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A test of the multiquadric method of interpolation using drumlin data from the Lunenburg, Nova Scotia and Peterborough, Ontario, fields

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A TEST OF THE MULTIQUADRIC METHOD OF INTERPOLATION USING DRUMLIN DATA FROM THE LUNENBURG, NOVA SCOTIA AND PETERBOROUGH, ONTARIO, FIELDS

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ABSTRACT

The multiquadric method of interpolation has been used to generate surfaces from irregularly distributed points of geophysical data, and has proven to be a successful means of mapping such data. There are four stages involved in the implementation of the method (Saunderson,1994): (1)solution of a system of simultaneous, linear equations; (2) interpolation of new z values, for a number of locations within the x,y area of an initial sample space of points; (3) plots of all the z values at their respective locations, using colour graphics instead of contouring; and (4) plots of the partial derivatives of z with respect to x and with respect to y.

This thesis uses drumlin data from two Canadian drumlin fields to test the applicability of Hardy's method of multiquadric interpolation by generating surfaces of sections of those fields. Contoured multiquadric surfaces are generated and compared to test surfaces derived from orthophoto maps. A working example leads the reader through the exact methods used, resulting in the final xyz and partialx and y plots. Drawbacks, such as the limited number of initial data points that can be used, as well as advantages, such as the unique nature of the methodology for drumlinized landscapes are outlined.

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TABLE OF CONTENTS

Page No.

Abstract.. ii

Acknowledgements.. iii

Table of Contents... iv

List of Tables... vi

List of Figures... vii

Chapter One Introduction.. 1

1.1 Drumlin Research.. 1
1.2 Spatial Interpolation... 2
1.3 Origin of Multiquadric Interpolation.. 4
1.4 Mathematical Basis of Multiquadric Interpolation.......................... 6
1.5 Applications of Multiquadric Interpolation.................................... 9
1.6 Comparison with other Techniques... 14
1.7 Thesis Objective.. 15

Chapter Two Regional Application and Methodology........................... 18

2.1 The Lunenburg Drumlin Field... 19
2.2 The Peterborough Drumlin Field... 19
2.3 Lunenburg Drumlin Field Data Collection..................................... 21
2.4 Peterborough Drumlin Field Data Collection.................................. 23
2.5 Computational Methodology... 24

Chapter Three Working Example... 27

3.1 Partialx Plot... 39
3.2 Partialy Plot... 43
3.3 Error Evaluation... 43

Chapter Four General results and Discussion of Research.............................. 48

4.1 Initial Lunenburg Data Collection.. 48
4.2 Initial Peterborough Data Collection.. 50
4.3 Reason for Errors in Initial Setup.. 52
4.3.1 Lunenburg Study Area........................ 52
4.3.2 Peterborough Study Area.................................... 54
4.4 Revised Lunenburg Data Collection and Results........................ 55
4.5 Revised Peterborough Data Collection and Results.................... 58
4.6 Comparison of Multiquadric Sections.. 62
4.7 Error Evaluation... 64

Chapter Five Conclusions... 68

5.1 Accuracy.. 69
5.2 Visual Aspect.. 70
5.3 Execution Time.. 70
5.4 Ease of Implementation.. 71

Appendix Raw xy and z Data Files.. 75

References 90

LIST OF TABLES

Table No. Page No.

3.1 Raw Data File *Ltest2xy.dru* 31

3.2 Values of z in File *Ltest2xy.dru* 32

3.3 Error Evaluation for Files *Ltest2xy.dru* and *Ltest2z.dru* 46

4.1 Error evaluation for several Lunenburg and Peterborough sections 67

LIST OF FIGURES

Figure No.		Page No.
2.1	Location of the major drumlin fields in mainland Nova Scotia	20
2.2	Location of Peterborough and other major drumlin fields in Southern Ontario	22
3.1	Location of the working example	28
3.2	Map of the working example with locations of all initial data points indicated	29
3.3	Drumlin heights, interpolated from a sample of 51 collected height values	34
3.4	Interpolated drumlin image and actual map of drumlin landscape	35
3.5	Plot of partialx derivatives at the same locations as interpolated heights in Figure 3.3	38
3.6	Plot of partialy derivatives at the same locations as interpolated heights in Figure 3.3	39
3.7	Interpolated xyz drumlin image and interpolated partialx image	40
3.8	Interpolated xyz drumlin image and interpolated partialy image	47
4.1	Initial Lunenburg drumlin field study area	49
4.2	Initial Peterborough drumlin field study area	51
4.3	Revised Lunenburg study area	57
4.4	Revised Peterborough study area	59

CHAPTER ONE

INTRODUCTION

1.1 Drumlin Research

Drumlins have fascinated researchers over the past decades due primarily to the mystery surrounding their genesis. Of all landscape features, drumlins are perhaps one of the most suitable for morphometric analysis. Their characteristic shapes and forms facilitate identification as well as data collection. Such analyses become evidence to test hypotheses of origin. In fact, one might go so far as to state that a geomorphological characterization of drumlin assemblages may be indicative of the glacial paleodynamics and suggest genetic hypotheses concerning drumlinization in relation to the local and regional environment (Coudé, 1989). A drumlin field may be considered a continuous surface, being the end result of one or several glacial advances and retreats. This thesis will use data collected from the Lunenburg, Nova Scotia, and Peterborough, Ontario drumlins to map such surfaces. Drumlins from sections of these drumlin fields will be used to test Hardy's method of multiquadric interpolation as a potential means of computationally mapping drumlin fields. They will also be used to test Saunderson's (1994) programmes, and are the application for a discussion detailing the use of these programmes.

Drumlins were recognized and given their name in 1867 by M.H. Close in Ireland, the name being derived from the Gaelic "druim", a hill (Charlesworth, 1957). No generally accepted theory exists as to their mode of origin. The ideal drumlin has been likened to "the inverted bowl of a spoon" (Heidenreich, 1964). Charlesworth (1957) describes them as "smooth, oval mounds or elongated ridges possessing straight major axes and having rounded summits, steep flanks, regular contours, and in profile double sigmoid curves". These are drumlins in their most ideal form, with differences reflecting changes in the characteristics of the ice or meltwater. Menzies (1979) points out that generally the steep, blunter end of a drumlin points in the up-ice direction and the gentler sloping, pointed end faces the down-ice direction, these two ends being respectively known as the stoss and lee sides. Drumlin orientations would therefore be indicative of the direction of movement of the agent which created them. It would seem, then, that although the exact mode or modes of their formation are not precisely known, drumlins are good indicators of the characteristics of the glacial forces which created them.

1.2 Spatial Interpolation

> "Interpolation, using a computer, is the
> performance of a numerical procedure
> that generates an estimate of functional
> dependence at a particular location, based
> upon knowledge of the functional

> dependence at some surrounding locations.
> It is only an informed estimate of the
> unknown." (Watson, 1992, p.101)

There are many spatial interpolation techniques, and theoretically, with sufficient precise data, any interpolation procedure will give good results because the sampled surface is known so well. It has been found, however, that some techniques work more accurately and efficiently than others (Shaw and Lynn, 1972; Franke, 1982). Features such as domes, basins, ridges, saddle points, and so on, may be poorly specified by sparse data collection, and most interpolation techniques would not be able to infer their presence (Watson, 1992). Unsatisfactory results may therefore be due either to the inappropriateness of the method for that application, or the data collection methods of the researcher.

Computer interpolation of topographical data can be obtained by two methodologically different approaches; fitted functions and weighted averages. Interpolation begins with the idea that a topographical measurement is a unit of information that describes that particular location and, with less certainty, a limited section of the surrounding area. This proximal region has been termed the kernel of influence. The surface represe[illegible] set of topographical data is therefore a collection of these kernels.

Computer interpolation methods are techniques to determine the sum of these influences, at particular locations. The difference between one method and another refers to the manner in ·.hich the influence of a datum is assumed to decline for more distant interpolation points, and the computational maneuvers necessary to process the elected influence function (Watson, 1992). Interpolation methods can be classified as those that first determine the parameters of an analytic bivariate function, then use the parameters to evaluate the height of the representative surface (fitted function methods), and those that directly sum the data influences (weighted average methods).

1.3 Origin of Multiquadric Interpolation

Multiquadric interpolation can be classed as a fitted function method. There are a number of other fitted function techniques, such as minimum curvature splines and kriging. The multiquadric method has been classified as a collocation procedure (Watson, 1992), which refers to two surfaces that are coincident at specific locations. If one were describing the variation of a spatial function by a representative surface which corresponds to the observations, this general surface would be said to collocate the data. The method has also been classified as global, as opposed to a local method of interpolation (Franke, 1982). The interpolant in a global method is dependent on all data points, and addition or deletion of a data point, or a change of one of the coordinates of a data point, will propagate throughout the domain of definition. A local method, on the other hand, is typically thought of as meaning that addition or deletion of a point, or a change in one of the coordinates of a datum , will affect the interpolant only at nearby points, and the interpolant will be unchanged at distances greater than some given distance. The multiquadric method may now be discussed in greater detail.

Multiquadric interpolation was developed by Dr. Rolland Hardy of Iowa State University (Hardy 1971, 1972). In the multiquadric method the surface model can be written as:

$$z=\sum_{j=1}^{n} c_j\,[Q(x,y,x_j,y_j)] \tag{1}$$

where Q represents any quadric basis function, c_j is the associated coefficient, and z is the observed quantity at x, y.

As a case for his multiquadric method, Hardy (1971) pointed out that there existed problems involving such map uses as determining unobstructed lines of sight, volumes of earth, and minimum length of surface curves. Fourier and polynomial series approximations have been applied to both surface and subsurface trend analysis in geologic mapping, but Hardy noted that these methods were not entirely efficient. Accuracy could be reached, with enough data points; however, with relatively few data points, the ordinary collocated Fourier series oscillates, with large variations between data points. The ordinary collocated polynomial series, limited to a few data points, was also found to be unmanageable in representing the sometimes rapid and sharp variations in real topographic surfaces (Hardy, 1971). There was a real need to improve efficient convergence on real topographic surfaces, which led to the development of variations of the Fourier and polynomial series approaches by many investigators. The frustrations of trying to use various harmonic and polynomial series to represent topography from relatively few data points led Hardy to search for a new series to represent this phenomenon. This search led him to discover the multiquadric method of interpolation (1971).

The construction of any irregular, continuous surface involves the interpolation of a large number of points (Saunderson, 1994). The control points may be sample locations with known x,y coordinates where some quantity at that location, z, has been observed. The method is therefore applicable to any continuous surface with "xyz" information. The known xyz data may then be used as a basis from which to interpolate new z values for any number of locations.

1.4 Mathematical Basis of Multiquadric Interpolation

In order to further explain the multiquadric method, the mathematical basis as it will apply to this research will be presented. In this case, locations in the drumlin field (x,y) and the height data (z) will provide input to a multiquadric system of equations (Hardy 1971, 1972, 1990) using a cone model with constant C=0. From a geometric point of view "C" simply changes the sharpness of each cone. If C is small, the cone could be described as "sharp-nosed". If C is large the cone could be described as "broad-nosed" (Hardy, 1990). Expansion of the equations generates a series of intersecting cones, producing an interpolated surface, approximating the original location of the data points as well. The following equations show the matrix (Lancaster and Salkauskas, 1990) and algebraic (Hardy 1971, 1972, 1990) forms.

$$A \cdot c = z, \tag{2}$$

with solution:

$$c = A^{-1} z, \tag{3}$$

and matrix form:

$$\begin{bmatrix} 0 & |p_1 - p_2| & \cdots & |p_1 - p_n| \\ |p_2 - p_1| & 0 & \cdots & |p_2 - p_n| \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ |p_n - p_1| & \cdot & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix} \tag{4}$$

where A is the matrix of terms calculated from the sampled x- and y- coordinates, c is the solution vector of coefficients c_j and z is the right-hand-side vector of sampled data (i.e. height), and

$$|p_i - p_j| = [(x_i - x_j)^2 + (y_i - y_j)^2]^{0.5} \quad (5)$$

The equation of a cone may be expressed as:

$$[(x^2 + y^2)\tan^2 \alpha]^{0.5} = z \quad (6)$$

where z is the elevation of the vertex of the cone above the xy plane and the $\tan\alpha$ is the slope of the surface of the cone (Saunderson, 1994). A series of multiquadric equations may also be set up from the known elevations of a sample of points (the vertices of a series of cones) at known locations, giving

$$\sum_{j=1}^{n} c_j[(x_i - x_j)^2 + (y_i - y_j)^2 + C]^{0.5} = z_i \quad (7)$$

where x, y are the Cartesian coordinates (sampled locations) of the vertex of each cone, z is the information (i.e. height) which has been collected from that location, and c_j is the coefficient. z_p values from intermediate locations (of the initial data input), x_p, y_p may then be interpolated from

$$\sum_{j=1}^{n} c_j[(x_j - x_p)^2 + (y_j - y_p)^2 + C]^{0.5} = z_p \quad (8)$$

Once the column vector (c_j) has been determined, a surface is plotted from the z values evaluated in equation (8).

The texture of the surface depends on the number of points plotted per unit area of the surface, the points being the vertices of a large number of cones (Saunderson, 1994). The variation of slopes on the surface may also be generated by evaluating the partial derivatives of z with respect to the x and y axes (Hardy, 1971). These partial derivatives are obtained by differentiation of equation (8), yielding

$$\partial z_p/\partial x = -\sum_{j=1}^{n} c_j[(x_j-x_p)^2+(y_j-y_p)^2]^{-0.5}(x_j-x_p) \quad \textbf{(9)}$$

and likewise

$$\partial z_p/\partial y = -\sum_{j=1}^{n} c_j[(x_j-x_p)^2+(y_j-y_p)^2]^{-0.5}(y_j-y_p) \quad \textbf{(10)}$$

1.5 Applications of Multiquadric Interpolation

Hardy's method of multiquadric interpolation has been used for terrain mapping (Hardy, 1971), computational fluid dynamics (Kansa, 1990), and mapping of fluid speeds in river cross-sections (Saunderson, 1992) to name but a few applications. The process of generating irregular, continuous surfaces is often necessary in geography and related sciences. Some of the specific applications which have used Hardy's method will be described in order to show the versatility as well as the viability of the method.

Hardy discovered the multiquadric approach, and tested it (1971). Contoured multiquadric surfaces were compared with topography and other irregular surfaces from which the multiquadric equation was derived. A model of simple fictitious topography was first used to test the feasibility of multiquadric analysis, with favourable results. Another model of fictitious topography, adjacent to the initial model, was then used to investigate the feasibility and accuracy of joining map edges, the result being a good fit along the common boundary between the two blocks. Hardy went on to determine a multiquadric equation of a fictitious contour model. This model was visualized as a subsurface problem, where the data points were selected on the basis of unillustrated surface conditions. Again, the results were favourable, with Hardy concluding that a random scanning mode will work reasonably well for an uncomplicated subsurface case (1971). A final test was based on a topographic model from a 9x10 grid sample of a part of a U.S. Geological Survey quadrangle map of McClure, Pennsylvania. A deficiency in this test was concluded to be mainly a result of poor elevation choices for representing the hills and saddles.

Hardy (1972) presented an application of multiquadric equations to surveying and mapping problems. In this paper, Hardy reiterated the first application of multiquadric analysis in representing topographic surfaces analytically in Cartesian coordinates. A later application of multiquadric analysis involved the determination of an equation of the world's topographic and bathymetric surface from a limited number of data points (Hardy, 1974). Further applications of the method are mentioned by Hardy, such as Shaw and Lynn's (1972) comparison of multiquadric analysis with the bi-cubic spline function as a method of representing areal rainfall. (The results of this study will be described in the following section comparing multiquadric interpolation to other surface fitting techniques). Hardy goes on to present the idea that traditional problems determining terrain corrections in gravimetric surveys may be overcome.

Schiro and Williams (1984) used an adaptive application of multiquadric interpolants to model large numbers of irregularly spaced hydrographic data. The authors note the need for a precise method for such an application, since a hydrographic survey is a basic component of a nautical chart, which is used for safe navigation. An accurate model of a hydrographic survey would therefore serve many practical and useful needs in nautical chart production. The three greatest areas which Schiro and Williams point out are survey reduction, survey verification, and the generation of accurate depth contours for nautical charts. Seven test areas were chosen for the iterative application of the multiquadric method in this study. The first five of the test areas came from hydrographic surveys of Swan Point to Dahlgren, Maryland, Virginia, and the Potomac River, the areas being chosen as representative of the majority of hydrographic surveys. The last two areas were surfaces defined by mathematical functions, as it was desired to test the multiquadric method on real and generated data. To briefly conclude their results, Schiro and Williams found that this method performed well when tested with a large set of real data and some common mathematical test surfaces. The multiquadric model "acceptably approximated the original mathematical surfaces even in areas in which the original data was not present. This approximation was done using only 50 percent of the original data. Hence, the tests show that the method can produce good results and be reasonably efficient, even with rather strict error criteria" (p.380).

Kansa (1990) published a paper presenting the application of multiquadrics to computational fluid dynamics. The first part of the paper investigates the new numerical technique of curve, surface and body approximations over an arbitrary data arrangement. The second part uses such techniques to improve parabolic, hyperbolic and elliptical partial differential equations. Kansa observed that multiquadric analysis is excellent in regions whose surfaces have gradients that are not too small. Two test functions were evaluated on the vertices of a uniform 5X5 grid, involving the interpolation of functions from a coarse grid onto a finer grid. The next set of experiments dealt with the problem of scattered data, involving a plot of 60 points over a unit square, resulting in the exact surface and the multiquadric surface being indistinguishable from one another. Kansa goes on to apply the extended Hardy scheme to fluid dynamics, hoping to uncover the underlying unknown continuous behaviour so that values of a variable anywhere in the domain may be predicted. Kansa found Hardy's multiquadric method to be very promising for such an application.

Saunderson (1992) applied multiquadrics to interpolating fluid speeds in a natural river channel. The paper describes the application of the method to the magnitude of fluid vectors taken from a meander. Known point speeds were reproduced accurately, and several thousand others were interpolated with good results. Saunderson points out that in many cases where new values of z_i are interpolated, there is still a need for a final contouring process to interpolate isolines. In this case, the colour coding of a closely-spaced set of points makes this final contouring process unnecessary.

Saunderson and Brooks (1994), researched multiquadric sections from a fluid vector field. Having already determined fluid speeds to be a viable application, these authors went on to show the distribution of fluid vectors by mapping the vectors observed at different locations in the flow through a river bend, and by plotting the vectors using the multiquadric method. It was concluded, through comparison of the isometric and multiquadric views of fluid vectors from the tested river bend, that the multiquadric method has considerable promise for the visualisation of vector fields in sinuous and meandering currents. It is pointed out that the full potential of this aspect of the method needs to be clarified by further research.

It seems appropriate to end this section by noting an article by Hardy (1990) which discusses the past twenty years of discovery in theory and application of the multiquadric method. The author makes subheadings of applications into those dealing with hydrology, geodesy, photogrammetry, surveying and mapping, geophysics and crustal movement, geology and mining, topography, and hydrography. The variety of these applications suggests that there are many others which have not been specifically mentioned in this thesis. Hardy makes the concluding remark that with the acceleration of attention by mathematicians to the study of the multiquadric method, other interesting discoveries will be made in the future, in theory as well as in practise.

1.6 Comparison with other Techniques

Perhaps one of the most detailed comparisons of scattered data interpolation techniques was conducted by Franke (1982). This extensive article evaluated 29 different algorithms for the scattered data interpolation problem on a variety of known data surfaces. Franke graded the various techniques according to the following criteria: accuracy, visual aspect, sensitivity to parameters, execution time, storage requirements, and ease of implementation. The methods he tested may be classified into the following groups: (1) inverse distance weighted methods, (2) rectangle based blending methods, (3) triangle based blending methods, (4) finite element based methods, (5) Foley's method, and (6) global basis function methods. Franke concluded that "In terms of fitting ability and visual smoothness, the most impressive method included in the tests is the multiquadric method, due to Hardy" (p.191).

Shaw and Lynn (1972) evaluated two surface fitting techniques. They compared multiquadric analysis with the bicubic spline function as a method of representing areal rainfall, noting that the bi-cubic spline function is limited to grid data. Since rainfall data is seldom acquired this way, their studies focused on multiquadric analysis, which has the flexibility of allowing the use of non-gridded data. Although the computation time for the multiquadric method was somewhat longer than for the bi-cubic spline, it was concluded that in applications involving changing networks and variable distribution of data the method of multiquadric analysis would be invaluable.

