                           'year
NEXT                              'i
```

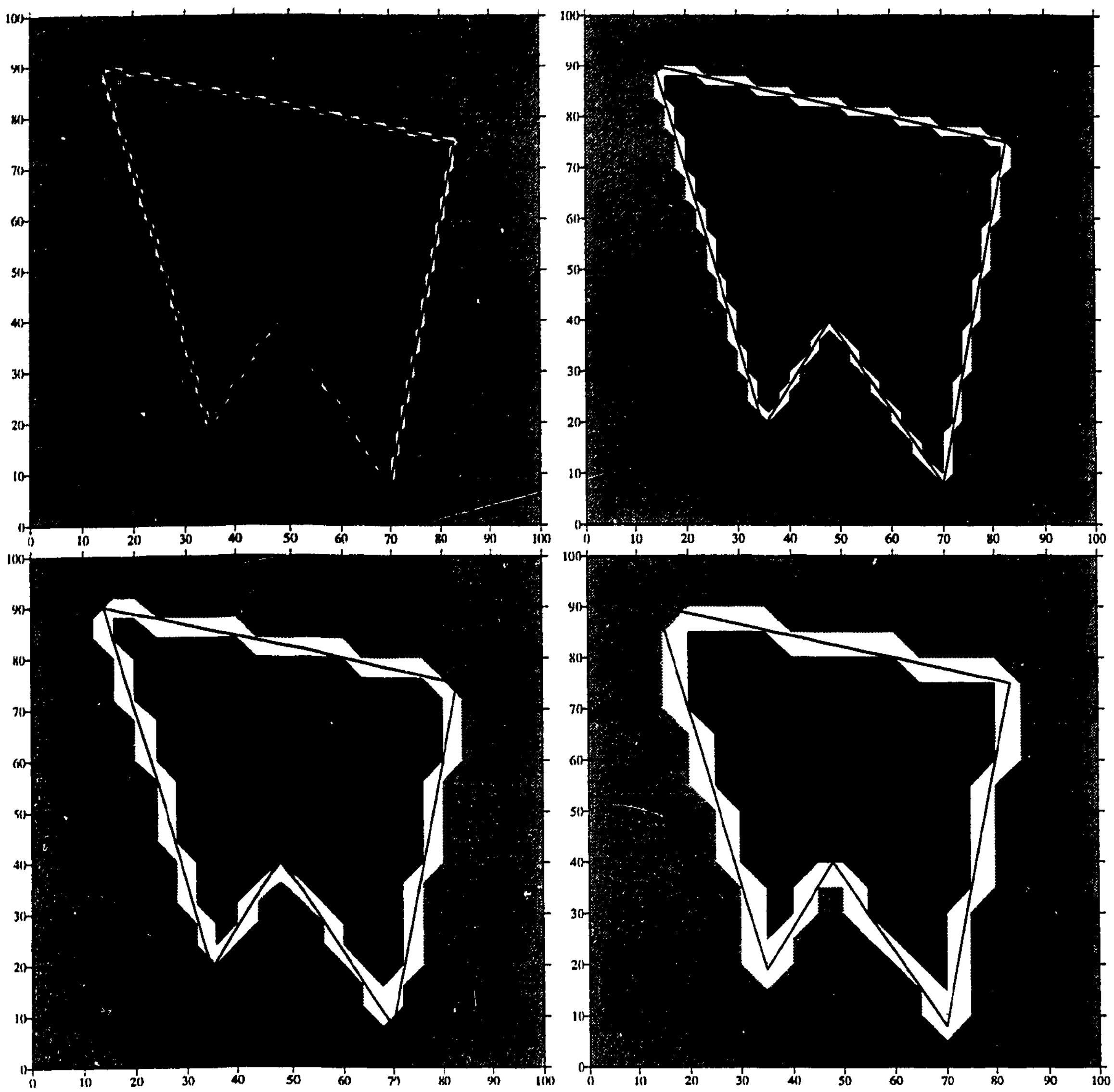
APPENDIX 8: MEASURING AREAS WITH *SURFER*

To measure area within a figure defined by a blanking file, *Surfer* divides the entire study area into a checkerboard pattern. The vertices of the squares are the grid nodes, having spacing defined by the user. With a fine grid spacing, there will be many squares; with coarse spacing, there will be few squares.

The area inside a border file is the sum of the grid squares that *Surfer* considers to be within the border. Thus, each square is checked to determine whether it lies entirely or partially within the blanking file. Squares entirely within the border are included in the area figure. If the border line crosses adjacent edges of a grid square such that a straight line connecting the two points of intersection encloses more than half the grid square, then half the area of the grid square is included. If the border crosses adjacent squares such that more than half of the grid square lies outside of the border, then the grid square is not included in the area figure.

Two figures were created to test *Surfer*'s calculation of area. These were a simple polygon (vertices (x,y) (14,90), (35,19), (48,40), (70,8), (83,75)) and an ellipse having a semi-major axis of 45 units and a semi-minor axis of 30 units, centred on the point (50,50); figures A8.1 and A8.2. The area of each was calculated geometrically outside the program *Surfer* and found to be 3098.01 units^2 for the polyhedron and 4241.15 units^2 for the ellipse.

Both figures were converted to blanking files within *Surfer*. An area with vertices (x,y) ((0,0), (100,0), (100,100), (0,100)) was gridded at differing grid spacings, and the blanking files were applied to them. The figures appear in figures A8.1 and A8.2. The areas that *Surfer* calculated for each polygon appear in table A8.1 in the column <u>Inside</u>. This table shows that for each border, the area listed is an underestimate of the true area of the figure. Figures A8.1 and A8.2 both show the area that *Surfer* considers to be inside the boundary for four different grid spacings as the zone of dark grey. As the grid spacing becomes finer, the estimate of area improves. However, even the finest grid

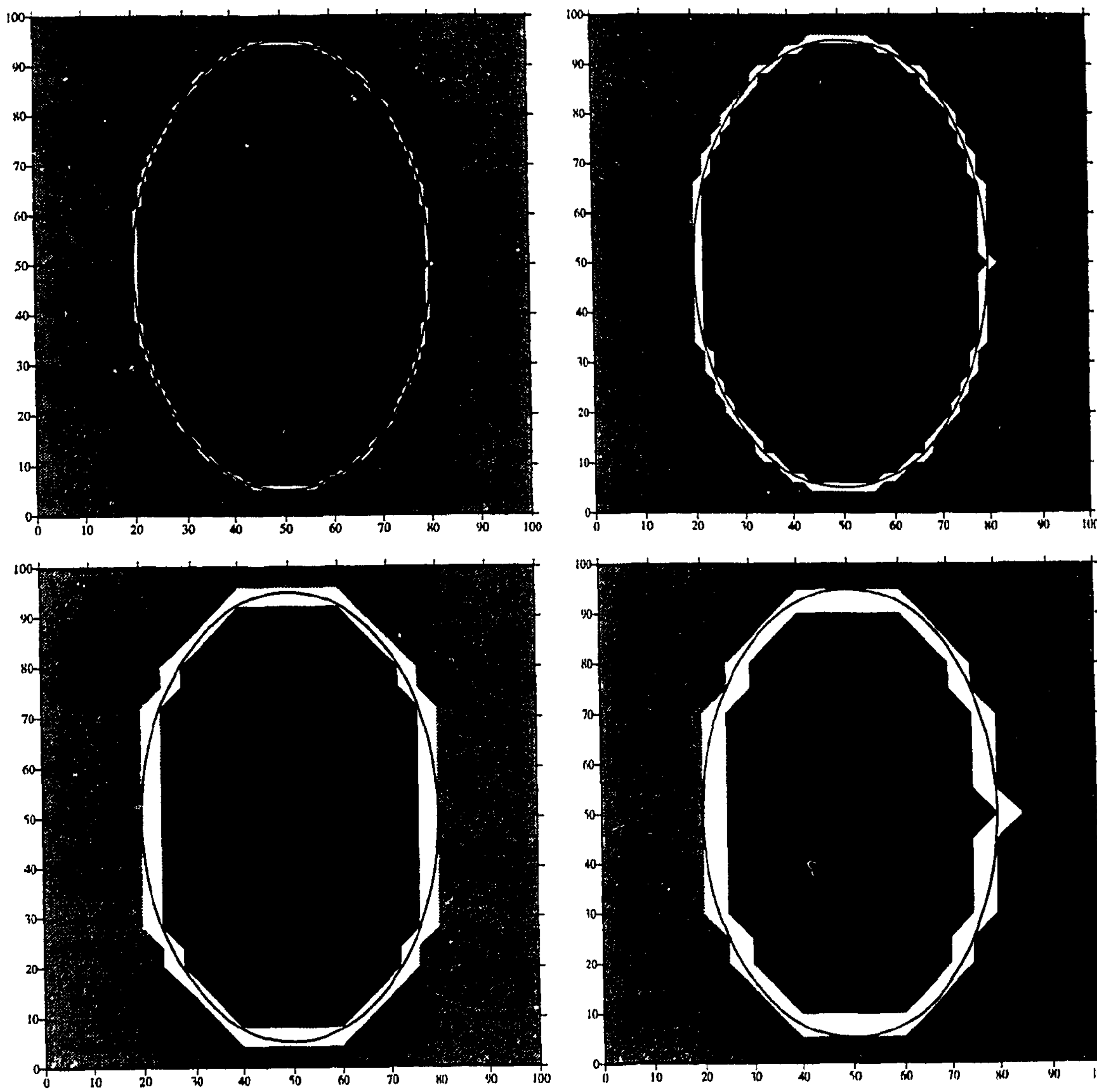


Inside ■ Outside Ellipse —

Space between grid nodes

a) 1 unit *b)* 2 units