1.7 Thesis Objective: Application of Multiquadric Interpolation to Drumlin Fields

The multiquadric method has been used for a wide variety of applications, and is very accurate, with advantages over many other surface fitting techniques (Franke, 1982). These advantages include improved accuracy, visual aspects, and a shorter computation time. This thesis will use drumlin data from two historically different drumlin fields to test their applicability to the multiquadric method of interpolation, and Saunderson's (1994) programmes, based on this technique, will be evaluated, and discussed.

Computational methods of mapping, such as the multiquadric method of interpolation, have not typically been used to map drumlins. This thesis will be the first to use drumlins as an application of Hardy's method. There are other benefits to conducting such a thesis, beyond being a new test of the multiquadric equations. It is believed that drumlin research may also substantially benefit from using such a method of mapping for several reasons.

Measures of the area of a drumlin have been traditionally calculated by measuring length and width from a topographic map and then using the traditional equation (A=l*w) to calculate the value. There are a couple of problems with this calculation (with regards to drumlin area). Human errors may be made in either the collection of the length or width or both values, resulting in an incorrect value. Even if these values are correct, the equation being used may be questioned, since drumlins have an elliptical, not rectangular, form. All of this is of significance to drumlin morphology since area is used in calculating Chorley's k-value (1959), a value which is used to describe the character of drumlin morphologies and quantify their differences. The multiquadric method would overcome this problem. By using height values alone as z values, surfaces would be generated portraying true drumlin form, with boundaries between drumlins being at some of the locations where $\partial z/\partial x$ and $\partial z/\partial y = 0$ (the other zero values being the tops of drumlins).

The ability to use such a means of mapping drumlin morphologies will also aid in this area of research. Instead of qualitatively describing differences across a drumlin field, the multiquadric method, which has proven advantages over other methods, will allow sections of drumlin fields to be quantitatively mapped. This would result in colour graphics of variations in drumlin morphologies. Such graphics will be created for both the Lunenburg and Peterborough drumlin fields.

Above all, the value and applicability of drumlins to this procedure (outlined in the following chapter) of Hardy's multiquadric method will be evaluated. Where necessary, drawbacks as w   is advantages of the method will be pointed out for this application. It will be pointed out that there are a number of factors which will influence the final results, such as the size and shape of the sample area, the method of sampling, the number of points sampled, and the scale used for producing the final image, among others.

CHAPTER TWO

REGIONAL APPLICATION AND METHODOLOGY

Two drumlin fields were chosen for this research project. These are the Lunenburg drumlin field in Nova Scotia, and the Peterborough drumlin field in Ontario. These two drumlin fields were chosen for a number of reasons. The Lunenburg drumlin field was chosen because of some previous data collection and a knowledge of the area (Schroeder (Conrad), 1993). It was also chosen because of the complexity of drumlin orientations in the area. The Peterborough drumlin field was chosen because it is located near to the study of research, and because it is such a large field, with maps available to easily observe and collect data from the drumlins. It was also chosen because the glacial history is very different than that of Nova Scotia. Not only will this show different results for each field, but it will also mean that Hardy's method will be used for two different areas, and testing the method for more than one drumlin field.

2.1 The Lunenburg Drumlin Field

Nova Scotia has a number of drumlin fields, of which the Lunenburg field, situated 80 kilometres southwest of Halifax, is the largest (Fig.2.1). Some of the largest "classic" drumlins in Nova Scotia are found here, with the slate areas of Lunenburg County forming "typical" drumlin country. The glacial history of Nova Scotia is characterized by external and local ice divides. The first ice flows were southeastward, then southward across Nova Scotia from external centres and divides (Stea and Brown, 1989). Very little can be said about the past drumlin research in Nova Scotia. Some general work was conducted by Gravenor (1974) although this work was aimed specifically at the Yarmouth field. Stea and Brown (1989) conducted research on the Lunenburg field, dealing with the complexity of drumlin orientations.

2.2 The Peterborough Drumlin Field

The Peterborough drumlin field is located approximately 110 kilometres northeast of Toronto. It covers 900 km^2 on a Paleozoic limestone plain and is one of the largest drumlin fields in southwestern Ontario (Fig.2.2). It is situated in a depressional area between the Algonquin upland to the north and the Niagara Escarpment to the southwest (Sharpe, 1987). Erosion of the limestone plain provided sediment in the overlying glacial drift. This drumlin field lies to the north of a major late Wisconsinan moraine system, the Oak Ridges moraine, that comprises large masses of stratified glaciofluvial/ glaciolacustrine sediment

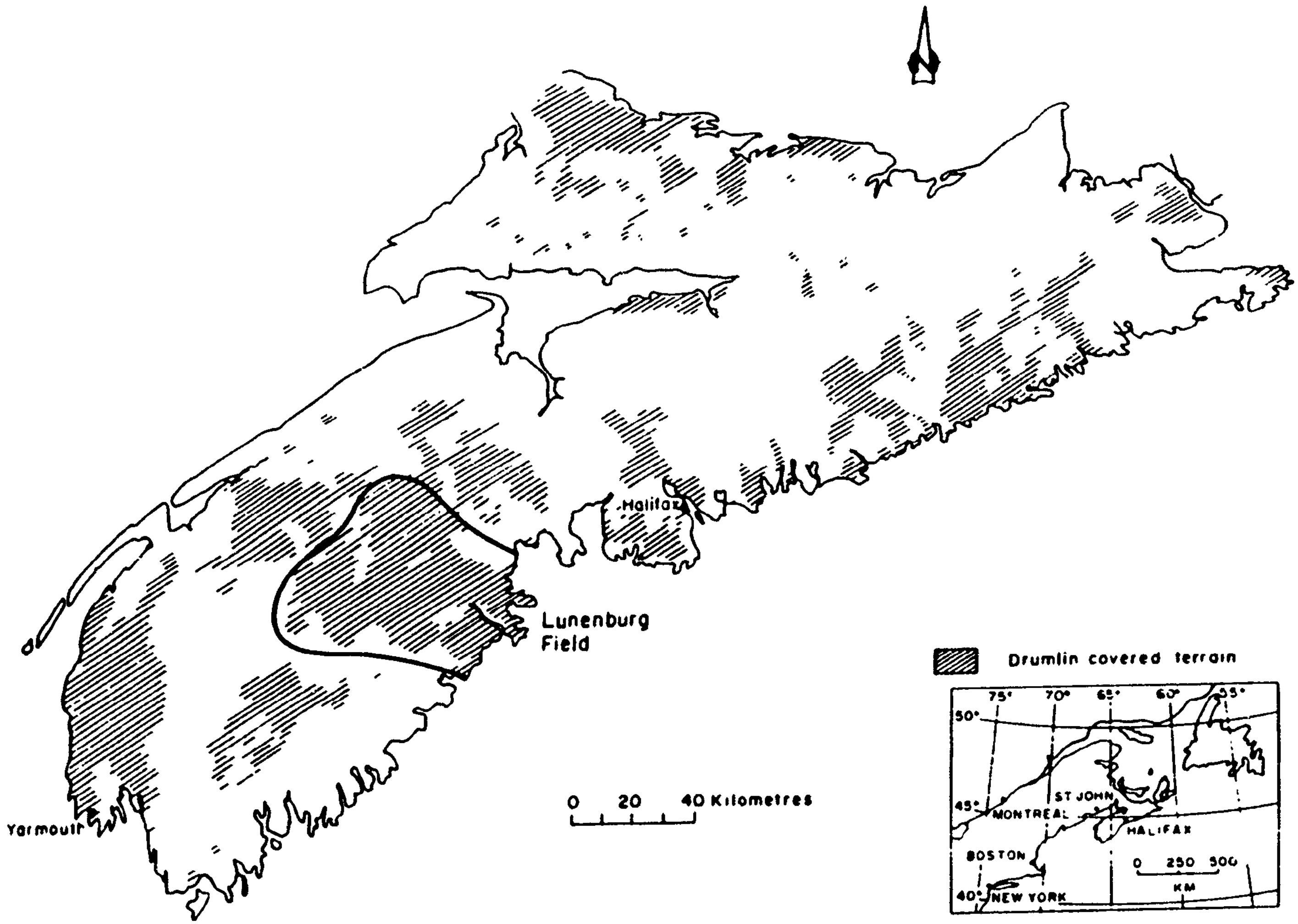

Figure 2.1 Location of the major drumlin fields in mainland Nova Scotia (Stea and Brown, 1989)

(Duckworth, 1979). Sharpe (1987) believes that the drumlin field may be a transitional set of landforms to the Oak Ridges moraine, stemming from a hypothesis that drumlins are but one landscape feature in a transitional series having been formed beneath an ice sheet. Gravenor (1957), however, believes that the Peterborough drumlin field was formed by erosion to the north part of the moraine.

2.3 Lunenburg Drumlin Field Data Collection

Drumlin data were collected using 1:10 000 orthophoto maps produced by the Nova Scotia Land Registration and Information Service. It is believed that data collected from these map sheets are accurate, due to the fact that the scale is relatively large, and the contour interval of 5 metres is well suited for the purpose. Most similar studies which have obtained drumlin data from maps have been those with 1:25 000 scales or smaller (Jauhianen, 1975; Rose and Letzer; 1975, Coudé, 1989). Such studies have also used aerial photographs in order to obtain more accurate information. Rose and Letzer tested the reliability of data derived from 1:25 000 scale topographic maps. They concluded that

> "...topographic maps [of this scale] fail to give information at the precision level required for the analysis of this type of glacier bedform." (1975, p.361)

It is believed that the Lunenburg data collected in this study are as accurate and detailed as possible for two reasons. First, they were obtained from large-scale map

Figure 2.2 Location of the Peterborough and other major drumlin fields in southern Ontario (Sharpe, 1987)

sheets with consistent criteria for data collection. Second, orthophoto maps, rather than topographic maps, were used. Orthophoto maps are produced from an assemblage of overlapping air photos which have been constructed to show natural features in their true planimetric positions. Precise measurement of distance can be made directly on the orthophoto map because the original photos were assembled to match a network of ground control points.

For the purpose of this study, drumlins were recognized by parallel groups of elliptical contour patterns. Once the drumlins were identified, height values were collected for each drumlin. Height was easily determined, due to the fact that spot heights were located at the highest point of each drumlin, as well as spot heights at the swales between drumlins. Following an extensive initial set-up of several test sections, it was determined that a certain criteria for selecting heights surrounding the drumlins would need to be followed. This meant that heights were consistently selected between adjacent drumlins, whether horizontally or vertically. This would ensure that the drumlin forms could be depicted. Without selecting heights at low elevations between drumlins would result in images that would not depict the drumlin forms. This was exactly what happened when the heights were initially non-systematically selected.

2.4 Peterborough Drumlin Field Data Collection

Drumlin data were collected using 1:10 000 topographic maps from the Ontario Ministry of Natural Resources Ontario Base Map series. The data collected from these maps are also accurate, again due to the fact that the scale is quite large and the contour interval is 5 metres, an appropriate number, given the heights of the drumlins (average height = 200m). The contours are distinctively visible, and data were easily obtained from the maps. Unfortunately, orthophoto maps do not exist for this area, and therefore could obviously not be used. It is believed, however, that the Ontario Base maps were sufficient to collect accurate data due to the quality and clarity of the cartography, contour interval, and scale. Height values were collected consistent to those methods described for the Lunenburg field.

2.5 Computational Methodology

The computational methods in this thesis are adapted from Saunderson (1994). The first step in this computational process is to obtain samples with xyz information. As it was noted, this application will be using gridded x and y locations for drumlin z heights. The program, *matrx1.c* (Saunderson, 1994) formats the sampled data, and then takes as input two data files, the first containing the x and y coordinates of the samples, and the second containing the sampled values for z. The square roots of the left hand side of equation (7) are calculated from the x,y coordinates and then printed, with the z values, to an output file called *matrx1.dat*, which is then used as input to the numerical recipe called "singular value decomposition with back substitution" (Press et al.,1988). This is also used to solve the column vector c_j.

A second program, *xyzpart.c*, (Saunderson, 1994) is used to obtain new, interpolated values. The program uses the x,y coordinates as input from which a large number of interpolated locations (x_p,y_p) are generated. These locations are determined by setting a for_loop, with a standard increment, which will change the x and y coordinates. The output from the singular value decomposition routine (Press, et al.,1988) is in a file called *cvec*, containing the column vector c_j. This program opens cvec and uses c_j to perform row-by-column multiplication to obtain the interpolated values for z_p of equation (8).The interpolated z values are then stored together with the x and y coordinates in a new file (xyz). In addition to interpolating new z values, *xyzpart.c* also calculates the partial derivatives of z with respect to x and with respect to y using equations (9) and(10). The $\partial z_p/\partial x$ values are stored in the output file partialx and the $\partial z_p/\partial y$ values in partialy (Saunderson, 1994). Both the xyz files and the partialx and y files can be split easily into smaller files for input to the plotting routines *xyzplot.c* and *prtlplx.c* or *prtlply.c* respectively.

A third program (*xyzplot.c*) takes blocks of 16 000 numbers from the xyz file as input files (a small awk program splits xyz files into smaller files). The input file names need only be typed as arguments to main() in order for the coloured graphics to emerge on the screen (ex. xyzplot xyzd1 xyzd2 xyzd3). The program draws a border and legend, establishes a real coordinate mode instead of the physical (default) system of coordinates, and plots the z values at their specified coordinates (Saunderson, 1994). Specific colours are assigned to each z value and are displayed within the border.

A fourth and final program (*prtlplx.c* or *prtlply.c*) takes either the values of $\partial z_p/\partial x$ from *partialx* or $\partial z_p/\partial y$ from *partialy* as input. Like xyzplot, the numbers are input as sets of 16 000. The plots of the partial derivatives provide information on local gradients on the surface generated by *xyzplot.c* and on the directions of those gradients parallel to the x and y axes (Saunderson, 1994).

The following chapter will discuss the specific procedure that was undertaken to obtain both xyz and partial derivative plots using one example from the Lunenburg drumlin field.

CHAPTER THREE

WORKING EXAMPLE

In order to understand the process of deriving interpolated xyz and partial derivative plots from the multiquadric method, an example from the Lunenburg drumlin field will be worked through in its entirety. This sample area was chosen due to the accurate nature of the maps. The methodology follows that of Saunderson (1994).

The sampled section of the Lunenburg field, which will be outlined, is located in the eastern portion of the drumlin field (Figure 3.1). Drumlins were identified, based on the criteria outlined in the previous chapter, and spot heights for those drumlins were collected. In order for the interpolated images to even remotely resemble the actual landscape, heights surrounding the drumlins needed to be collected. These heights were collected between drumlins in most cases, and lowest elevations were chosen. Without a geomorphological background, the best heights to chose in order to get the best resulting images would not have been known. Figure 3.2 shows the exact locations of the data points (heights) which

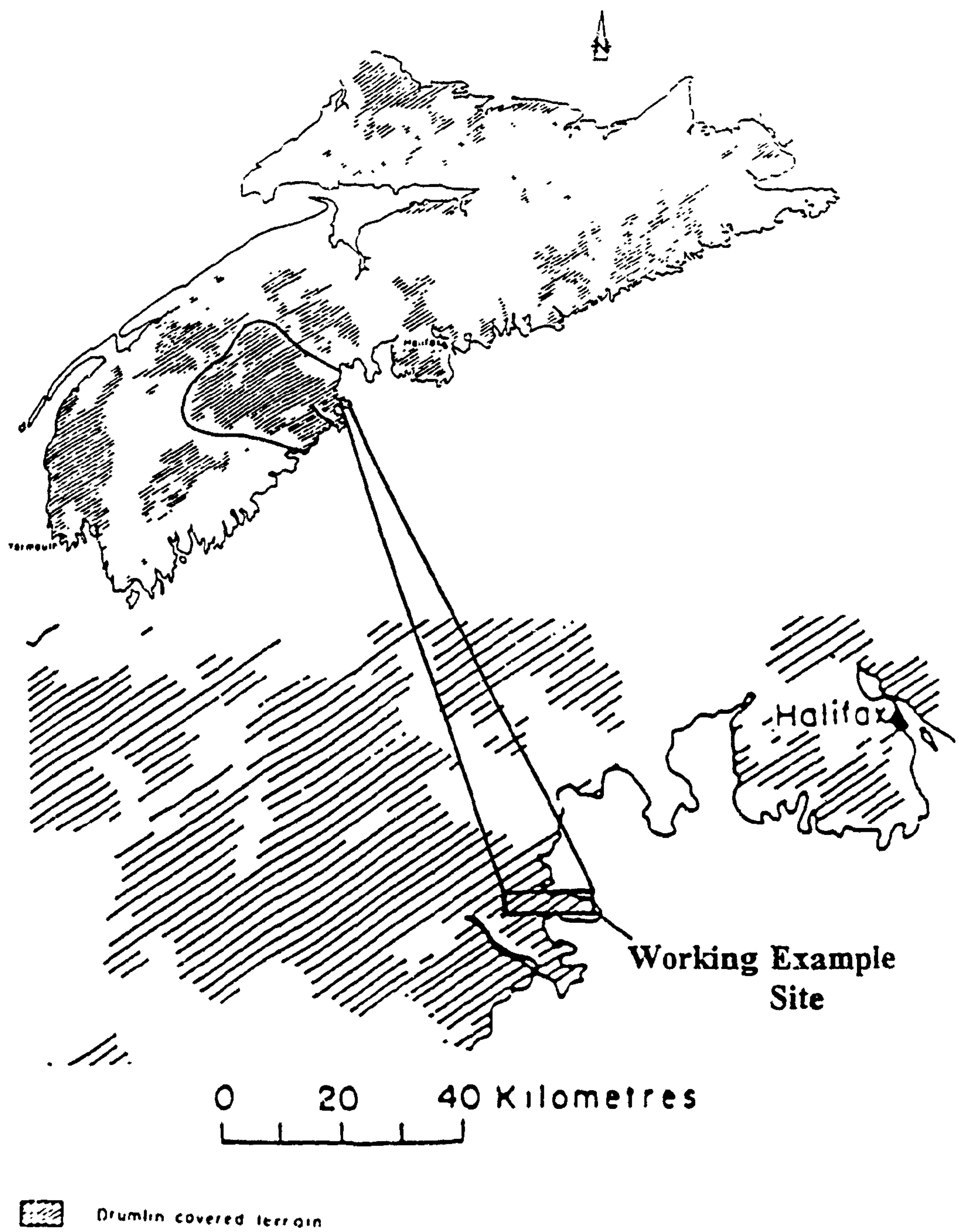

Figure 3.1 Location of the working example outlined in this chapter

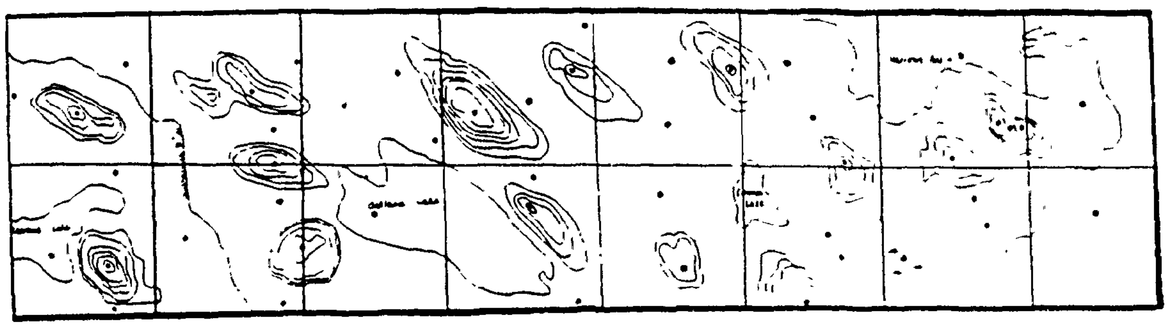

Figure 3.2 Map of the working example with locations of all initial data points indicated (drumlins and intermittent heights).

were initially collected.