c) 4 units *d)* 5 units

Figure A8.1: Polyhedron area. As the grid spacing becomes more fine, the Surfer-calculated area within the figure becomes more accurate.



Inside ■ Outside ▩ Ellipse —

Space between grid nodes

a) 1 unit *b)* 2 units
c) 4 units *d)* 5 units

Figure A8.2: Ellipse area. As the grid spacing becomes more fine, the Surfer-calculated area within the figure becomes more accurate.

spacing shown here is unsatisfactory. To improve the accuracy of area estimation, an additional step must be taken.

Table A8.1: Test of *Surfer* Determination of Area

Orthogonal space between grid nodes	POLYHEDRON			ELLIPSE		
	Inside	NOT Outside	Average	Inside	NOT Outside	Average
0.25	3061.63	3132.94	3097.29	4206.66	4271.16	4238.91
0.5	6026.50	3168.75	3097.62	4171.13	4300.13	4235.63
1	2957.00	3240.00	3098.50	4091.50	4349.00	4220.25
2	2828.00	3376.00	3102.00	3974.00	4494.00	4234.00
4	2576.00	3648.00	3112.00	3728.00	4736.00	4232.00
5	2462.50	3862.00	3162.25	3387.50	4637.50	4012.50
True Areas:			**3098.01**			**4241.15**

Surfer can also calculate the area outside a figure. The procedure for determining area is the same: area is the sum of grid squares lying outside the boundary. Grid squares on the boundary are tested using the same procedure as listed above. The area that *Surfer* considers to be outside the boundary is shown on figures A8.1 and A8.2 as the zone of light grey. The gridded area that is not light grey (the sum of the dark grey and white areas) is given in table A8.1 in the column "NOT Outside". As can be seen, values appearing in that column overestimate the true area value of each polygon.

Between the dark grey and light grey on each figure there is an uncoloured zone that *Surfer* does not consider to be either inside or outside the boundary line. In all figures it can be seen that the white zone is bisected by the boundary line. The magnitude of the white area varies directly with the spacing of the grid nodes.