These data were then entered into the computer, using QuickC for DOS (Tables 3.1 and 3.2). As the origin of the plot was placed at the upper right corner of the study area, the x,y coordinates are negative, x being the east-west distance across the drumlin field, and y the north-south distance down the field. Saunderson (1994) points out that users with a preference for positive numbers could place the origin at the lower left corner and make small changes to the programs. Once these two files were set up, the computational methodology that was outlined in Chapter two could begin.

The program, called *matrx1.c* , is the first in a series of four which result in the final, plotted products. The only change to this program was the definition of "N" (the number of records) to whatever the number of data points there are. In this case, N was changed to 51. *Matrx1.c* could then be compiled on a command line and run using this command;

matrx1 Ltest2xy.dru Ltest2z.dru

Ltest2xy.dru and Ltest2z.dru were the names of the two input files for this application. These would change depending on whatever the user called his/her files.

The output created from this is called *matrx1.dat.* A number of numerical

-0.1	-0.1
-0.5	-0.6
-0.2	-1.4
-1.1	-0.7
-1.2	-0.8
-1.3	-0.7
-1.6	-0.5
-1.6	-1.1
-1.4	-1.3
-1.3	-1.4
-1.8	-1.6
-2.7	-0.4
-2.5	-0.8
-2.3	-1.1
-2.5	-1.5
-2.4	-1.6
-2.5	-1.9
-2.5	-2.2
-3.1	-0.5
-3.5	-0.4
-3.6	-1.3
-3.5	-1.7
-3.5	-0.8
-4.2	-0.5
-4.1	-0.3
-4.5	-0.6
-4.8	-0.7
-5.1	-0.9
-4.2	-1.3
-4.4	-1.4
-4.1	-1.9
-4.1	-2.2
-5.4	-0.5
-5.7	-0.6
-5.5	-1.5
-6.1	-0.4
-6.4	-0.5
-6.2	-0.7
-6.8	-0.7
-6.3	-1.1
-6.2	-1.3
-6	-1.5
-6.2	-1.9
-6.7	-1.5
-7.2	-0.4
-7.6	-0.7
-7.9	-0.5
-7.3	-1.1
-7.3	-1.7
-7.3	-1.9
-7.6	-1.5

Table 3.1 Raw data file *Ltest2xy.dru* containing x,y coordinates of sampled points in the Lunenburg drumlin field.

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Table 3.2: Values of z in file *Ltest2z.dru* which are actual heights in metres, collected at the coordinates in Table 1.

recipes (Press et. al.,1988) were then used, inputting *matrx1.dat*, and then outputting the column vector c_j. In the numerical recipes *xsvbksb.c*, *svdcmp.c*, and *svbksb.c* were then compiled with *nrutil.h* and *pythag.c*. Output to *cvec* could then be accomplished by simply typing;

xsv > cvec

at the dollar sign ($) prompt.

A number of changes were then made to the program *xyzpart.c*. Again, N needed to be changed to 51, and then the length and width of the study area were changed to fit this application. A final change made to this program was the memory allocation function *malloc()*. This function was used in a multiple "for" construct to overcome the 64k limit to the size of data segments in large arrays (Saunderson, 1994). The numerical value of *malloc()* was therefore required to be changed to 192 to suit this application.

Once these changes had been made, the program could be compiled on the command line. The program could then be run using the original *xy* data file as input:

xyzpart Ltest2xy.dru

Depending on the speed of the hardware on which this software is being used, running *xyzpart.c* could take from a couple of hours to a couple of minutes to run. This work was run on a 486DX, and therefore only took about 3-4 minutes to execute. This program then created three output files; *xyz, partialx* and *partialy*. Changes were required to be made to all of these files, so they will each be described separately, resulting in the final images which were derived from the computed information in each file.

The *xyz* files store the new x and y locations as well as the z (height) values. Using a small *awk* program, the maximum value of z was calculated in order to be added into the eventual plotting program *xyzplot.c.* The number of records was then determined, again using a small *awk* program, in order that the *xyz* file could be divided into a couple of smaller files. The program *xyzplot.c* takes blocks of 16 000 records from the *xyz* file as input files (again, by using *awk*). A choice of 16 000 or less enables the program to compile within the 64k limit for the array size (Saunderson, 1994). Once those changes were made to the *xyz* file, a few changes were required of the program, *xyzplot.c.*

First, the maximum value was changed to 77.0 metres. The width and length of the study area were also changed (in this case to 7.80 and 1.80) and it is noted in a comment statement in the program for users to change this to suit their application. The coordinates for the window within which the plot was to be drawn also needed to be changed, and this is again noted in comment statements in the program. A change which was required for this application (and might be for other applications) was the intervals of the percentages of the maximum value, which assigned the z values to different colour classes. The interval, which was 5, was changed to an interval of 10 to enhance the image and display the best results, this being a trial and error process. A final change to this program was simply to change the labels and the intervals for the legend.

Once these changes had been made, the program could be compiled and the input file names then typed on the command line as arguments to main()

xyzplot Lun3 Lun4

(Lun3 and Lun4 being derived from the *xyz* file). The program sets and scales VGA mode, draws a border and legend for the plot, establishes a real coordinate mode instead of the physical system of coordinates, and plots the z values (heights) at their specified locations (Figure 3.3).

When looking at this plot, upon comparison to the actual map of the landscape, it can be seen that the interpolated image closely corresponds to the actual landscape (Figure 3.4). It was an initial goal to determine whether this specific procedure would actually map the drumlin features, and these can clearly be observed in this image.

The next step at this stage was to plot the partial derivatives. Similar to the *xyz* data, changes were made to *partialx* and *partialy* data. Once those

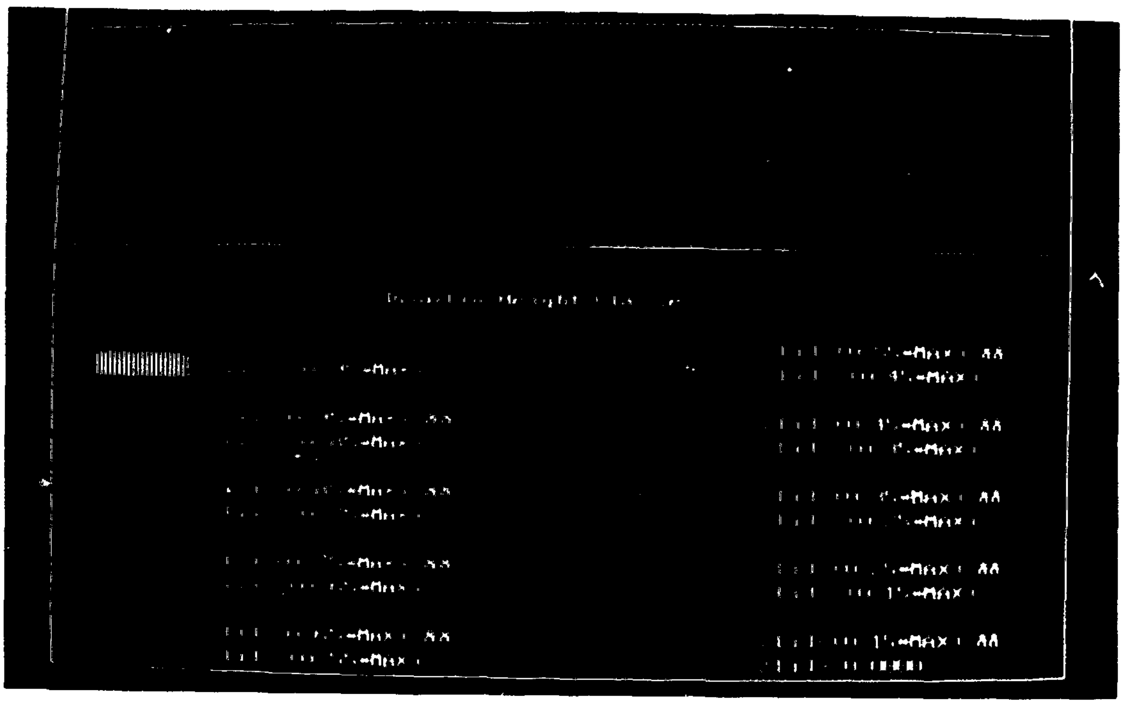


Figure 3.3 Drumlin heights, interpolated from a sample of 51 collected height values.

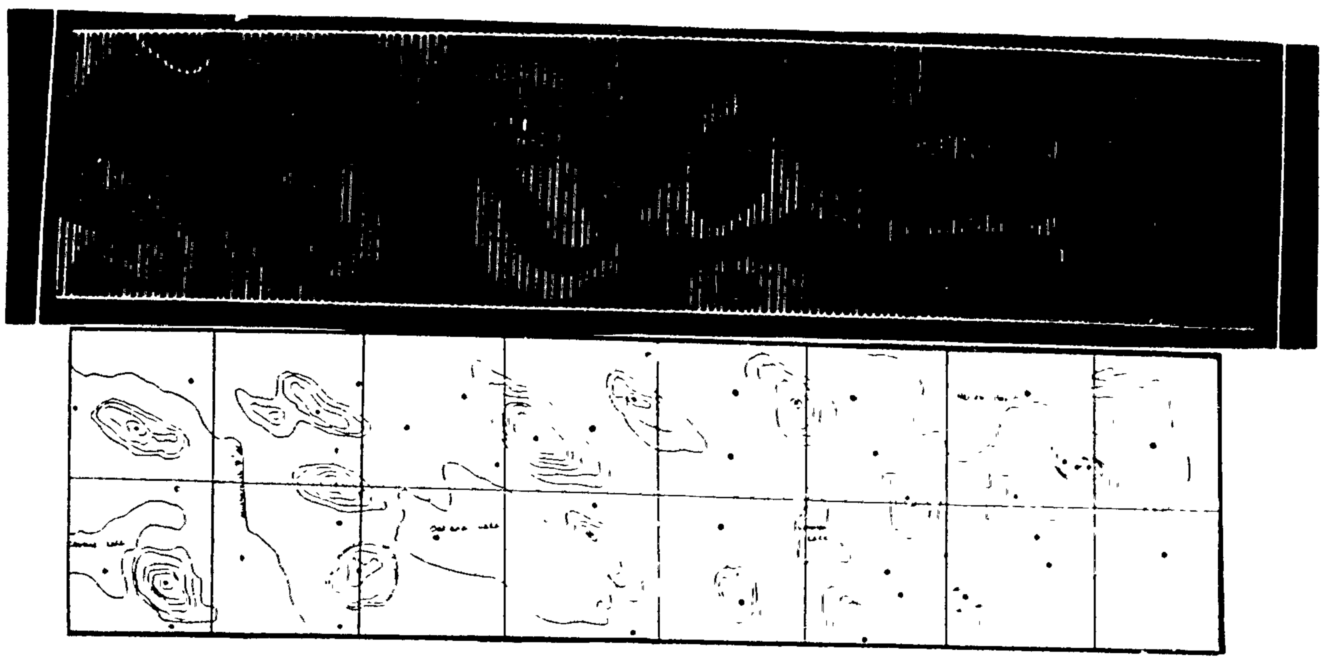


Figure 3.4 Interpolated drumlin image and actual map of drumlin landscape

changes were made, changes were required of *prtlplt.c*, the plotting program for the partial derivatives, similar to *xyzplot.c*.

The first thing that needed to be done to both the *partialx* and *partialy* files was to eliminate the fourth column, since this column was not required for this application. Then, both the maximum and minimum values were determined, since the slopes have both positive and negative values. For this example, the maximum of *partialx* was (+)146 and the minimum was (-)146, so the maximum value used (for ease in plotting) was (+)146. The maximum slope of the *partialy* file was (+)189 and the minimum slope was (-)168.

Both of these files, similar to the *xyz* file, were also divided into blocks of 16 000 for input to the *prtlplt.c* program. Once these changes had been made, and values determined, the changes to *prtlplt.c* could be made. Although two plots were determined from the *prtlplt.c* program (one for the *partialx* values and one for the *partaily* values), the changes to the program will only be discussed once, since the only difference between the two is the maximum value.

The changes to *prtlplt.c* are similar to those that were made to *xyzplot.c*. For the *partialx* plot, the maximum value was changed to (+)146, and for the *partialy* plot, the maximum value was changed to (+)189. The length and width and coordinates of both plots were changed to the same values as those for the *xyzplot.c* program, since all of these values came from the same sample area. The interval was then changed to a range of (+,-).99 to (+,-).50 and (+,-).50 to (+,-)1. The .99 and .50 values are percentages of MAX (the maximum height), while the (+,-)1 values are absolute. This last class ((+)1 to (-)1) was crucial in picking out those areas where the slope is near zero. For values closer to zero to have been used, the reduced number of points would have meant that those areas on the plot (Figure 3.5 and Figure 3.6) would not have been visible. Once these changes were made, the program could be compiled, and then plotted, by typing the input file names on the command line as arguments to main;

prtlplt Lprtlx3 Lprtlx4 (for *partialx*'s)

and

prtlplt Lprtly3 Lprtly4 (for the *partialy*'s)

When initially glancing at either the *partialx* or *partialy* plot, it might be assumed that neither corresponds to the *xyz* plot. Upon closer investigation and explanation of the images, however, it may be seen that they correspond quite nicely. The *partialx* plot will be explained first.

3.1 Partialx Plot

The plots of the partial derivatives provide information on gradients on the surface generated by *prtlplt.c* and on the directions of those gradients parallel to the x and y axes. The plot of the *partialx* derivatives shows those gradients parallel

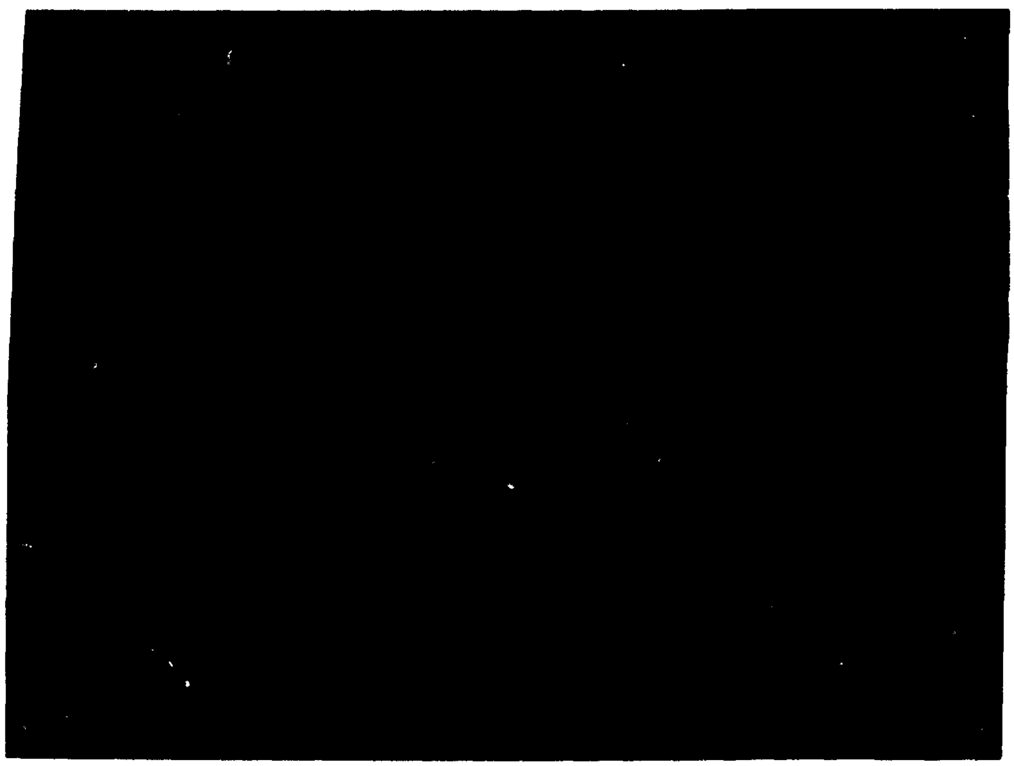


Figure 3.5: Plot of partialx derivatives at the same locations as interpolated heights in Figure 3.3

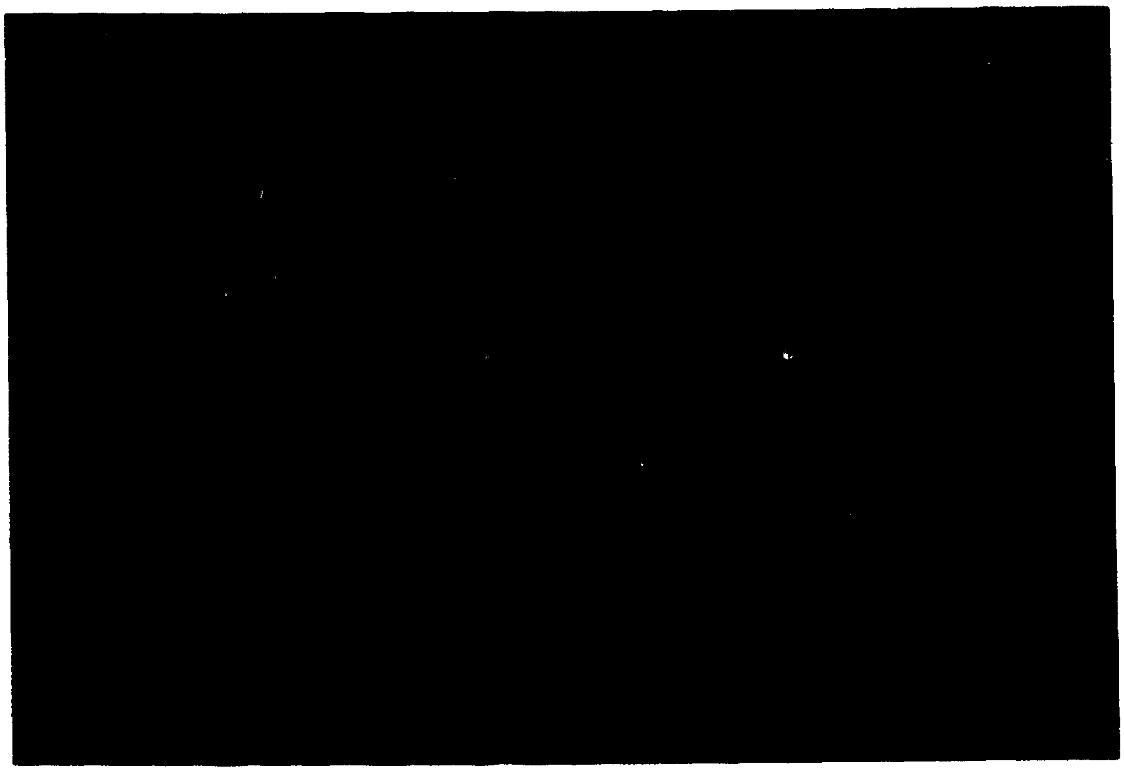


Figure 3.6 Plot of partialy derivatives at the same locations as interpolated heights in Figure 3.3

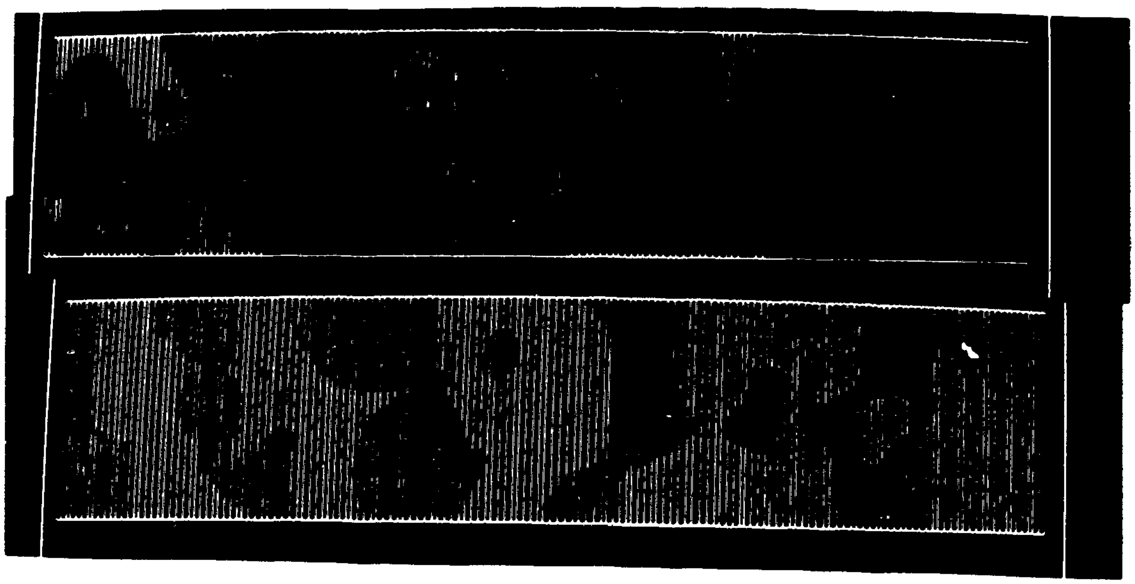


Figure 3.7 Interpolated xyz drumlin image and interpolated partialx image

to the x axis. If one were to look at the *xyz* and *partialx* plots together (Figure 3.7), moving the eye from left to right, sense can be made from the image. Green is a positively sloping section, blue is a negatively sloping section, and purple gives an indication of those areas where the slope flattens out.