A much more accurate measure of the area inside a boundary is the mean average of the area inside the boundary and the area NOT outside the figure: that is, the difference between the total area gridded and the area outside the boundary:

$$\bar{A} = \frac{A_i + A_{No}}{2} = \frac{A_i + A_t - A_o}{2}$$

where

$\bar{A}$ = Mean area
A_i = Area inside
A_{No} = Area NOT outside
A_t = Total area
A_o = Area outside

This calculation produces an area figure that includes *Surfer* calculations of the area inside the boundary line and half of the white area. These values were calculated for each of the rows in table A8.1. These values show that even at relatively coarse grid sizes, the mean average produced values that are very close to the geometrically calculated value. When the grid spacing becomes finer, the mean average becomes very close to the true value.

This is illustrated in figures A8.3*a,b*, which show the relationship between grid spacing and area calculation. On both of these graphs, the *x*-axis is logarithmic and the relationship between the inside, NOT outside, mean area and the geometric area values of the figure they measure is seen to be asymptotic. As the spacing between grid nodes approaches zero, the closer the two area values come to represent the true geometric value of area.

This procedure was used each time an area figure was calculated in the program *DVOL.BAS*.

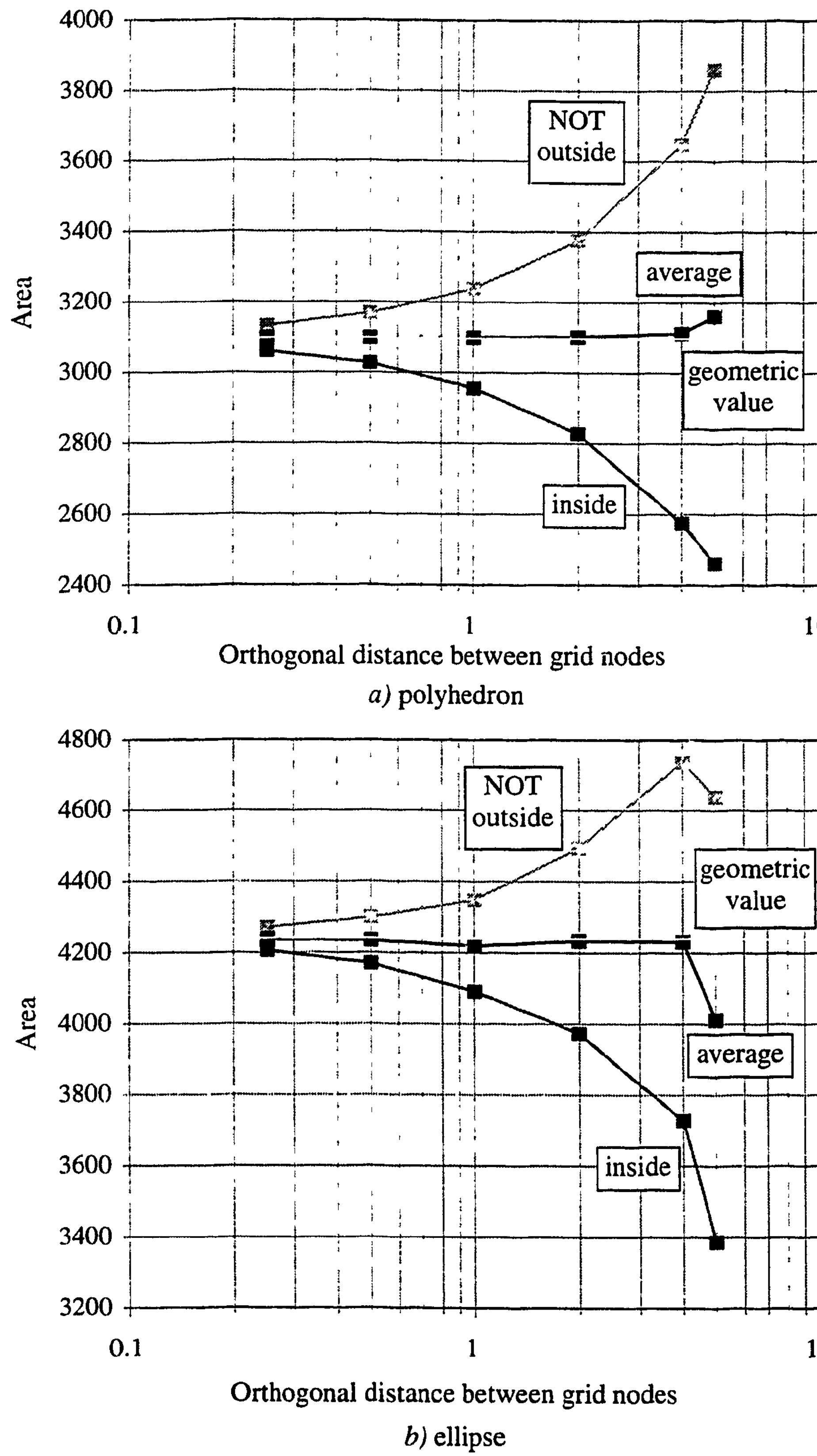


Figure A8.3: Relationship between grid spacing and area calculation

APPENDIX 9: CALCULATION OF SAMPLE RESULTS FROM RAW DATA

The program *SURELEV.BAS* produced a large number of DEMs, and many report files were produced from each DEM. To keep track of which file contained which information, each file had a unique code which revealed the information the file contained. Each letter and position of each letter in the title of the file denotes the information the file contains.