3.2 Partialy Plot

The plot of the *partialy* derivatives shows those gradients parallel to the y axis. It is easier to visualize the correspondence of the *partialy* plot to the xyz plot when looking at them together, turning them so that the left-hand side of the window faces the eye (as shown in Figure 3.8). Again, those areas with a positive slope are green and those areas with a negative slope are blue, with purple indicating those areas where the slope flattens out. The pattern formed by the positive and negative derivatives generally indicates the stoss and lee of the drumlins, however the fit is not a perfect one.

3.3 Error Evaluation

In order to numerically estimate whether the interpolators are actually calculating values that resemble the real heights, a reliability test was conducted. Every second value from the original *Ltest2xy.dru* and *Ltest2z.dru* was eliminated and the interpolation procedure, previously outlined, was conducted for this new set of files. The original *Ltest2xy* and *z.dru* files contained fifty-one records. With every second record removed, this meant that a total of twenty-five records were deleted, leaving the new test files with a total of twenty-six records. It must be noted that every one of the fifty-one points in the initial files were crucial for determining the final, accurate image. It would therefore be expected that by removing almost half of the initial heights, substantial errors would result. Even though this was expected, it was considered a valuable evaluation, with the desire being an estimate of the errors. Table 3.3 (all *Ltest2* error data) contains the results of the error evaluation for these files. This table has both the observed (o) and interpolated (i) values for elevation, as well as the relative error (expressed as a percentage). The first and second columns contain the x and y coordinates of sampled points, x being east-west distance and y being north-south distance.

It might also be noted that the size of the errors also depends on the range of heights for the file, since the larger the range, the greater the expected error. The errors also depend on the size of the file (number of records), and, as stated, the number of records which were removed (this will be discussed further in the following chapter). The z file (height file) from which new heights were intepolated, had a seventy-five meter range of heights. The maximum error was (+)40.4384 m, the minimum error was 0.0000, with the average error being about (+,-)15.7495 m. The context of the results of these errors will be discussed with the results of some of the other data files in the following chapter.

With refinement of the plotting, and control of error, multiquadric interpolation could aid in the calculation of actual drumlin area, which has been a problem or ignored by researchers in the past. Those sections of the plot where the purple colour is located, indicate a break in slope, which could either be at the actual edges or on the crest of a drumlin. An example can be seen when looking at Figure 3.7, in the lower left-hand corner of both images. The xyz plot shows a drumlin, and the partialx plot shows a band of purple, indicating that this may be the flattened crest of the hill.

The following chapter will discuss the general results and provide an overview of the entire research procedure.

X	Y	(0)	(i)	(i-0)	i-o/o (%)
-0.5	-0.6	21	9.8261	-11.1739	53.21
-1.1	-0.7	34	19.8844	-14.1156	41.52
-1.3	-0.7	27	17.755	-9.245	34.24
-1.6	-1.1	37	12.7116	-24.2884	65.64
-1.3	-1.4	28	15.6813	-12.3187	43.99
-2.7	-0.4	29	29.7468	0.7468	2.58
-2.3	-1.1	38	11.4272	-26.5728	69.93
-2.4	-1.6	8	21.4551	13.4551	68.19
-2.5	-2.2	23	15.6743	-7.3257	31.85
-3.5	-0.4	26	28.2284	2.2284	8.57
-3.5	-1.7	48	24.0678	-23.9322	49.86
-4.2	-0.5	66	35.2494	-30.7506	46.59
-4.5	-0.6	42	54.5253	12.5253	29.82
-5.1	-0.9	32	55.2223	23.2223	72.57
-4.4	-1.4	57	33.5572	23.4428	41.13
-4.1	-1.9	27	27	0	0
-5.7	-0.6	31	42.8969	-11.8969	38.38
-6.1	-0.4	31	52.1451	21.1451	68.21
-6.2	-0.7	31	45.4381	14.4381	46.57
-6.3	-1.1	58	27.4258	30.5742	52.71
-6	-1.5	52	21.8879	30.1121	57.91
-6.7	-1.5	23	33.2605	10.2605	44.61
-7.6	-0.7	59	18.5616	40.4384	68.54
-7.3	-1.1	32	25.1455	6.8545	21.42
-7.3	-1.9	32	26.7611	5.2389	16.37

Table 3.3 Error evaluation for files *Ltest2xy.dru* and *Ltest2z.dru*. (o) is obsereved height, and (i) is interpolated height

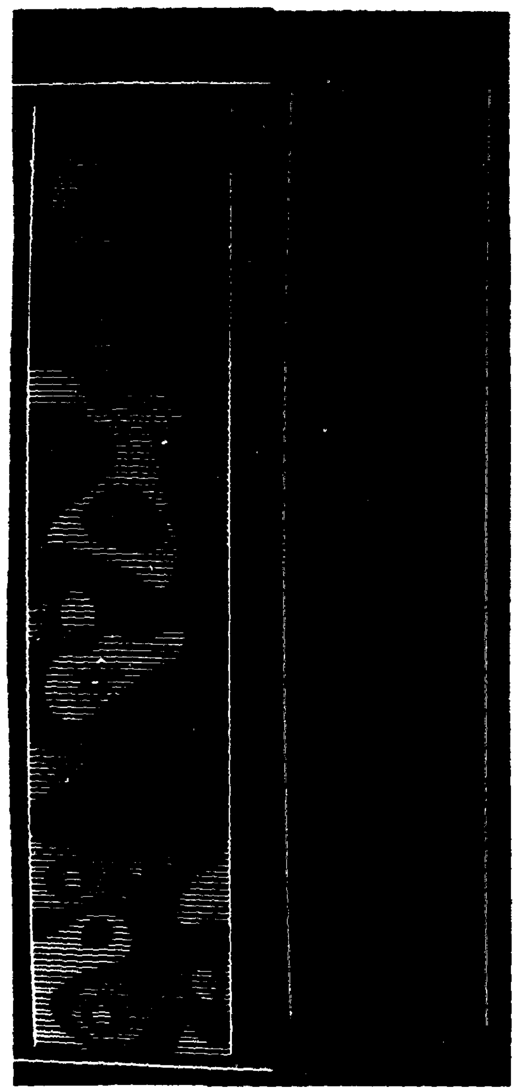


Figure 3.8 Interpolated xyz drumlin image and interpolated partialy image

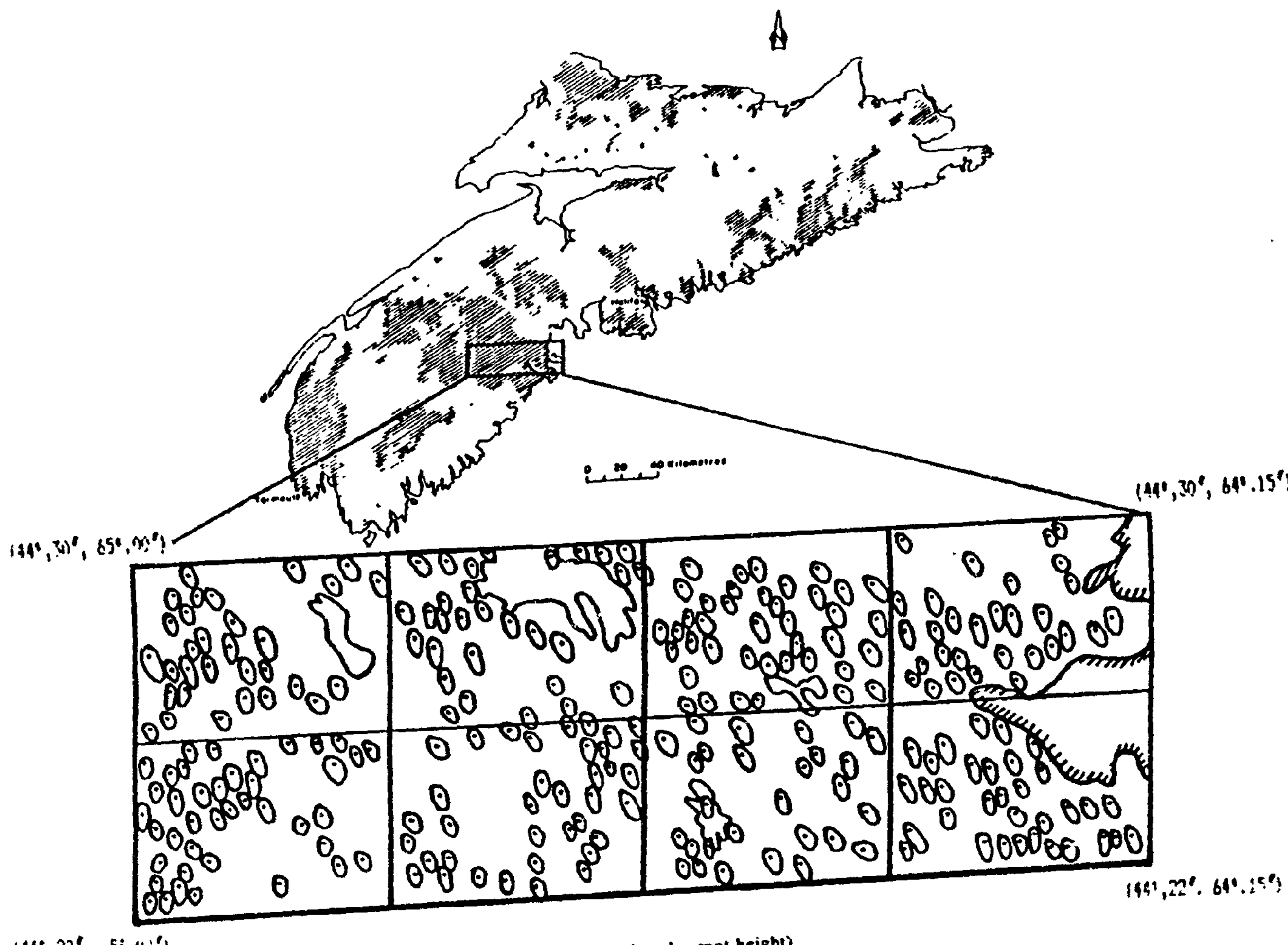
CHAPTER FOUR

GENERAL RESULTS AND DISCUSSION OF RESEARCH

An example of how the multiquadric method of interpolation can work for drumlin landscapes has been outlined. There was a long process which had to be worked through before that viable, final working product was reached, however. This chapter will outline that process, indicating the favourable aspects of as well as the drawbacks of the multiquadric method in this application.

4.1 Initial Lunenburg Data Collection

Initially, eight orthophoto map sheets were used (Figure 4.1) and two hundred and seventy-three drumlins were identified. Two hundred and eighty-eight non-systematically selected heights between and around the drumlins (swales) were also collected, resulting in a total of five hundred and sixty-one heights to be used. The eight orthophoto map sheets formed the sampling sizes of the Lunenburg area Both the actual drumlin heights as well as the random heights between the drumlins were then divided into seventeen computer files. Seventeen xy files



(Each dot represents one drumlin spot height)

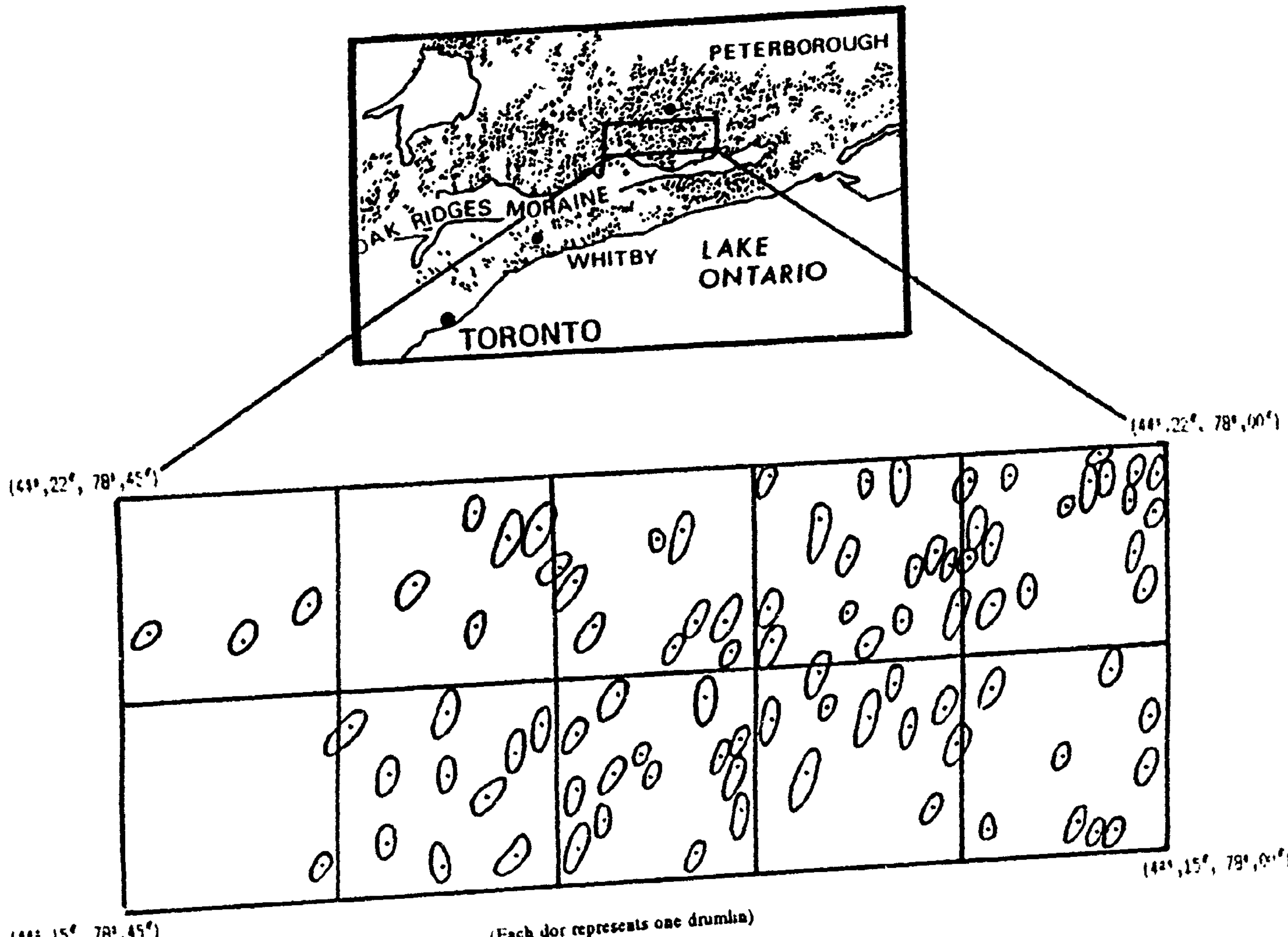
Figure 4.1 Initial Lunenburg drumlin field study area

included an xy and z coordinate which locates each drumlin and its sampled heights. Seventeen z files included the heights of the drumlins and intervening swales. Changes needed to be made with regards to sample size, both numerically and aerially to this initial set-up.

4.2 Initial Peterborough Data Collection

Initially, ten topographic map sheets were used and ninety-two drumlins were identified (Figure 4.2). One hundred and eleven spot heights between the drumlins (swales) were also collected, resulting in a total of two hundred and three heights to be used. Drumlins and drumlin heights were collected consistently with those methods described for the Lunenburg field. Both the actual drumlin heights as well as the random heights between the drumlins were then divided into six computer files. Each record in the x,y files included an x and a y coordinate to locate each drumlin and other sampled elevations. Six z files included the elevations of the drumlins and intervening heights. As with the initial Lunenburg sample areas, the Peterborough sample sections needed to be revised.

There were a number of reasons as to why these sampled sections did not work out. By not working out, this doesn't mean that *xyzpart.c* would not run, and images could not be produced, because each section generated a surface. The



(Each dot represents one drumlin)

Figure 4.2 Initial Peterborough drumlin field study area

surfaces did not, however, reflect the initially sampled surfaces from the orthophoto maps. Resampling then was done, with the goal to generate more accurate images. Some of the reasons why the initial set-up did not work, resulting in changes that were made for the second set-up, will be discussed.

4.3 Reasons for Errors in Initial Set-up

4.3.1 Lunenburg Study Area

It was discovered very early on that *xsvbksb.c* would not run if the number of records in the xy file was greater than fifty-seven (this will be discussed more completely at the end of the chapter). With an average number of thirty-four drumlins in each study area, this allowed for only eleven intermittent heights to be collected, in most cases. This was not initially discovered, however. An average of thirty-six random heights (per study area) were collected.Each study area was then divided in half (through the middle, from left to right), meaning that there was an average of seventeen drumlins and eighteen random heights per study area (now sixteen, since the initial eight were divided). Saunderson (1994) had noted that interpolation of new z values, using the multiquadric equation could be used for *any* number of locations within the x,y domain of an initial sample space of points. In this initial research, as pointed out, it was discovered that only up to approximately fifty-seven data points could be used, otherwise *xsvbksb.c* would not run, meaning that new z values would not be interpolated, and *partialx* and *y* values not output.

There was another problem with the initial sampling method used. The intermittent heights, around the drumlins, were selectednon-systematically. Shaw and Lynn (1972) stated that an advantage of the multiquadric method was its suitability for dealing with nongridded (scattered) points, selected at random. This research found that to be completely untrue for this application. Selecting random heights meant that care was not taken to ensure that lows between drumlins were sampled, or if a drumlin was located near an edge of the study area, care was not taken to include a lower elevation closer to the edge of the section.

Hardy (1971) pointed out that ambiguity in data collection can lead to the method doing a number of unreliable things, especially if significant initial data points are left out. Random heights were purposely selected in this initial set-up. and a number of unreliable things, as Hardy put it, did occur. In his own terrain analysis, Hardy discovered that when more data (including crucial points) were added, the multiquadric surface became a reasonably good generalization of the terrain.

Franke (1982), in his test of interpolating techniques, similarly noted that terrain data may be sparse in certain regions, or exist in clumps. He hypothesized (although did not test) that methods based on quadratic approximations would not work in a reasonable fashion for this type of data. The current research found this to be the case, although the problem could be resolved (as will be shown in the second set-up).

A final problem which was discovered at this stage was determining the correct elevation interval. A trial-and-error process was used, in the event that the *xyz* images from this initial set-up might be accurate, the problem being that the incorrect interval was being used. After this trial-and-error process had nearly been exhausted, and yet the images were still not correct, it was concluded that the data would need to be resampled.

4.3.2 Peterborough Study Area

Many of the reasons why the initial Peterborough data needed to be resampled are the same as those for the Lunenburg study area. There were reasons specific to this field, however, and therefore Peterborough is being discussed separately from Lunenburg.

Unfortunately, the problems which were discovered for the Lunenburg field were not observed until after this initial Peterborough data had already been collected. All of the data, for both fields, was collected, and then computer images were generated. It was then that the problems were discovered. The problem of having too many data points for each sample area was not an issue for the Peterborough sections, since the average number of drumlins was twelve, and the average number of random heights was fifteen, with an overall average of twenty-seven original data points per section. This fell well below the discovered limit of approximately fifty-seven original data points. The original test sections were not, therefore, divided into half, because the *xyz* and *partialx* and *y* files were successfully generated from *xyzpart.c*. The major errors that resulted in these images were primarily due to randomly sampling the intermittent heights, and not systematically looking for sections of the terrain that could erroneously be interpolated.

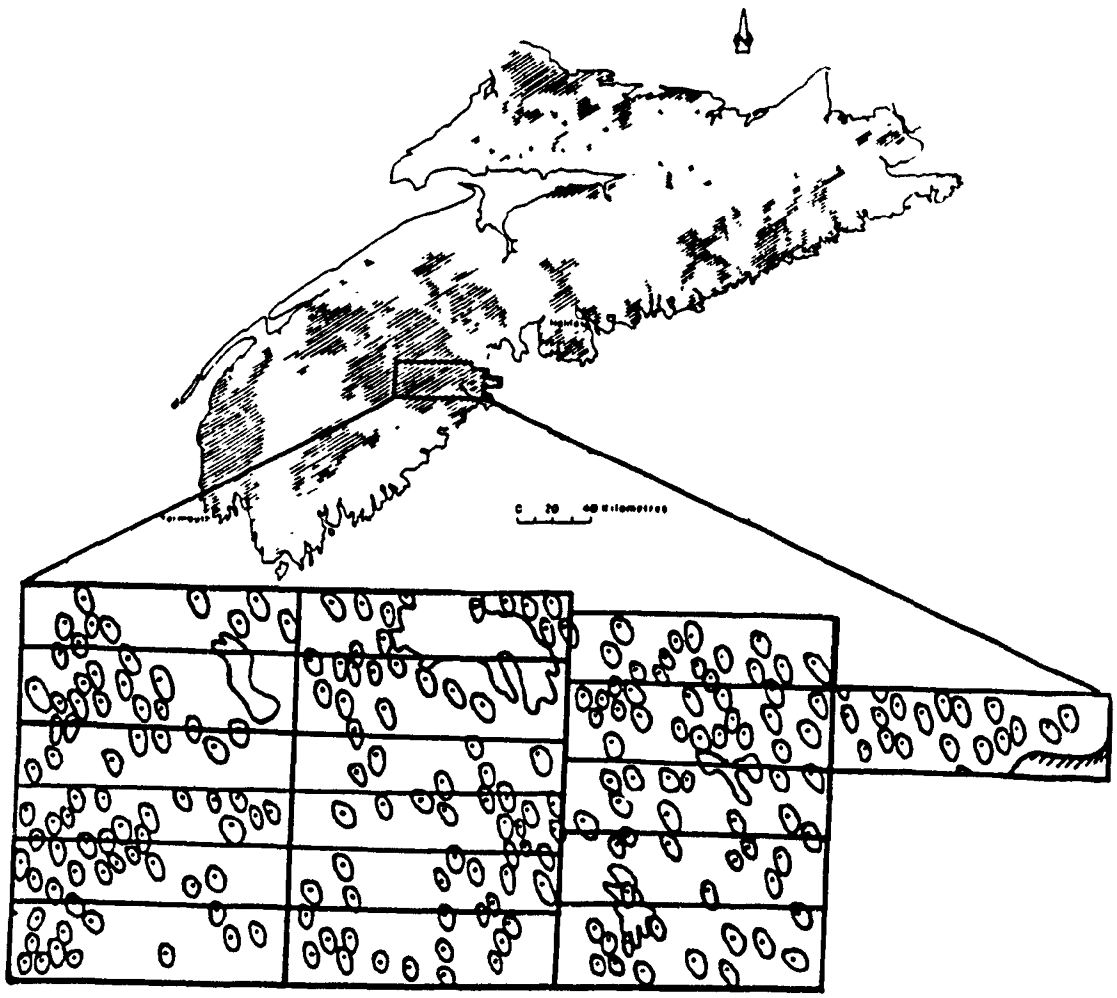
As with the Lunenburg sections, it was discovered that there had been a problem with the data collection. This was discovered when the *xyz* data was plotted and looked absolutely nothing like the actual drumlinized landscape. The ranges for the percentages of MAX (the maximum drumlin heights for each image) were changed, again using trial and error, until it was concluded that the data would have to be more systematically collected.

4.4 Revised Lunenburg Data Collection and Results

Although the initial test sections did not work out, they are not regarded as having been a waste of time. Without the entire process of failure, success could not have been reached.

New sections were set up within the same study area as the initial samples (Figure 4.3). Eighteen sample areas with approximately two hundred and fifteen drumlins and six hundred and eighty-eight systematically sampled heights surrounding the drumlins were then set up. These new sample sizes were chosen, with the number of drumlins within each in mind, so that the number of intermittent heights could be optimized. This means that the sections had to be small enough to allow for a large number of heights around the drumlins to be collected (keeping in mind that more than fifty-seven values would not run). The heights (excluding drumlin heights) were strategically chosen this time. Lowest elevations between drumlins were collected, and more heights were therefore collected in areas where there was a cluster of drumlins. Data was then entered into the computer, creating *xy* and *z* files of information, similar to that outlined in Chapter three.

Of the eighteen sets of data that were collected, fourteen successfully ran, and had *xyz* and *partialx* and *y* files created. All of the four that did not run had more than fifty-seven records of data, as a second attempt. Of those fourteen, there were five that were especially worked on to see if the best possible results could be reached, and they were. This does not mean that the other nine sections could not generate successful results, but that five were quickly determined to exhibit accurate results. This was also a workable number of sections for getting the



(Each dot represents one drumlin spot height)

Figure 4.3 Revised Lunenburg study area

optimal results. By changing the intervals of the drumlin height classes, the results for each of these sections was a close approximation to the actual landscape, and drumlins could easily be picked out on the images.

It might seem like this has been a long process to get to this stage, but once the criteria for data selection were understood, the computational steps were quite successful. The only obvious limitations were the trial and error process for getting the best interval for optimal image results and the fact that no more than fifty-seven points of original data could be used. As Franke (1982) discovered, the more original data points used, the more accurate the image. An original one hundred point sample, in his study of the multiquadric method, interpolated an actual image almost exactly.

4.5 Revised Peterborough Data Collection

New sections were set up within the same study area as the initial samples (Figure 4.4). Four smaller sample sections were set up, with approximately thirty drumlins and one hundred systematically sampled intermittent heights collected. All four of these sets of data ran, and had *xyz* and *partialx* and *y* files created. Of these four, one was especially successful. This again meant several attempts at getting the best drumlin height class interval to make the image resemble the actual

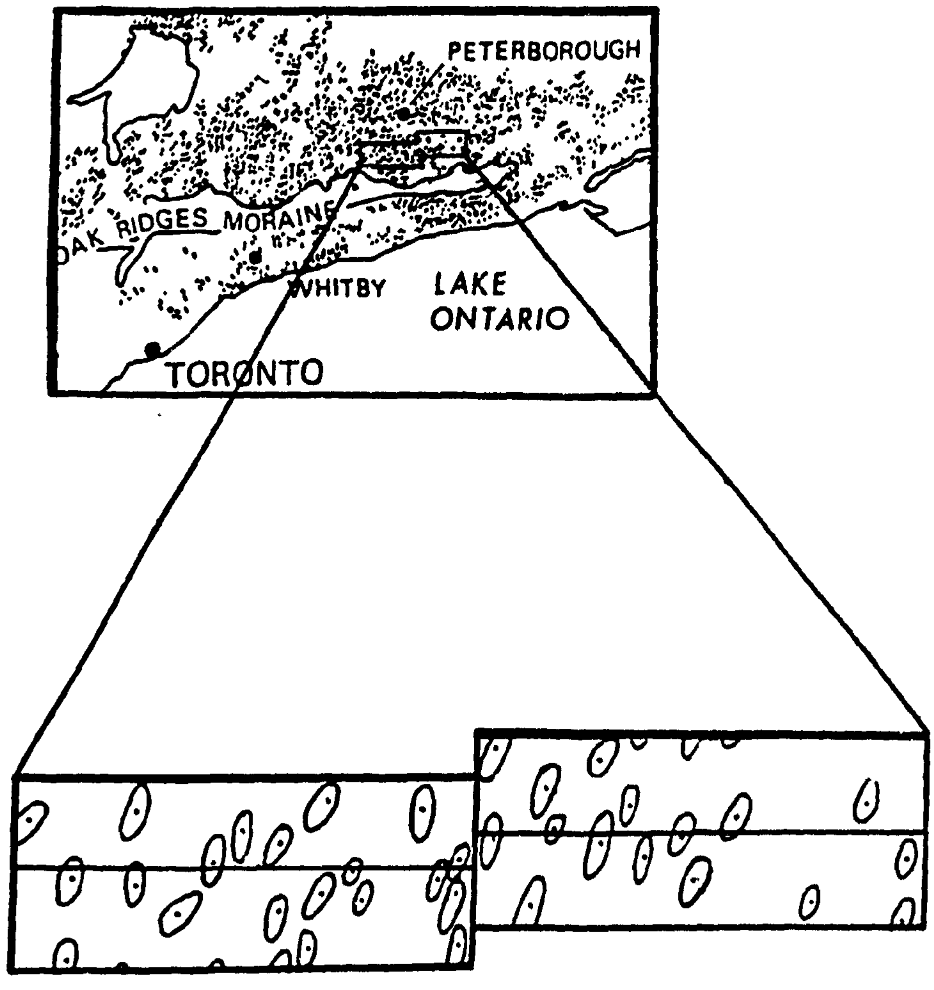


Figure 4.4 Revised Peterborough study area

landscape. This was successfully completed, and the section was a close approximation of the actual landscape.

In general, these final five Lunenburg sections and one Peterborough section interpolated images that very closely approximated the actual landscapes. Although time was concentrated on these sections, it was because the images generated from them were accurate representations. It is possible that with more time, better results could have been generated for the other sections as well. It might be questioned why these sections were interpolated so accurately, however, while others did not. Given the fact that the method of sampling the heights for all of the revised sample sections was consistent, all or no images should have accurately turned out. Hardy (1971) concluded in his research that the deficiency in size and shape of the hills resulted mainly from poor elevation choices for representing the hills and the saddles between them. In this case, several elevation choices for each section were tested, and the image was not abandoned as unsuccessful until many ranges had been applied. For those images that were successful, a variety of elevation choices were tested. If only one elevation interval had been used for every section, it is likely that no successful test areas would have been determined. Therefore, Hardy is correct in stating that the size and shape of landscape features can depend highly on the best elevation choices; it is believed that those study sections (for this research) which did not resemble the test landscape, were not a result of poor elevation choices. There are many other reasons why the images did not resemble the test surfaces, however. The surfaces which Hardy tested were not as complex as the ones in this study, which would mean that more initial data points would need to be included for the images to sufficiently resemble the test surfaces. Unfortunately, only fifty-seven points could be used. Also, those initial sections which were tested resulted in poor images due to the method of randomly selecting the data. Once the data was selected, with geomorphological thought going into that selection, the images being generated were much better.

Even though the heights were consistently collected during the revised data collection, for both the Peterborough and Lunenburg fields, the terrain was different for each of those sections, and therefore each test area would have had initial data points in different patterns within the section. For example, a section with a number of drumlins together would have meant that several intermittent heights would have been collected around all of these drumlins, resulting in a number of data points. In his terrain analysis, Hardy (1971) noted that a few additional points on the hilltops would have helped form elliptical rather than circular contours. It could be that in those sections where more data was collected around the drumlins, the images turned out to better resemble the actual landscapes. This suggests that if the multiquadric method could perhaps be adapted for use as a local interpolator, rather than just a global interpolator, the resulting surfaces might be better.

Franke (1982) discovered that the more initial data points that were used, the more accurate multiquadric interpolation worked out to be. It was determined that the average number of initial data points for the most successful sections was 48, while the average number of initial heights for the other sections was 45. It is believed that this is not significant, however. Franke conducted his test with twenty-five, thirty-three, and one hundred points of data, with the best results coming exclusively from the one hundred point initial data sample. Hopefully, the fifty-seven point limit may be overcome and then more initial data points can be used to see if this holds true. A possible way of doing this in the future might be to use the multiquadric method as a local interpolator, enabling one to integrate the results over the entire surface.

4.6 Comparison of Multiquadric Sections

Without exception, all of the images that were generated (for both the Lunenburg and Peterborough fields) from the initial set-ups were poor. There was a number of reasons for this, which, once realized, will allow images that resemble the actual landscapes to be generated much quicker. It has already been mentioned that heights surrounding the drumlins are required to be systematically collected, with low elevations around all drumlins to be included, for a good representation to result. This was not done in the initial set-up. In the subsequent data collection, more intermittent heights were collected, with drumlin shapes visible in the resulting images, in the locations where they would be expected. Of the images in the second set-up, there were not ones that were substantially better than others. In order to make these images even closer to the actual landforms, however, would require the collection of even more initial heights.

Another problem with the initial study sections for both drumlin fields was the size of the study areas themselves. Not only were they too large to allow for more detailed data collection (due to the N=57 limit), but the sizes and shapes did not plot well with the programmes as they exist. The legend is located at given coordinates, and with larger plot sizes and shapes, the legend would either need to be left out or the programmes substantially modified. This was not realized until the initial plots had been tested. In the second set-up this problem was overcome by making sure that the sizes of the study area would fit above the legend and that the x axis was approximately four times as long as the y axis.

In general, the initial study sections of both the Lunenburg and Peterborough drumlin fields were not successful due to the size, shape, and data collection methods used. By following those methods in the second set-up, however, the study sections were all successful, in that the drumlin forms could easily be seen, in locations that correspond to the map sheets from which the data was collected. By following this method, it is believed that accurate representations of a given surface can be generated. It is also believed that even more accurate and more detailed surfaces may be generated by collecting more data for smaller sample areas.

4.7 Error Evaluation

Although evaluation of interpolated errors could be a thesis in itself, a brief discussion of interpolated errors is considered to be useful for this analysis. As with the working example in the previous chapter, an error evaluation was conducted for many of the *xy* and *z* files, in order that some of the resulting errors could be quantified. Fourteen of the re-tested Lunenburg sections (including the working example), and two of the re-tested Peterborough sections were evaluated, by removing every third value from the original files (Table 4.1).

Table 4.1 gives observed (o) and interpolated (i) values,and average values of elevation errors. Range of height, number of records, and number of records removed, are also included, so that an overall analysis of the errors could be conducted. One might easily assume that by looking for that set of files with the largest mean error, the greatest inaccuracies in interpolation would be found, and therefore this set of files would generate the most inaccurate image. There are a number of factors which need to be considered, however, since each set of files is not being generated with the same conditions. The range of heights, the number of records, as well as the number of records removed from each set of files, are all factors which should be taken into consideration. It would be expected that the best interpolated heights (those files with the lowest mean errors (i-o)) would be derived from those files with the lowest range of heights, the largest number of records, and the smallest number of records removed.

The smallest mean error (+,-12.6225), however, comes from files with only an average number of records and quite a large height range (100.0 m). The largest mean error (+,-20.2930), was derived from files with one of the smallest (but not the smallest) number of records, but the height interval was less than that for the files which generated the lowest mean error. The files with the smallest number of records (19) generated a relatively low mean error. The files with the largest number of records (38) generated an average mean error, but these files also have a large height range; therefore it is possible that the advantage of a large number of records could have been offset by the large range of heights. Those files with the largest height range (114.0 m) generated quite a large mean error, while those files with the smallest height range (43.0 m) generated one of the smallest mean errors.

From these results it could be concluded that the errors which are generated are a result of the complexity of a number of factors. The working example, outlined in the previous chapter, was one of the set of files which generated one of the best (and most accurate-looking) images. When looking at the mean error for these files, however, it falls into the middle range, with several files having mean errors that are smaller than this one. It might be noted, however, that given the fact that nine to twenty-five crucial records have been removed from original data files, the results are quite good.

In conclusion, even though problems were discovered, the method by no means should be over-looked, because the advantages greatly out-weigh the disadvantages. Hardy (1990) put this nicely when he stated that

> "Multiquadric...methods are not foolproof but the mathematical proofs that the MQ systems of equations is always solvable, should encourage those who may have problems with solutions to try again,..." (p.205)

	max (i-o)	min (i-o)	mean (i-o)	height range (m)	N=	# of records removed
Ltestxy.dru	40.6250	-0.213	19.1382	97	22	10
Ltestz.dru						
Ltest2xy.dru	-40.4384	0.0000	15.7495	75	26	25
Ltest2z.dru						
Ltest3xy.dru	31.2925	-4.5635	18.6781	95	32	16
Ltest3z.dru						
Ltest4xy.dru	34.9739	0.1336	13.9895	68	35	17
Ltest4z.dru						
Ltest5xy.dru	-36.2068	-1.2958	14.0172	63	33	15
Ltest5z.dru						
Ltest6xy.dru	-18.1095	-1.2533	12.6225	100	33	15
Ltest6z.dru						
Ltest7xy.dru	49.6887	1.2447	15.590	104	38	19
Ltest7z.dru						
Ltest8xy.dru	-47.674	-4.9113	17.5751	114	31	15
Ltest8z.dru						
Ltestx.dru	-33.1697	-6.0773	20.2930	93	29	14
Ltest11z.dru						
Ltest12x.dru	48.0438	-1.1921	18.9143	95	32	14
Ltest12z.dru						
Ltest13x.dru	-47.4622	1.8414	18.1439	109	26	13
Ltest13z.dru						
Ltest14x.dru	-26.3843	-1.2884	14.2556	92	28	13
Ltest14z.dru						
Ltest15x.dru	-40.2720	-0.312	12.8947	65	34	16
Ltest15z.dru						
Ltest16x.dru	-45.3404	-0.3811	14.4252	86	36	17
Ltest16z.dru						
Ptest1xy.dru	26.6535	3.8070	13.0244	43	32	15
Ptest1z.dru						
Ptest2xv.dru	-38.24	0.647	13.2244	78	19	9

Table 4.2 Error evaluation for several Lunenburg and Peterborough sections

CHAPTER FIVE

CONCLUSIONS

Hardy's multiquadric method of interpolation was used as a basis for mapping sections of the Lunenburg, Nova Scotia, and Peterborough, Ontario, drumlin fields. Saunderson's (1994) specific methodology was used tc test drumlin mapping as a potential application. The method, in general, has already been determined by several authors (Saunderson 1992,1994; Hardy 1971,1990; Kansa 1990; Krohn 1976) to be an accurate and reliable method for interpolating data, with several advantages over other interpolating techniques. The concluding discussion of this thesis will be based on the results of the application of data from drumlin landscapes to Saunderson's model (based on Hardy's multiquadric method).

As it was discussed and worked through in Chapter three, a series of programs (Saunderson,1994) was modified and tested using drumlin and swale elevations. In a similar manner to Franke's (1982) evaluation and comparison of several interpolation techniques, Saunderson's programmes of Hardy's method will be evaluated based on four criteria: (1) accuracy, (2) visual aspect, (3) execution time, and (4) ease of implementation.

5.1 Accuracy

The accuracy of this method for drumlins, based on height data, depends highly on the initial method of data collection. The data for this study was initially randomly collected. Upon generating images for these initial test sections, it was quickly determined that the data would need to be collected in a different manner if the results were even to be somewhat accurate. After the data had been re-sampled and run in the computer, the resulting images were immediately recognized, in most cases, to be substantially better.

An error evaluation was conducted (Table 4.2) with the highest overall error being (+) 49.6887 and the lowest overall error being 0.0000. As was discussed in the previous chapter, there was a variety of factors which contributed to the errors. In some cases, the errors were quite large, with an overall mean error of 14.7210 for a mean height range of 86.063. The fact that there are such errors indicates a further need for the study of errors in multiquadric interpolation.

For some drumlins, then, it may be concluded that this methodology can be used with drumlin and swale elevations to produce relatively accurate results, but for others, large errors may occur. Care needs to be taken in the initial sampling stages, in order that the best and most accurate images possible can be generated.

5.2 Visual Aspect

One of the unique features of Saunderson's methodology is the fact that his C programs reduce the number of stages necessary in mappings of z values by eliminating the need for using a contouring routine, substituting instead a selection of colours for classes of z values (Saunderson, 1994). The one problem which was discovered was in determining the appropriate colour class interval. Any method of consistently finding the most appropriate interval (i.e.the range/10) was not viable. Therefore a trial and error process had to be used until the best image was determined. A user who was not aware of this, might generate an image and conclude that the method did not work, without realizing that the colour class interval should need to be adjusted. Once manipulation of this is mastered, however, it is only a process of inspecting the values until the best interval is discovered. The resulting images are then quite elegant and unique, thereby removing the additional step of having to use a contouring routine.