There are two groups of files, divided implicitly by the extent of the boundary of the DEM. The first consists of the subarea DEMs that have as boundaries the vertices listed in table 3.10*a*. The second are those that use the boundaries listed in table 3.10*b*. The 1955 DEM, since it appears in both groups, is listed twice. This and the codes for other DEM years in the text files is shown in table A9.1. The elevation zone codes are also divided into two groups, depending on the interval being measured. The codes for the elevation zones are listed in table A9.2.

The individual contour line calculated at 10 m intervals begins with the lower boundary of the elevation zone, having a code of *A*. Each subsequent 10 m elevation zone above that has the next letter in the alphabet as its code until the upper limit of the elevation zone. Therefore, in a 50 m elevation zone, codes will go from *A* through *F*. In a 100 m elevation zone, codes go from *A* to *K*.

Additionally, the title of the file must display information relevant to the calculation of volumetric uncertainty. The codes of the surface as calculated, and the lower and upper surfaces of uncertainty as described in section 3.3.3.6 are listed in table A9.3.

The remaining codes denote which blanking file is to be used. These are the blanking file type, which describes whether the entire ice surface or clear ice only is being enclosed (table A9.4); whether the file blanks inside or outside the border (table A9.5); and from what year the blanking file is from (table A9.6). The year of blanking file can be either 'trailing' which is from the previous map in the series, from the year of mapping itself, or 'leading,' which is from the next map in the series.

The codes that describe the various attributes of the DEM surface appear in a fixed order as follows:

i) year code
ii) contour line code
iii) elevation zone code
iv) blanking: in or out
v) blanking: file year
vi) blanking: file type
vii) surface code

The surface code is followed by the extension of the file, which was invariably ".TXT".

Thus, the file describing the calculated values from the unmodified 1975 DEM with blanking that retained the information inside the border for the complete ice surface using the 1977 border for the elevation 2230 m would be titled: **HDOOCMT.TXT**.

Table A9.1: Year codes for raw data files

A Group				B Group	
1955	A	1973	G	1919	L
1959	B	1975	H	1955	M
1965	C	1977T	I		
1967	D	1979	J		
1969	E	1977A	K		
1971	F				

Table A9.2: Elevation zone codes for raw data files

Group A	
Code	Elevation (metres)
H	1910-2000
J	2000-2050
K	2050-2100
L	2100-2150
N	2150-2200
O	2200-2250
P	2250-2300
Q	2300-2350
R	2350-2400

Group B	
Code	Elevation (metres)
V	1900-2000
W	2000-2100
X	2100-2200
Y	2200-2300
Z	2300-2400

Table A9.3: DEM surface

Estimate Used	Code
Low	S
Unmodified	T
High	U

Table A9.4: Blanking File Type

Ice Surface Enclosed	Code
All Ice	M
Clear Ice	N
Not blanked	U

Table A9.5: Blanking Inside or Outside Border

Direction	Code
Inside	O
Outside	I
Unblanked	U

Table A9.6: Blanking File Year

Year in relation to DEM	Code
Trailing	A
Year	B
Leading	C
Not blanked	D
1977T blanked with 1977A	E
1977A blanked with 1977T	F

The following sections present how final results were produced from raw data files. The interval selected was 1971 to 1973 for the contour zone 2140-2150 metres. The raw data is presented in table A9.7.