5.3 Execution Time

Franke (1982) gave the multiquadric method a B-/C- in his evaluation of timing. Franke's testing was conducted, however, on an IBM 360/67. This research was conducted on a 486DX (with a math coprocessor) and therefore the speed of compiling and running the programs, and then generating an image, was greatly enhanced. Speed was of great importance for this research since a large number of files were being used. If one were to use this method for only a small number of files, a slower computer would be alright. It is safe to say, however, that without a faster computer with a math coprocessor, this research would still be in the testing stages.

5.4 Ease of Implementation

Since the programs are all workable, compilable programs, only minor changes need to be made to each to suit one's specific application. Time is required, however, for a novice in C programming to grasp all of the steps required to reach the end result. Anyone familiar with programming will be able to implement their application with much greater ease.

It might be noted again that to date *xsvbksb.c* would not run with an initial data file with more than approximately fifty-seven records. This could pose no problem for some applications, but it would have greatly aided in getting better, faster results for this application. One can only assume that the more initial data points used as input, the more accurate and realistic the results will be.

To conclude, then, it is believed that this multiquadric method is a viable and visually interesting way of generating drumlin field images. There are many ways in which this research could be expanded. Further work on the partial derivatives to improve the plots, in order that calculations of actual drumlin area be determined, is but one example.

The research outlined in this thesis has by no means been exhausted. There is a lot of work that can and will follow from the knowledge gained from this work. Further investigation into the problem of errors, outlined briefly in the error evaluation noted in the previous chapters, will begin. This will be done using several data sets, with such geophysical applications including vertical crustal and tidal movements, ocean current directions, further terrain analysis as well as atmospheric information. Testing may then proceed in order to overcome some of the errors which were not only discovered in this research, but in the ensuing research as well.

A comparison of the multiquadric method with other interpolation procedures may be another step in further validating and understanding the multiquadric and other interpolation methods. Although a study has been conducted by Franke (1982), the technology and research has sufficiently changed to warrant further investigation and comparison of interpolation techniques. This would include a comparison of both local and global methods. The multiquadric method might then be studied not only as a global procedure, but as a local procedure as well. This alone might eliminate the problems of limited data input and undetailed images being generated.

The multiquadric method of interpolation, then, has by no means been exhausted. There is a lot more research to be done, and a lot more to be discovered. Upon future and further investigation of this and other interpolation techniques, it is hoped that the method will become even more "user-friendly" and may be given consideration for use by any researcher who studies geophysical data.

APPENDIX

Raw xy and z Data Files

-0.10	-0.10	1.0
-0.70	-2.20	18.0
-1.10	-0.80	34.0
-1.30	-0.90	16.0
-1.50	-2.30	28.0
-1.60	-1.10	37.0
-1.90	-1.20	22.0
-2.30	-1.10	38.0
-2.30	-1.60	8.0
-2.50	-1.30	11.0
-2.60	-0.40	24.0
-2.60	-1.40	29.0
-3.10	-0.40	46.0
-3.30	-1.70	49.0
-3.60	-0.50	32.0
-3.90	-1.50	7.0
-3.90	-2.40	36.0
-4.10	-0.50	66.0
-4.40	-1.30	57.0
-4.50	-0.60	42.0
-4.50	-2.30	15.0
-4.60	-0.30	62.0
-4.70	-1.20	28.0
-4.80	-0.70	77.0
-5.20	-0.60	38.0
-5.40	-1.50	28.0
-5.50	-0.60	57.0
-5.60	-2.10	41.0
-5.70	-2.20	35.0
-5.80	-0.90	31.0
-5.90	-2.30	51.0
-6.00	-1.60	52.0
-6.30	-1.10	58.0
-6.40	-0.40	28.0
-6.40	-2.30	59.0
-6.60	-1.60	21.0
-6.70	-0.60	36.0
-6.70	-1.30	20.0
-6.70	-0.20	32.0
-6.80	-2.40	23.0
-6.90	-0.80	12.0
-7.30	-1.70	68.0
-7.40	-0.70	56.0
-7.40	-2.10	59.0
-7.50	-2.30	26.0
-7.60	-0.30	18.0
-7.60	-1.10	39.0
-7.90	-2.40	74.0

Lunxy1.dru

Lunz1.dru

-0.10	-0.10	1.0
-2.20	-2.40	26.0
-2.40	-2.10	16.0
-2.50	-1.90	28.0
-2.50	-2.70	14.0
-2.70	-0.60	33.0
-2.80	-1.20	12.0
-2.90	-0.80	34.0
-3.30	-2.30	24.0
-3.80	-1.50	12.0
-4.20	-2.20	49.0
-5.70	-1.20	63.0
-5.70	-1.70	52.0
-6.80	-2.80	72.0
-7.70	-2.80	46.0
7.90	-0.50	66.0

Lunxy2.dru

Lunz2.dru

-0.30	-0.10	18.0
-0.30	-2.40	71.0
-0.50	-2.40	84.0
-0.60	-0.50	57.0
-0.60	-1.10	74.0
-0.70	-0.60	33.0
-0.90	-0.30	23.0
-1.00	-1.10	46.0
-1.00	-2.50	49.0
-1.40	-1.20	49.0
-1.40	-1.50	87.0
-1.50	-0.60	74.0
-1.50	-2.50	86.0
-1.60	-0.50	25.0
-1.60	-1.70	61.0
-1.70	-0.10	72.0
-1.70	-1.20	57.0
-2.20	-2.30	67.0
-2.40	-1.80	87.0
-2.40	2.40	103.0
-2.60	-1.10	89.0
-2.90	-1.80	58.0
-3.20	-1.50	97.0
-3.40	-0.20	82.0
-3.50	-1.50	92.0
3.50	-2.30	58.0
3.70	0.20	48.0
-3.70	0.80	44.0
-3.90	-1.50	108.0
-4.10	-0.30	90.0
-4.10	-2.40	86.0
-4.60	-0.60	58.0
-4.70	-1.10	88.0
-5.20	-0.40	61.0
-5.20	-2.30	122.0
-5.40	-0.90	59.0
-5.40	-1.60	91.0
-5.60	-2.40	69.0
-6.10	-0.90	112.0
-6.20	-2.30	118.0
-6.40	-1.20	75.0
-6.50	-0.50	78.0
-6.50	-0.90	80.0
-6.60	-2.30	90.0
-6.70	-0.20	111.0
-6.70	-0.80	106.0
-6.80	-1.40	112.0
-6.90	-1.60	92.0
-6.90	-1.80	115.0
-7.00	-0.70	96.0
-7.10	-0.30	105.0
-7.10	2.30	108.0
7.30	0.50	110.0
7.50	0.90	112.0
-7.60	-0.50	81.0

Lunxy3.dru

Lunz3.dru

-0.20	-0.10	55.0
-0.60	-2.20	61.0
-0.80	-2.30	31.0
-1.30	-2.50	68.0
-1.40	-2.20	38.0
-1.50	-2.80	41.0
-2.10	-2.30	82.0
-2.20	-1.50	74.0
-2.20	-1.90	50.0
-2.50	-2.50	51.0
-3.20	-2.10	87.0
-3.60	-2.60	82.0
-3.80	-2.60	74.0
-3.90	-2.50	89.0
-4.10	-2.10	57.0
-4.20	-1.50	96.0
-4.60	-1.50	71.0
-4.60	-2.50	71.0
-4.80	-1.60	86.0
-5.40	-2.30	113.0
-5.50	-2.40	98.0
-5.50	-2.60	118.0
-5.50	-1.50	89.0
-6.60	-2.50	102.0
-6.70	-2.10	113.0
-6.70	-2.40	91.0
-6.80	-1.50	112.0
-6.80	-1.80	88.0
-6.80	-2.70	72.0
-7.30	-1.50	887.0
-7.50	-2.50	91.0

Lunxy4.dru

Lunz4.dru

-0.20	-0.10	80.0
-0.20	-1.10	86.0
-0.80	-2.10	126.0
-1.40	-2.20	76.0
-1.60	-1.30	83.0
-2.30	-2.40	93.0
-2.50	-2.30	82.0
-2.70	-0.90	116.0
-2.80	-1.30	86.0
-2.80	-1.90	107.0
-3.40	-0.50	96.0
-3.40	-2.30	78.0
-3.70	-1.30	87.0
-3.90	-0.20	116.0
-4.20	-1.10	99.0
-4.50	-0.00	129.0
-4.50	-0.50	86.0
-4.80	-1.40	78.0
-5.30	-0.30	77.0
-5.50	-0.90	98.0
-5.60	-2.00	94.0
-5.70	-1.50	58.0
-5.70	-2.30	78.0
-6.30	-0.50	72.0
-6.30	-2.30	102.0
-6.50	-1.40	83.0
-6.50	-1.70	48.0
-6.70	-0.70	112.0
-6.90	-1.20	51.0
-7.00	-0.50	89.0
-7.30	-0.40	109.0
-7.50	-1.50	63.0

Lunxy5.dru

Lunz5.dru

-0.20	-0.10	65.0
-0.40	-0.60	111.0
-0.40	-1.40	122.0
-0.50	-0.90	91.0
-0.50	-1.50	94.0
-0.60	-0.50	91.0
-0.90	-1.70	117.0
-1.40	-0.60	121.0
-1.50	-0.60	110.0
-1.50	-1.50	79.0
-1.50	-2.50	79.0
-1.60	-0.50	126.0
-2.20	-0.70	79.0
-2.50	-1.50	79.0
-2.60	-0.70	97.0
-3.50	-0.50	79.0
-4.50	-1.50	79.0
-5.50	-0.60	79.0
-5.50	-1.50	79.0
-5.50	-1.60	118.0
-5.60	-0.50	112.0
-5.60	-0.80	99.0
-5.60	-1.80	102.0
-5.80	1.50	112.0
-6.20	-0.20	79.0
-6.20	-2.40	122.0
-6.30	-1.40	111.0
-6.50	-1.50	79.0
-6.50	2.60	88.0
-6.60	-0.30	109.0
-6.70	-0.40	79.0
-6.80	-0.70	112.0
-6.80	-1.00	123.0
-6.90	-0.80	99.0
-6.90	-2.40	111.0
-6.90	-2.50	102.0
-7.10	-2.50	114.0
-7.20	-1.20	96.0
-7.20	-2.70	86.0
-7.50	-0.50	86.0
-7.50	-0.70	123.0
-7.50	-2.40	100.0
-7.50	-1.50	63.0
-7.70	-2.30	127.0

Lunxy6.dru

Lunz6.dru

-0.10	-0.00	80.0
-0.10	-0.40	66.0
-0.60	-0.10	108.0
-1.30	-1.30	49.0
-1.50	-0.50	32.0
-1.80	-1.90	89.0
-2.00	-1.60	72.0
-2.20	-1.50	81.0
-2.60	-1.50	71.0
-2.60	-0.80	74.0
-3.10	-0.10	119.0
-3.10	-2.30	76.0
-3.20	-1.40	111.0
-3.30	-0.70	136.0
-3.50	-0.40	104.0
3.50	-0.60	125.0
-3.50	-1.40	86.0
-3.60	-0.60	132.0
-3.90	-1.40	102.0
-4.10	-1.70	78.0
-4.10	-1.00	86.0
-4.50	-2.30	124.0
-4.70	-1.00	127.0
-5.10	0.30	89.0
-5.10	-0.80	114.0
-5.40	-2.20	79.0
-5.50	-0.60	92.0
-5.70	-0.40	98.0
-5.80	-0.20	116.0
-5.90	-0.40	112.0
-6.10	-0.70	106.0
-6.30	-1.40	108.0
-6.30	-1.90	87.0
-6.40	-0.80	95.0
-6.50	-1.30	99.0
-6.60	-0.80	113.0
-6.60	-1.60	88.0
-6.60	-1.50	126.0
-6.60	-2.30	112.0
-6.80	-2.40	99.0
-7.00	-0.70	100.0
-7.10	-2.40	114.0
-7.30	-0.30	99.0
-7.40	-1.30	92.0
-7.70	-0.20	129.0

Lunxy7.dru

Lunz7.dru

-0.10	-0.10	100.0
-0.30	-1.90	142.0
-0.50	-2.50	68.0
-0.80	-1.40	91.0
-1.00	-0.40	142.0
-1.50	-1.50	82.0
-1.60	-0.90	56.0
-2.10	-1.30	59.0
-2.50	-2.50	32.0
-2.70	-0.70	82.0
-3.50	-1.50	32.0
-4.10	-2.30	131.0
-4.50	-2.80	109.0
-4.50	-2.10	116.0
-4.70	-1.60	139.0
-5.10	-1.60	118.0
-5.40	-1.40	147.0
-5.50	-0.80	109.0
-5.70	-0.30	139.0
-5.80	-1.40	125.0
-6.00	-1.80	101.0
-6.10	-0.80	111.0
-6.10	-1.30	143.0
-6.40	-2.20	123.0
-6.50	-2.40	98.0
-6.60	-2.60	128.0

Lunxy8.dru

Lunz8.dru

-0.30	-0.10	[illegible]
-0.50	-1.[illegible]	43.0
-0.70	1.20	16.0
-0.90	-1.90	28.0
-1.00	1.[illegible]	[illegible]
-1.30	1.80	13.0
-1.50	0.[illegible]	[illegible]
-1.50	1.10	[illegible]
-1.80	0.30	[illegible]
-2.10	-0.30	36.0
-2.30	1.80	10.0
-2.50	-0.30	[illegible]
-2.70	0.30	31.0
-2.80	-1.20	[illegible]
-2.80	-1.80	[illegible]
-3.00	0.50	[illegible]
-3.00	1.80	29.0
-3.10	1.80	36.0
-3.20	-0.60	[illegible]
-3.60	-1.70	10.0
-3.70	0.10	[illegible]
-3.70	-1.20	37.0
-3.90	1.30	16.0
-4.10	-0.10	51.0
-4.30	1.50	58.0
-4.40	1.50	50.0
-4.60	-1.50	67.0
-5.00	-1.10	10.0
-5.30	-1.50	71.0
-5.50	-0.50	38.0
-6.20	-0.60	57.0
-6.50	-1.50	39.0
-6.80	-0.70	38.0
-7.40	-1.10	97.0
-7.50	-1.20	58.0
-7.60	-1.30	76.0

Lunxy10.dru

Lunz10.dru

-0.20	-0.20	5.0
-0.20	-2.40	50.0
-0.10	-1.80	3.0
-0.50	-1.10	18.0
-0.60	-2.30	4.0
-1.10	-1.30	3.0
-1.10	-2.70	46.0
-1.20	-2.10	43.0
-1.20	-2.30	21.0
-1.30	-2.80	23.0
-1.40	-2.90	44.0
-1.60	-2.80	33.0
-1.80	-2.90	68.0
-2.00	-0.80	42.0
-2.10	-1.10	48.0
-2.40	-0.60	29.0
-2.50	-0.80	26.0
-2.50	-2.60	29.0
-2.60	-0.50	39.0
-2.70	-1.50	27.0
-2.90	-0.70	59.0
-3.70	2.70	84.0
-3.80	-1.60	52.0
-4.00	-2.60	74.0
-4.10	-1.60	38.0
-4.10	-1.90	39.0
-4.30	-2.60	82.0
-4.40	-1.80	56.0
-4.50	-1.50	38.0
-4.50	-2.50	53.0
-4.60	-1.20	62.0
-4.70	-2.60	66.0

Lunxy11.dru

Lunz11.dru

-0.10	-0.10	59.0
-0.60	-1.20	87.0
-0.70	-1.80	71.0
-0.70	-2.30	89.0
-1.30	-0.10	92.0
-1.40	-0.20	101.0
-1.70	-2.20	72.0
-1.80	-1.80	94.0
-2.10	-0.50	82.0
-2.10	-1.90	74.0
-2.70	-1.70	114.0
-2.70	-2.40	66.0
-2.70	-2.90	88.0
-3.50	-2.30	64.0
-3.60	-0.60	116.0
-3.80	-2.50	109.0
-4.20	-1.70	64.0
-5.10	-0.80	66.0
-5.30	-1.40	91.0
-5.40	-1.60	75.0
-6.10	-0.90	108.0
-6.10	-1.90	98.0
-6.10	-2.90	103.0
-6.20	-1.50	57.0
-6.40	-2.00	56.0
-6.60	-2.60	56.0
-6.70	-2.20	77.0
-6.80	-2.60	54.0
-6.90	-1.80	56.0
-6.90	-2.70	64.0
-7.20	-1.50	81.0

Lunxy12.dru

Lunz12.dru

-0.20	-0.20	50.0
-0.60	-1.90	72.0
-0.80	-1.80	55.0
-0.90	-2.50	43.0
-1.10	-1.50	76.0
-1.40	-2.90	104.0
-1.80	-1.90	70.0
-1.90	-1.60	47.0
-2.20	-2.10	72.0
-2.50	-2.50	72.0
-2.80	-2.80	116.0
-2.90	-1.40	86.0
-3.50	-1.30	58.0
-3.60	-2.50	82.0
-4.00	-1.10	92.0
-4.80	-1.50	68.0
-4.90	-2.10	126.0
-5.40	-1.00	62.0
-5.50	-2.30	66.0
-6.40	-1.30	71.0
-6.50	-2.20	101.0
-6.60	-2.40	73.0
-6.70	-0.70	116.0
-6.90	-1.50	98.0
-7.10	-1.90	65.0
-7.80	-1.60	111.0

Lunxy13.dru

Lunz13.dru

-0.20	-0.10	64.0
-0.40	-0.60	94.0
-0.60	-1.10	68.0
-0.90	-1.30	111.0
-0.90	-1.70	88.0
-0.90	-1.80	101.0
-1.10	-0.30	54.0
-1.50	-0.20	61.0
-1.70	-0.10	54.0
-1.80	-1.60	36.0
-1.90	-0.10	63.0
-2.20	-0.20	50.0
-2.40	-1.90	65.0
-2.40	-2.10	82.0
-2.50	-1.50	76.0
-2.60	-1.10	28.0
-2.70	-0.10	78.0
-3.20	-1.70	21.0
-3.50	-1.50	68.0
-4.00	-1.60	65.0
-4.50	-1.70	81.0
-4.60	-1.80	64.0
-5.50	-1.20	101.0
-5.50	-1.50	77.0
-5.50	-1.80	101.0
-5.80	-1.20	80.0
-5.90	-1.30	101.0
-6.10	-1.40	108.0
-6.30	-1.60	94.0
-6.60	-0.90	109.0
-6.70	-0.40	

Lunxy14.dru

Lunz14.dru

-0.20	-0.10	95.0
-0.20	-0.70	116.0
-0.20	-1.60	92.0
-0.40	-1.40	63.0
-0.40	-2.20	58.0
-0.40	-2.80	76.0
-0.50	-3.50	105.0
-0.60	-1.80	57.0
-1.00	-0.90	75.0
-1.10	-1.60	81.0
-1.10	-2.40	81.0
-1.10	-2.60	61.0
-1.30	-3.30	62.0
-1.10	-1.90	92.0
-1.50	-1.90	80.0
-1.60	-2.60	82.0
-1.80	-1.80	92.0
-1.80	-2.50	71.0
-1.80	-3.10	101.0
-1.90	-0.80	106.0
-2.20	-3.20	55.0
-2.50	-0.70	111.0
-2.60	-3.10	93.0
-2.70	-0.90	99.0
-2.80	-3.00	60.0
-3.10	-1.20	78.0
-3.20	-2.80	92.0
-3.50	-1.30	106.0
-3.50	-3.20	64.0
-3.60	-3.70	82.0
-3.70	-2.40	61.0
-3.80	-1.30	81.0
-4.60	-1.00	100.0
-4.60	-3.30	20.0
-4.80	-1.30	63.0
-5.50	-1.20	67.0
-6.20	-1.40	4.0
-6.30	-3.90	128.0
-6.50	-2.40	22.0
-6.50	-3.60	94.0
-6.60	-3.20	113.0
-6.80	-1.50	69.0
-7.40	-3.50	82.0
-7.60	-3.90	116.0

Lunxy15.dru

Lunz15.dru

-0.10	-0.10	72.0
-0.90	-1.80	126.0
-1.00	-1.80	111.0
-1.20	-1.90	121.0
-1.50	-1.50	91.0
-1.80	-0.70	86.0
-1.80	-1.20	112.0
-1.90	-0.40	113.0
-2.30	-0.60	85.0
-2.50	-0.60	106.0
-2.90	-2.40	142.0
-3.00	-0.30	64.0
-3.30	-0.10	81.0
-4.20	-0.20	72.0
-4.60	-0.10	91.0
-4.60	-1.50	78.0
-4.70	-0.30	82.0
-4.90	-0.30	103.0
-5.20	-1.10	101.0
-5.50	-0.30	83.0
-5.60	-1.30	69.0
-5.70	-2.30	74.0
-5.80	-2.60	118.0
-5.90	-0.40	111.0
-5.90	-1.10	93.0
-6.00	-1.10	67.0
-6.10	-1.90	104.0
-6.30	-0.10	85.0
-6.30	-2.20	68.0
-6.50	-0.80	108.0
-6.50	-2.60	108.0
-6.60	-1.80	79.0
-6.70	-0.50	84.0
-6.70	-2.70	70.0
-6.90	-2.70	92.0
-7.10	-0.40	103.0
-7.10	-1.60	111.0
-7.10	-2.20	70.0
-7.20	-1.20	78.0
-7.20	-2.20	84.0
-7.50	-2.60	74.0
-7.60	-0.40	80.0
-7.80	-0.20	102.0
-7.80	-2.70	94.0

Lunxy16.dru

Lunz16.dru

-0.10	-0.10	25.0
-0.30	-1.40	82.0
-0.60	-1.30	55.0
-0.90	-1.50	68.0
-1.10	-0.80	56.0
-1.40	-1.50	56.0
-1.50	-0.80	60.0
-1.70	-0.80	81.0
-1.80	-1.80	91.0
-2.40	-0.80	75.0
-2.70	-0.80	89.0
-3.30	-1.20	69.0
-3.30	-2.10	71.0
-3.70	-1.40	114.0
-3.70	-2.40	91.0
-3.70	-2.80	69.0
-3.80	-2.80	81.0
-4.20	-1.40	79.0
-4.20	-2.30	107.0
-4.20	-2.70	79.0
-4.50	-2.50	79.0
-4.80	-2.60	102.0
-5.10	-0.90	118.0
-5.10	-2.60	87.0
-5.50	-0.90	85.0
-5.60	-2.40	116.0
-6.00	-2.40	88.0
-6.10	-1.10	107.0
-6.20	-1.30	89.0
-6.60	-2.40	113.0
-6.70	-1.60	117.0
-6.80	-1.70	111.0
-6.80	-2.40	86.0
-6.90	-1.80	118.0
-7.10	-2.10	94.0
-7.20	-2.20	113.0
-7.50	-1.90	89.0
-7.60	-1.40	114.0

Lunxy17.dru

Lunz17.dru

-0.10	-0.10	95.0
-0.80	-0.50	79.0
-0.60	-0.50	71.0
-0.70	-1.10	126.0
-0.60	-1.50	71.0
-1.50	0.30	83.0
-1.70	-0.70	79.0
-1.30	-1.30	76.0
-1.70	-1.20	91.0
-2.30	-0.40	93.0
-2.80	-0.30	86.0
-2.40	-1.30	82.0
-3.50	-0.50	87.0
-3.20	-0.90	85.0
-3.50	-0.90	104.0
-3.50	-1.40	75.0
-3.80	-1.50	104.0
-4.70	-0.40	79.0
-4.50	-1.40	66.0
-5.50	-0.50	74.0
-5.90	-0.70	54.0
-5.60	-0.50	94.0
-5.10	-1.30	78.0
-6.10	-0.50	57.0
-6.50	-0.50	83.0
-6.50	-0.70	48.0
-6.30	-1.30	102.0
-6.10	-1.60	68.0
-6.70	-1.30	78.0
-7.40	-0.30	56.0
-7.20	-0.80	68.0
-7.70	-0.60	54.0
-7.40	-1.30	29.0

Ltestxy.dru

Ltestz.dru

-0.10	-0.10	3.0
-0.50	-0.60	21.0
-0.20	-1.40	5.0
-1.10	-0.70	34.0
-1.20	-0.80	24.0
-1.30	-0.70	27.0
-1.60	-0.50	3.0
-1.60	-1.10	37.0
-1.40	-1.30	17.0
-1.30	-1.40	28.0
-1.80	-1.60	2.0
-2.70	-0.40	29.0
-2.50	-0.80	15.0
-2.30	-1.10	38.0
-2.50	-1.50	17.0
-2.40	-1.60	8.0
-2.50	-1.90	44.0
-2.50	-2.20	23.0
-3.10	-0.50	46.0
-3.50	-0.40	26.0
-3.60	-1.30	17.0
-3.50	-1.70	48.0
-3.50	0.80	8.0
-4.20	-0.50	66.0
-4.10	-0.30	28.0
-4.50	0.60	42.0
-4.80	-0.70	77.0
-5.10	-0.90	32.0
-4.20	-1.30	28.0
-4.40	-1.40	57.0
-4.10	-1.90	27.0
-4.10	-2.20	16.0
-5.40	-0.50	41.0
-5.70	-0.60	31.0
-5.50	-1.50	28.0
-6.10	-0.40	31.0
-6.40	-0.50	59.0
-6.20	-0.70	31.0
-6.80	-0.70	12.0
-6.30	-1.10	58.0
-6.20	-1.30	21.0
-6.00	-1.50	52.0
-6.20	-1.90	20.0
-6.70	-1.50	23.0
-7.20	-0.40	22.0
-7.60	-0.70	59.0
-7.90	-0.50	18.0
-7.30	-1.10	32.0
-7.30	-1.70	68.0
-7.30	-1.90	32.0
7.60	1.50	26.0

Ltest2xy.dru

Ltest2z.dru

-0.10	-0.10	60.0
-0.60	-0.60	54.0
-0.50	-1.30	61.0
-0.50	-1.70	23.0
-0.90	-1.40	38.0
-1.50	-0.70	63.0
-1.50	-1.40	38.0
-1.40	-1.50	68.0
-1.50	-1.80	41.0
-2.20	-0.50	74.0
-2.80	-0.10	51.0
-2.30	-0.80	54.0
-2.20	-1.30	82.0
-2.10	-1.60	56.0
-2.20	-1.80	79.0
-2.10	-2.10	66.0
-2.50	-2.50	51.0
-3.50	-0.50	64.0
-3.30	-1.10	87.0
-3.30	-1.50	46.0
-3.80	-1.10	54.0
-3.90	-1.60	89.0
-3.80	-1.90	53.0
-4.00	0.40	58.0
-4.20	-0.70	96.0
-4.40	-0.70	63.0
-4.60	-0.60	71.0
-4.70	-0.70	86.0
-4.50	-1.20	54.0
-4.20	-1.80	58.0
-5.20	-0.40	74.0
-5.10	-0.80	62.0
-5.40	-1.30	113.0
-5.40	-1.40	98.0
-5.50	-1.60	118.0
-5.40	-2.00	75.0
-5.80	-1.50	72.0
-6.30	-0.50	61.0
-6.70	-0.30	67.0
-6.80	-0.50	112.0
-6.80	-0.70	88.0
-6.20	-1.20	72.0
-6.70	-1.10	113.0
-6.70	-1.40	91.0
-6.10	-1.60	86.0
-7.30	-0.20	98.0
-7.20	-0.60	88.0
-7.30	-1.20	86.0

Ltest3xy.dru

Ltest3z.dru

-0.30	-0.70	81.0
-0.50	-0.50	79.0
-0.60	-1.40	102.0
-0.20	-1.70	136.0
-0.20	-1.90	98.0
-0.70	-1.70	93.0
-1.20	-0.50	91.0
-1.20	-0.30	79.0
-1.10	-0.70	79.0
-1.50	-0.50	79.0
-1.30	-1.20	79.0
-1.40	-1.70	126.0
-1.30	-1.90	105.0
-1.70	-1.60	81.0
-2.20	-0.20	79.0
-2.50	-0.60	88.0
-2.60	-0.70	79.0
-2.50	-1.40	89.0
-2.70	-1.90	116.0
-2.90	-1.70	112.0
-3.60	-0.40	91.0
-3.50	-1.60	96.0
-3.10	-1.30	87.0
-3.60	-1.70	92.0
-3.90	-1.20	116.0
-4.20	-0.80	94.0
-4.20	-1.30	92.0
-4.40	-0.20	81.0
-4.60	-0.50	102.0
-4.50	-1.00	129.0
-4.50	-1.50	86.0
-5.10	-0.40	79.0
-5.50	-0.50	118.0
-5.70	-0.70	93.0
-5.50	-1.30	78.0
-5.80	-1.90	72.0
-5.90	-0.50	112.0
-6.20	-0.40	122.0
-6.40	-0.30	111.0
-6.80	-0.10	123.0
-6.80	-0.30	112.0
-6.90	-0.90	88.0
-6.70	-1.60	112.0
-6.70	-1.90	99.0
-7.00	-1.50	89.0
-7.30	-1.30	109.0
-7.60	-1.50	69.0
-7.30	-0.70	86.0
-7.10	-0.50	114.0
-7.50	-0.40	96.0
-7.70	-0.70	79.0

Ltest4xy.dru

Ltest4z.dru

-0.10	-0.10	57.0
-0.50	-0.70	76.0
-0.80	-0.70	63.0
-0.50	-1.00	67.0
-0.50	-1.50	105.0
-0.70	-1.70	64.0
-0.80	-1.60	68.0
-1.30	-1.30	62.0
-1.80	-1.10	110.0
-1.90	-1.70	52.0
-1.70	-0.70	62.0
-1.60	-0.50	82.0
-1.20	-0.40	74.0
-2.50	-0.50	74.0
-2.60	-0.90	86.0
-2.60	-1.10	93.0
-2.60	-1.80	52.0
-2.80	-1.10	70.0
-3.20	-0.90	92.0
-3.60	-0.50	61.0
-3.50	-0.90	68.0
-3.40	-1.20	69.0
-3.60	-1.70	82.0
-3.70	-1.90	64.0
-4.50	-1.50	20.0
-4.50	-0.50	50.0
-5.20	-1.30	50.0
-5.50	-1.50	75.0
-5.50	-0.50	25.0
-6.30	-1.90	128.0
-6.60	-1.80	92.0
-6.50	-1.50	94.0
-6.60	-1.20	113.0
-6.80	-0.70	75.0
-6.50	-0.50	50.0
-7.50	-0.50	50.0
-7.50	-1.50	82.0
-7.60	-1.80	116.0
-7.90	-1.80	100.0

Ltest5xy.dru

Ltest5z.dru

-0.10	-0.10	100.0
-0 10	-0.70	100.0
-0.30	-0.80	116.0
-0.30	-1.20	75.0
-0.60	-1.60	58.0
-0.20	-1.90	92.0
-0.50	-1.80	57.0
-1.10	-1.80	78.0
-1.10	-1.70	73.0
-1.10	-1.60	81.0
-1.20	-1.50	71.0
-1.10	-1.90	71.0
-1.40	-1.90	92.0
-1.50	-1.80	71.0
-1.70	-1.90	92.0
-1.40	-1.40	77.0
-1.40	-0.90	64.0
-1.90	-0.80	106.0
-2.10	-0.90	89.0
-2.60	-0.70	111.0
-2.60	-0.90	99.0
-2.50	-1.50	75.0
-3.50	-1.40	106.0
-3.70	-1.50	81.0
-4.00	-1.40	94.0
-3.60	-0.90	72.0
-4.30	-1.30	83.0
-4.50	-1.50	64.0
-4.50	-1.10	100.0
-4.50	-0.70	63.0
-5 20	-0.90	62.0
-5.50	-1.20	67.0
-5.50	-1.50	50.0
-6.30	-1.50	46.0
-6.80	-1.40	69.0
-7.00	-1.80	25.0
-6.80	-1.80	35.0
-7.50	-0.80	29.0
-7.50	-1.50	62.0
-7.90	-1.50	45.0
-7.50	-1.90	30.0

Ltest6xy.dru

Ltest6z.dru

-0.10	-0.10	69.0
-0.70	-0.20	87.0
-0.50	-0.50	69.0
-0.70	-0.80	71.0
-0.40	-1.30	67.0
-0.70	-1.40	89.0
-0.50	-1.50	75.0
-0.40	-1.80	61.0
-1.30	-1.20	77.0
-1.70	-1.30	72.0
-1.70	-0.80	94.0
-1.50	-0.50	72.0
-2.40	-0.30	78.0
-2.20	-0.50	74.0
-2.70	-0.70	114.0
-2.70	-0.90	75.0
-2.80	-1.10	67.0
-2.70	-1.40	66.0
-2.70	-1.80	88.0
-2.90	-1.80	75.0
-3.50	-0.50	72.0
-3.30	-0.90	64.0
-3.50	-1.20	64.0
-3.80	-1.60	109.0
-3.70	-1.90	98.0
-4.20	-1.80	87.0
-4.40	-1.40	98.0
-4.50	-0.50	64.0
-4.90	-0.40	75.0
-5.30	-0.40	91.0
-5.60	-0.40	73.0
-5.30	-0.70	77.0
-5.80	-0.70	82.0
-5.40	-1.30	116.0
-5.40	-1.50	86.0
-5.90	-1.80	89.0
-6.20	-1.90	103.0
-6.60	-1.90	52.0
-6.40	-1.50	57.0
-6.70	-1.20	77.0
-6.80	-0.80	56.0
-6.50	-0.50	56.0
-7.20	-0.50	81.0
-7.40	-0.50	62.0
-7.40	-0.30	64.0
-7.60	-0.70	74.0
-7.90	-0.50	68.0
-7.40	-1.20	61.0
-7.50	-1.50	53.0
-7.70	-1.70	96.0

Ltest7xy.dru

Ltest7z.dru

-0.50	-0.50	49.0
-0.80	-0.80	77.0
-0.60	-1.30	56.0
-0.30	-1.80	73.0
-1.40	-0.90	104.0
-1.20	-0.30	41.0
-1.80	-0.20	49.0
-1.80	-1.10	78.0
-1.50	-1.50	76.0
-2.30	-0.30	62.0
-2.50	-0.50	74.0
-2.80	-0.80	116.0
-2.90	-1.10	104.0
-2.50	-1.50	85.0
-3.10	-1.50	81.0
-3.60	1.60	116.0
-3.80	-1.80	87.0
-3.40	-1.30	81.0
-3.20	-0.40	74.0
-3.60	-0.70	82.0
-4.20	-0.60	101.0
-4.10	-0.80	85.0
-4.80	-0.80	81.0
-4.90	-0.20	126.0
-4.60	-1.20	79.0
-4.50	-1.50	81.0
-5.20	-1.60	73.0
-5.40	-1.70	76.0
-5.60	-1.50	58.0
-5.30	-0.40	75.0
-5.50	-0.50	64.0
-5.90	-0.30	81.0
-6.10	-0.60	75.0
-6.10	-0.90	63.0
-6.30	0.10	108.0
-6.40	-0.20	98.0
-6.50	-0.30	107.0
-6.50	-0.10	95.0
-6.60	-0.50	73.0
-6.90	-0.50	98.0
-6.90	-0.10	64.0
-6.50	-0.80	73.0
-6.20	-1.90	108.0
-6.10	-1.50	58.0
-6.50	-1.50	58.0
-6.80	-1.70	56.0
-7.50	-1.50	57.0
-7.70	-1.80	81.0
-7.20	-1.10	56.0
-7.50	-0.50	56.0
-7.30	-0.30	93.0
-7.70	-0.10	63.0

Ltest8xy.dru

Ltest8z.dru

-0.10	-0.10	105.0
-0.50	-0.40	81.0
-0.80	-0.70	126.0
-1.20	-0.30	80.0
-1.20	-0.90	121.0
-1.10	-0.80	111.0
-1.50	-0.60	91.0
-1.70	-0.30	112.0
-1.70	-0.10	95.0
-1.90	-0.20	104.0
-2.10	-0.60	113.0
-2.50	-0.40	126.0
-2.50	-0.10	95.0
-2.60	-0.90	117.0
-3.10	-0.70	138.0
-3.50	-0.30	102.0
-3.60	-0.60	134.0
-3.20	-0.80	131.0
-3.50	-0.90	142.0
-4.50	-0.40	78.0
-4.70	-0.20	102.0
-4.80	-0.10	80.0
-4.90	-0.70	77.0
-5.20	-0.50	101.0
-5.50	-0.20	72.0
-5.10	-0.70	75.0
-5.70	-0.50	69.0
-6.10	-0.80	104.0
-6.30	-0.50	74.0
-6.60	-0.70	79.0
-7.10	-0.70	111.0
-7.30	-0.30	78.0
-7.90	-0.50	74.0
-7.10	-0.90	70.0

Ltest9xy.dru
Ltest9z.dru

0.10	-0.10	56.0
-0.70	-0.70	43.0
-0.40	-0.80	64.0
-0.50	-1.60	114.0
-0.30	-1.90	95.0
-0.80	-1.30	39.0
-1.20	-1.80	73.0
-1.80	-1.60	86.0
-1.90	-1.40	113.0
-1.20	-1.20	39.0
-1.50	-0.80	53.0
-1.70	-0.60	79.0
-1.60	-0.40	61.0
-2.50	-0.40	49.0
-2.70	-0.80	87.0
-2.70	-1.40	78.0
-2.80	-1.50	86.0
-2.50	-1.60	106.0
-2.30	-1.90	95.0
-2.20	-1.40	75.0
-3.60	-1.40	99.0
-3.70	-1.50	95.0
-3.80	-1.60	102.0
-3.40	-1.80	86.0
-3.90	-1.50	70.0
-3.40	-1.20	70.0
-3.30	-1.10	81.0
-3.30	-0.80	70.0
-3.50	-0.50	69.0
-3.70	-0.40	91.0
-3.80	-0.10	81.0
-3.90	-0.50	77.0
-4.30	-0.40	107.0
-4.50	-0.40	79.0
-4.80	-0.60	102.0
-4.90	-0.40	84.0
-4.70	-0.90	82.0
-4.60	-1.10	91.0
-4.70	-1.30	82.0
-4.90	-1.30	103.0
-4.50	-1.50	65.0
-5.50	-1.50	83.0
-5.90	-1.50	111.0
-5.90	-1.80	79.0
-5.80	-1.30	87.0
-5.20	-0.50	87.0
-5.50	-0.10	84.0
-5.40	-0.40	106.0
-5.50	-0.50	104.0
-5.60	-0.50	116.0
-5.90	-0.50	88.0
-6.40	-0.20	93.0
-6.60	-0.60	113.0
-6.30	-0.70	93.0
-6.70	-0.80	86.0
-6.50	-1.50	94.0
-6.50	-1.80	108.0
-6.50	-1.90	90.0
-6.70	-1.70	88.0
-7.20	-1.50	103.0
-7.40	-1.10	84.0
-7.20	-1.50	85.0
-7.80	-1.80	114.0
-7.90	-1.90	76.0
-7.80	-1.40	84.0
-7.80	-1.20	102.0
-7.60	-0.70	89.0
-7.30	-0.30	113.0
-7.10	-0.20	94.0

Ltest10x.dru
Ltest10z.dru

-0.10	-0.70	25.0
-0.30	-1.40	82.0
-0.20	-1.90	51.0
-0.70	-1.40	56.0
-0.90	-1.80	60.0
-1.30	-1.40	51.0
-1.80	-1.80	91.0
-1.90	-1.90	75.0
-1.70	-1.40	56.0
-1.70	-0.80	81.0
-1.50	-0.70	62.0
-1.10	-0.80	56.0
-2.00	-0.80	75.0
-2.30	-0.80	98.0
-2.50	-0.80	75.0
-2.70	-0.80	89.0
-2.70	-1.30	68.0
-2.80	-1.50	106.0
-2.30	1.70	82.0
-3.40	-1.50	101.0
-3.90	-1.80	88.0
-3.70	-1.40	114.0
-3.40	-1.20	69.0
-3.70	-0.80	71.0
-4.50	-1.50	94.0
-4.50	-0.80	75.0
-5.20	-0.90	118.0
-5.40	-0.90	87.0
-5.70	-0.80	116.0
-5.60	-1.40	96.0
-5.50	-1.70	87.0
-6.10	-0.80	99.0
-6.30	-1.30	109.0
-6.30	-1.40	89.0
-6.60	-0.80	94.0
-6.70	-0.70	117.0
-6.80	-0.80	111.0
-6.90	-0.90	118.0
-7.40	-1.40	98.0
-7.60	-1.50	111.0
-7.90	-1.80	85.0
-7.30	-1.20	94.0
-7.60	-0.90	97.0