Table A9.7: Raw Data for calculation of final results

Border Type	Year	Elevation (metres)	In or Out	File Name	Positive Volume	Positive Planar Area (m^2)
Entire Ice	1971	2140	in	FELOBMT	6.75767×10^6	307245
			out	FELIBMT	9.77521×10^6	149687
		2150	in	FFLOBMT	4.02051×10^6	241031
			out	FFLIBMT	8.38046×10^6	130141
	1973	2140	in	GELOAMT	7.20512×10^6	314820
			out	GELIAMT	8.95611×10^6	151371
		2150	in	GFLOAMT	4.38958×10^6	247789
			out	GFLIAMT	7.56393×10^6	126467
Not Blanked	1971	2140	not blanked	FELUDUT	1.65329×10^7	458431
		2150		FFLUDUT	1.23890×10^7	371479
	1973	2140		GELUDUT	1.61612×10^7	468122
		2150		GFLUDUT	1.19535×10^7	374633

A9.1 Example of area calculation

The area values in the raw data represent total planar area above the elevation listed. As discussed in appendix 8, the total area within a blanked file is calculated as the mean of the area inside and the area NOT outside. For both years, this leads to the total glacier surface area figures that appear in table A9.8.

Table A9.8: Total Area for Entire Ice Surface Greater than Given Elevations (m^2)

Elevation (metres)	1971	1973
2140 m	**Total Area =** ½·(307245 + 458431 - 149687) = **307994.5**	**Total Area =** ½·(314820 + 468122 - 151371) = **315785.5**
2150 m	**Total Area =** ½·(241031 + 371479 - 130141) = **241184.5**	**Total Area =** ½·(247789 + 374633 - 126467) = **247977.5**

From these figures, area and mean area values can be produced for elevation zones. As stated in section 3.3.3.3, the area between two elevation zones is the total area at or above the lower elevation zone subtract the area at or above the upper elevation zone. When this procedure has been carried out on both years of record, the mean area in the elevation zone in that interval between mapping can be calculated. The results of area and mean area calculations are shown in table A9.9.

Table A9.9: Area and mean area calculation example

Elevation Zone (metres)	Area (m^2) in each year		Mean Area (m^2)
	1971	1973	
2140	307994.5	315785.5	
2150	241184.5	247977.5	
2140-2150	66810	67808	67309

When rounded to four significant digits, the mean area value reported in Table A9.9 is identical with that listed in Table 4.1*g*.

A9.2 Example of volume calculation using *Surfer* surfaces

The positive volume values listed in the raw data given in table A9.7 represent the total volume contained between the DEM surface, the level plane of elevation, the border of the glacier, and the boundaries of the DEM.

Values of volume and volumetric change above given elevations could not be calculated in the same way that values of area were. In the *Surfer* calculation of volume the values of the volume inside the border of the glacier are identical to corresponding values of volume NOT outside the border.

It was assumed that multiplying the value of the volume inside the boundary by the ratio of the corrected area and the uncorrected area for the corresponding elevation, a volumetric value that took the additional area of the border into the account would be produced:

$$Vol_{est} = Vol_{raw}\left(\frac{Area_{est}}{Area_{raw}}\right)$$

The values for the 1971-1973 interval for the 2140-2150 m elevation zone appear in table A9.10. These values can be used to calculate the volume contained within the boundaries of the DEM, the elevation zones and the surface of the DEM, which also appear in table A9.10.

Table A9.10: Calculation of Volumetric Change Figures from Raw and Modified Volumetric Data

Elevation (metres)	Unmodified Raw Values (m^3)		Modified Raw Values (m^3)	
	1971	1973	1971	1973
2140	6.75767×10^6	7.20512×10^6	6.77318×10^6	7.22722×10^6
2150	4.02051×10^6	4.38958×10^6	4.02307×10^6	4.39292×10^6
2140-2150	2.73716×10^6	2.81554×10^6	2.76180×10^6	2.84818×10^6
Differences	78380		86382	

This table shows that substantial differences exist between calculations of volumetric change with modified and unmodified data. The values of volumetric change that appear in the tables of chapter four are all calculated from the modified raw data values.

Checking the value of volumetric change in table 4.1*f* for the 2140-2150 elevation zone, it will be seen that it does not match the value produced in table A9.10. This is because, as discussed in section 3.3.3.6, the volumetric change value is the mean of the nine values of volumetric change produced when the different surfaces of uncertainty are taken into account. Thus, the modified volume value that appears in table A9.10 is only one of nine data inputs to the volumetric change calculation for that elevation zone. The eight other values of volumetric change were calculated in an identical manner to the example presented above.

A9.3 Using Surfer to calculate the Haumann method

As discussed in section 3.3.3.4.2, *Surfer* was used to calculate volumetric change using the Haumann method. This was calculated from raw data produced by blanking the subarea files with clear ice data. As discussed in section 2.4.1.1 and shown in figure 2.3, this method calculated volumetric change from the planar surface area of the various elevation zones. The raw data for the area calculations appears in table A9.11.

Table A9.11: Raw Data for calculation of final results

Border Type	Year of Data Acquisition	Blanking File From Year	Elevation (metres)	In or Out	File Name	Positive Planar Area (m^2)
Clear Ice	1971	1971	2140	in	FELOBNT	268367
				out	FELIBNT	186806
			2150	in	FFLOBNT	218027
				out	FFLIBNT	151355
	1971	1973	2140	in	FELOCNT	264800
				out	FELICNT	190221
			2150	in	FFLOCNT	214876
				out	FFLICNT	154227
	1973	1973	2140	in	GELOBNT	271899
				out	GELIBNT	192734
			2150	in	GFLOBNT	219566
				out	GFLIBNT	152714
Not Blanked	1971	not blanked	2140	not blanked	FELUDUT	458431
			2150		FFLUDUT	371479
	1973		2140		GELUDUT	468122
			2150		GFLUDUT	374633

The raw area measurements are modified in a manner identical to that described in section A9.1 to produce measurements of the planar area inside the glacier borders at or above the given elevation that take the extra area of the borders into consideration. These values appear in table A9.12.

Table A9.12: Modified Glacial Area Values for DEMs at or above given Elevations, Clear Ice Surface (m^2)

Elevation (metres)	1971 DEM	1971 DEM	1973 DEM
	1971 border	1973 border	
2140	269996.0	266505.0	273643.5
2150	219075.5	216064.0	220742.5

From these values, the Haumann method calculations can be made. These appear in table A9.13. In that table, the first five rows in the column labelled 'equation' contain two eight-digit values. Each of these eight-digit values represents a value in table A9.12. The first four digits represent the elevation of the contour, in metres, above which area is measured. The remaining four digits are in subscript. The first two of these represent the year of measurement of the DEM. The second two digits represent the year of origin of the blanking file applied. The final three rows in the body of the table represent values which are calculated from the results of previous calculations in that table.

The value of volumetric change produced in this table appears, rounded to four significant digits, in table 4.8*b*.

Table A9.13: Calculation of Volumetric Change using the Haumann Method

Variable	Equation	Value
F_1 =	$2140_{7173} - 2150_{7173}$	50441
F_2 =	$2140_{7373} - 2150_{7373}$	52901
ΔF_1 =	$2140_{7373} - 2140_{7173}$	7138.5
ΔF_2 =	$2150_{7373} - 2150_{7173}$	4678.5
F_1'' =	$2140_{7171} - 2150_{7171}$	50920.5
F_m =	$\frac{1}{2}(F_2 + F_1'')$	51910.75
dh =	$\Delta h(\Delta F_1 + \Delta F_2)(F_1 + F_2)^{-1}$	1.14348474
ΔV =	$F_m \cdot dh$	**59359.1**