Ltestx.dru
Ltest11z.dru

-0.10	-0.10	64.0
-0.50	-0.50	59.0
-0.70	-0.60	82.0
-0.90	-0.70	70.0
-0.80	-1.20	68.0
-0.50	-1.50	75.0
-1.40	-0.20	54.0
-1.60	-0.60	94.0
-1.70	-0.70	62.0
-1.40	-1.30	66.0
-1.90	-1.40	111.0
-1.90	-1.70	8.0
-2.10	-0.30	53.0
-2.40	-0.20	61.0
-2.60	-0.10	54.0
-2.80	-0.10	63.0
-2.50	-0.50	51.0
-2.80	-1.60	36.0
-3.20	-0.10	50.0
-3.70	-0.10	78.0
-3.60	-1.10	28.0
-3.50	-1.60	76.0
-3.50	-1.80	65.0
-3 90	-1.70	16.0
-4.50	-0.50	17.0
-4.20	-1 50	33.0
-4.60	-1.60	68.0
-5.40	-1.70	81.0
-5.30	-1.10	61.0
-5.10	-0.20	50 0
-5 60	-0.60	83.0
-6.40	-1.80	101.0
-6.50	-1.90	74.0
-6.60	-1.60	77.0
-6.60	-1.30	101.0
-6.70	-1.10	93.0
-6.30	-0.90	93.0
-6.50	-0.50	103.0
-7.30	-0.30	92.0
-7.70	-0.50	109.0
-7 90	-0.10	101.0
-7.90	-0.70	91.0
-7.50	-1.00	91.0
-7.30	-1.60	109.0
7.20	-2.00	79.0
-7.70	-1.30	95.0

Ltest12x.dru
Ltest12z.dru

-0.10	-0.10	57.0
-0.50	-0.70	76.0
-0.80	-0.70	63.0
-0.50	-1.00	67.0
-0.50	-1.50	105.0
-0.70	-1.70	64.0
-0.80	-1.60	68.0
-1.30	-1.30	62.0
-1.80	-1.10	110.0
-1.90	-1.70	52.0
-1.70	-0.70	62.0
-1.60	-0.50	82.0
-1.20	-0.40	74.0
-2.50	-0.50	74.0
-2.60	-0.90	86.0
-2.60	-1.10	93.0
-2.60	-1.80	52.0
-2.80	-1.10	70.0
-3.20	-0.90	92.0
-3.60	-0.50	61.0
-3.50	-0.90	68.0
-3.40	-1.20	69.0
-3.60	-1.70	82.0
-3.70	-1.90	64.0
-4.50	-1.50	20.0
-4.50	-0.50	50.0
-5.20	-1.30	50.0
-5.50	-1.50	75.0
-5.50	-0.50	25.0
-6.30	-1.90	128.0
-6.60	-1.80	92.0
-6.50	-1.50	94.0
-6.60	-1.20	113.0
-6.80	-0.70	75.0
-6.50	-0.50	50.0
-7.50	-0.50	50.0
-7.50	-1.50	82.0
-7.60	-1.80	116.0
-7.90	-1.80	100.0

Ltest13x.dru
Ltest13z.dru

-0.10	-0.10	100.0
-0.10	-0.70	100.0
-0.30	-0.80	116.0
-0.30	-1.20	75.0
-0.60	-1.60	58.0
-0.20	-1.90	92.0
-0.50	-1.80	57.0
-1.10	-1.80	78.0
-1.10	-1.70	73.0
-1.10	-1.60	81.0
-1.20	-1.50	71.0
-1.10	-1.90	71.0
-1.40	-1.90	92.0
-1.50	-1.80	71.0
-1.70	-1.90	92.0
-1.40	-1.40	77.0
-1.40	-0.90	64.0
-1.90	-0.80	106.0
-2.10	-0.90	89.0
-2.60	-0.70	111.0
-2.60	-0.90	99.0
-2.50	-1.50	75.0
-3.50	-1.40	106.0
-3.70	-1.50	81.0
-4.00	-1.40	94.0
-3.60	-0.90	72.0
-4.30	-1.30	83.0
-4.50	-1.50	64.0
-4.50	-1.10	100.0
-4.50	-0.70	63.0
-5.20	-0.90	62.0
-5.50	-1.20	67.0
-5.50	-1.50	50.0
-6.30	-1.50	46.0
-6.80	-1.40	69.0
-7.00	-1.80	25.0
-6.80	-1.80	35.0
-7.50	-0.80	29.0
-7.50	-1.50	62.0
-7.90	-1.50	45.0
-7.50	-1.90	30.0

Ltest14x.dru
Ltest14z.dru

-0.10	-0.10	69.0
-0.70	-0.20	87.0
-0.50	-0.50	69.0
-0.70	-0.80	71.0
-0.40	-1.30	67.0
-0.70	-1.40	89.0
-0.50	-1.50	75.0
-0.40	-1.80	61.0
-1.30	-1.20	77.0
-1.70	-1.30	72.0
-1.70	-0.80	94.0
-1.50	-0.50	72.0
-2.40	-0.30	78.0
-2.20	-0.50	74.0
-2.70	-0.70	114.0
-2.70	-0.90	75.0
-2.80	-1.10	67.0
-2.70	-1.40	66.0
-2.70	-1.80	88.0
-2.90	-1.80	75.0
-3.50	-0.50	72.0
-3.30	-0.90	64.0
-3.50	-1.20	64.0
-3.80	-1.60	109.0
-3.70	-1.90	98.0
-4.20	-1.80	87.0
-4.40	-1.40	98.0
-4.50	-0.50	64.0
-4.90	-0.40	75.0
-5.30	-0.40	91.0
-5.60	-0.40	73.0
-5.30	-0.70	77.0
-5.80	-0.70	82.0
-5.40	-1.30	116.0
-5.40	-1.50	86.0
-5.90	-1.80	89.0
-6.20	-1.90	103.0
-6.60	-1.90	52.0
-6.40	-1.50	57.0
-6.70	-1.20	77.0
-6.80	-0.80	56.0
-6.50	-0.50	56.0
-7.20	-0.50	81.0
-7.40	-0.50	62.0
-7.40	-0.30	64.0
-7.60	-0.70	74.0
-7.90	-0.50	68.0
-7.40	-1.20	61.0
-7.50	-1.50	53.0
-7.70	-1.70	96.0

Ltest15x.dru
Ltest15z.dru

-0.10	-0.10	49.0
-0.50	-0.50	49.0
-0.80	-0.80	77.0
-0.60	-1.30	56.0
-0.30	-1.80	73.0
-1.40	-0.90	104.0
-1.20	-0.30	41.0
-1.80	-0.20	49.0
-1.80	-1.10	78.0
-1.50	-1.50	76.0
-2.30	-0.30	62.0
-2.50	-0.50	74.0
-2.80	-0.80	116.0
-2.90	-1.10	104.0
-2.50	-1.50	85.0
-3.10	-1.50	81.0
-3.60	-1.60	116.0
-3.80	-1.80	87.0
-3.40	-1.30	81.0
-3.20	-0.40	74.0
-3.60	-0.70	82.0
-4.20	-0.60	101.0
-4.10	-0.80	85.0
-4.80	-0.80	81.0
-4.90	-0.20	126.0
-4.60	-1.20	79.0
-4.50	-1.50	81.0
-5.20	-1.60	73.0
-5.40	-1.70	76.0
-5.60	-1.50	58.0
-5.30	-0.40	75.0
-5.50	-0.50	64.0
-5.90	-0.30	81.0
-6.10	-0.60	75.0
-6.10	-0.90	63.0
-6.30	0.10	108.0
-6.40	-0.20	98.0
-6.50	-0.30	107.0
-6.50	-0.10	95.0
-6.60	-0.50	73.0
-6.90	-0.50	98.0
-6.90	-0.10	64.0
-6.50	-0.80	73.0
-6.20	-1.90	108.0
-6.10	-1.50	58.0
-6.50	-1.50	58.0
-6.80	-1.70	56.0
-7.50	-1.50	57.0
-7.70	-1.80	81.0
-7.20	-1.10	56.0
-7.50	-0.50	56.0
-7.30	-0.30	93.0
-7.70	-0.10	63.0

Ltest16x.dru
Ltest16z.dru

-0.10	-0.10	18.0
-0.80	-0.40	23.0
-0.40	-0.50	57.0
-0.20	-0.90	22.0
-0.50	-0.60	43.0
-0.50	-1.10	74.0
-0.50	-1.50	34.0
-0.10	-1.90	31.0
-1.40	-1.50	87.0
-1.00	-1.80	65.0
-1.60	-1.70	61.0
-1.70	-1.30	57.0
-1.30	-1.20	49.0
-1.40	-0.90	74.0
-1.50	-0.80	45.0
-1.50	-0.50	41.0
-1.40	-0.50	68.0
-1.50	-0.30	38.0
-1.70	-0.50	50.0
-2.20	-0.40	82.0
-2.50	-0.10	60.0
-2.70	-0.30	51.0
-2.40	-1.40	73.0
-2.70	-1.70	78.0
-2.90	-1.30	38.0
-3.50	-1.80	43.0
-3.40	-1.30	82.0
-3.70	-1.30	48.0
-3.30	-1.10	57.0
-3.40	-0.70	69.0
-3.30	-0.10	87.0
-3.80	-0.40	66.0
-3.90	-0.50	89.0
-3.80	-0.90	53.0
-4.20	-1.40	90.0
-4.50	-1.50	51.0
-4.10	-1.70	44.0
-4.60	-0.90	62.0
-4.50	-0.50	71.0
-5.20	-0.30	86.0
-5.40	-0.30	113.0
-5. 0	-0.40	98.0
-5.50	-0.70	113.0
-5.50	-0.90	75.0
-5.80	-0.30	72.0
-5.90	-0.80	87.0
-5.20	-1.30	62.0
-5.50	-1.50	61.0
-6.20	-1.80	112.0
-6.30	-1.70	75.0
-6.70	-1.70	106.0
-6.90	-1.90	96.0
-6.60	-1.50	78.0
-6.60	-1.20	121.0
-6.70	-0.70	72.0
-6.10	-0.60	86.0
-6.20	-0.20	72.0
-6.70	-0.10	113.0
-6.70	-0.30	91.0
-6.60	-0.50	102.0
-7.50	-0.50	91.0
-7.40	-1.30	81.0
-7.40	-1.60	106.0
-7.50	-1.70	81.0

Ltest17x.dru

Ltest17z.dru

-0.10	-0.10	35.0
-0.80	-0.30	47.0
-0.40	-0.50	34.0
-0.80	-0.50	43.0
-1.50	-0.50	86.0
-1.10	-0.10	53.0
-1.70	-0.30	73.0
-1.70	-0.60	63.0
-2.20	-0.40	67.0
-2.50	-0.50	103.0
-2.40	-0.60	90.0
-2.80	-0.30	56.0
-3.50	-0.30	56.0
-4.30	-0.40	91.0
-5.20	-0.30	122.0
-5.10	-0.10	80.0
-5.60	-0.30	69.0
-5.30	-0.60	62.0
-6.30	-0.30	113.0
-6.10	-0.10	80.0
-6.40	-0.60	69.0
-7.10	-0.30	108.0
-7.50	-0.40	75.0
-7.20	-0.50	66.0
-0.70	-0.80	42.0
-0.40	-1.60	39.0
-0.60	-1.90	72.0
-0.80	-1.70	55.0
-1.00	-1.50	76.0
-1.10	-1.20	54.0
-1.40	-1.60	69.0
-1.70	-1.50	46.0
-2.50	-1.40	73.0
-3.10	-1.20	86.0
-3.30	-1.30	59.0
-3.70	-0.90	56.0
-3.90	-1.80	81.0
-4.50	-1.50	65.0
-5.00	-1.10	[illegible]
-5.50	-1.20	[illegible]
-5.70	-1.50	68.0
-6.50	-1.60	[illegible]
-6.30	-1.90	[illegible]
-6.80	-1.70	64.0
-6.40	-1.30	71.0
-6.20	-0.80	69.0
-6.70	-0.80	116.0
-7.40	-0.30	64.0
-7.50	-1.20	82.0
-7.90	-1.50	[illegible]
-7.90	-1.80	62.0
-7.90	-1.50	[illegible]

Ltest18x.dru

Ltest18z.dru

-0.10	0.20	
-0.20	1.70	[illegible]15.0
-0.30	0.80	23[illegible].0
-0.40	0.60	238.0
-0.40	-[illegible].50	215.0
-0.40	[illegible].30	205.0
-0.10	[illegible].50	19[illegible].0
-0.60	[illegible].50	219.0
-0.70	0.60	216.0
-0.70	[illegible].20	248.0
-0.80	0.50	208.0
-0.90	-1.30	230.0
-1.10	-0.90	240.0
-1.20	-[illegible].00	223.0
-1.40	0.30	208.0
-1.50	-1.50	268.0
-1.60	0.80	186.0
-1.70	-0.60	261.0
-1.70	0.20	241.0
-1.90	0.[illegible]0	244.0
-2.20	-1.20	258.0
-2.30	1.30	225.0
-2.70	2.40	258.0
-2.80	-0.20	204.0
-3.30	-3.50	190.0
-3.60	2.70	229.0
-3.60	3.60	206.0
-3.80	-0.50	202.0
-3.90	-1.10	232.0
-4.10	-4.80	212.0
-4.20	-2.20	236.0
-4.20	-3.50	248.0
-4.30	-0.70	218.0
-4.40	-4.30	201.0
-4.50	-2.50	219.0
-4.60	-1.80	220.0
-4.70	-1.40	244.0
-4.90	-0.80	221.0
-4.90	-2.60	238.0

Petxy1.dru

Petz1.dru

-0.10	-0.10	208.0
-0.10	-3.70	238.0
-0.20	-2.90	210.0
-0.20	-3.30	204.0
-0.40	-2.80	236.0
-0.50	-2.80	215.0
-0.70	-2.50	236.0
-0.70	-3.60	202.0
-0.80	-2.60	224.0
-1.20	-2.90	249.0
-1.40	-3.50	224.0
-1.60	-1.50	214.0
-1.60	-4.00	248.0
-1.70	-0.30	230.0
-1.90	-0.40	212.0
-1.90	-4.30	213.0
-2.40	-0.60	248.0
-2.40	-4.60	240.0
-2.60	-4.40	206.0
-2.70	-1.30	210.0
-2.80	-2.30	240.0
-2.90	-3.80	234.0
-3.10	-3.50	206.0
-3.40	-2.10	216.0
-3.50	-1.60	242.0
-4.10	-2.80	206.0
-4.20	-1.40	205.0
-4.60	-4.40	224.0
-4.70	-0.30	224.0
-4.80	-3.70	228.0
-4.80	-4.30	200.0

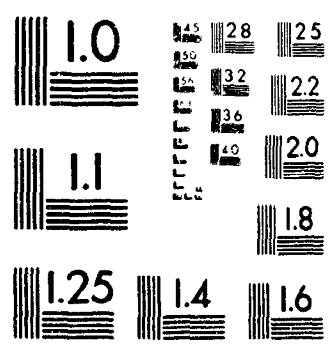
Petxy2.dru

Petz2.dru

0.10	0.10	[illegible]
0.[illegible]0	1.[illegible]	[illegible]
0.20	1.20	[illegible]10.0
-0.40	4.[illegible]0	[illegible]
-0.60	[illegible].90	[illegible]
0.60	1.10	[illegible]
0.[illegible]0	[illegible]	[illegible]
-1.10	[illegible]	[illegible]
1.50	[illegible]	[illegible]
-1.60	1.[illegible]0	[illegible]
-1.[illegible]0	1.10	[illegible]
-1.80	-1.[illegible]0	[illegible]
2.00	1.[illegible]	[illegible]
-2.30	-1.[illegible]	210.[illegible]
-2.50	1.50	19[illegible].0
-2.80	[illegible].80	[illegible]0[illegible].0
-3.80	[illegible].90	[illegible]
-4.40	-[illegible]	[illegible]
4.50	[illegible]	[illegible]
4.60	-[illegible]	[illegible]
-4.80	[illegible].10	[illegible]11.0
-5.00	1.80	1[illegible]1.0
-5.10	-1.10	[illegible]
-5.50	1.30	1[illegible]1.0
-5.50	3.[illegible]0	1[illegible]5.0
-5.80	-1.20	224.0
-6.10	-0.80	19[illegible].0
-6.50	-2.50	1[illegible]5.0
-6.60	-0.70	208.0
-6.60	-3.10	212.0
-7.30	-1.70	192.0
-7.60	-2.10	188.0
-8.00	-2.50	204.0
-9.00	-2.80	192.0
-10.50	-2.80	213.0
-11.50	-3.50	201.0
-12.10	[illegible].60	[illegible]
-13.20	3.60	200.0
14.50	3.30	[illegible]74.0

Petxy3.dru

Petz3.dru


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

0.10	0 10	186.0
-0 30	1 60	197 0
0.30	[illegible].00	186.0
-0.40	[illegible].90	234.0
-0.50	0 50	186.0
0.70	3.90	188.0
-1.40	-0.30	218.0
-1.50	1.50	186.0
-1.50	4.60	218.0
-1.60	-1.30	209.0
-1.70	-4.40	216.0
-1.90	1.30	204.0
-2.10	-1.30	208.0
-2.50	-[illegible].50	200.0
-2.50	-[illegible].50	186.0
-3.50	1 50	186.0
-3.50	1.50	186.0
-4.10	0.90	216.0
-4.30	-1.20	196.0
-5.00	1 30	190.0
-5 10	-2.10	200.0
-5.20	-3.90	186.0
-5.30	-1.50	190.0
-5.40	-1 40	218.0
-5.50	-2.70	187.0
-5.80	-0.60	198.0
-5.80	-3.70	198.0
-5.90	-1.50	188.0
-6.40	-1.40	192.0
-6.40	-1.80	204.0
-6.60	-0.60	226.0

Petxy4.dru

Petz4.dru

-0 10	-0.10	205 0
-0.10	-0.10	200 0
-0.10	-1 20	194 0
-0.40	-1.50	300.0
-0.80	-1 50	189 0
-1.30	-2.30	189 0
-1.40	-0.80	201 0
-1.50	-0 30	230 0
-1.50	-1.20	21[illegible] 0
-1.90	-2.80	220 0
-2.30	-2.20	186 0
-2.40	-0.90	192.0
-2.70	-0 10	195 0
-2.70	-1.40	231.0
-3.30	-3.80	216 0
-3.40	-3.50	194 0
-3.50	-1.90	206.0
-3.50	-2.80	208.0
-3.50	-4.10	187.0
-3 60	-2.20	194 0
-3.60	-2.80	192.0
-3.70	-1.40	188.0
-3.80	-2.10	214.0
-4.10	-0.90	226 0
-4.30	-4.60	218 0
-5.50	-2.50	215.0
-5.60	-1.40	192.0
-5.60	-2.10	210.0
-5.70	-2.30	195.0
-5.80	-2.20	195.0
-6.30	-2.40	202.0
-6.40	-0.50	239.0
-6.60	-3.10	188.0
-6.70	-1.60	187 0
-6.70	-3.70	228 0
-6.80	-2.60	187 0
-6.90	-0.90	193 0
-6.90	-3.30	186 0
-6.90	-3.90	210.0
-7.20	-4.10	218 0
-7.30	-2.90	208 0
-7.40	-2 10	187 0
-7.50	-1.40	224 0

Petxy5.dru

Petz5.dru

0 [illegible]0	-0 10	205 0
0 30	-1 [illegible]0	216.0
-0 [illegible]0	4.30	186 0
-0.60	-1.40	190 0
0.80	-1.90	222.0
-0.80	-4.30	211 0
-1.20	-1.50	192.0
-1.20	-3.10	186.0
-1.30	-2.20	192.0
-1.40	-2.80	211.0
-1.70	-4.50	186 0
-1.80	-2.30	192.0
-2.20	-3.30	187.0
-2.40	-1.40	191.0
-2.40	-2 20	221.0
2 50	-0.50	224.0
-2.50	-4.30	214.0
-3.10	-2.20	188.0
-3 50	-0 50	185.0
-3.50	-4.40	192.0
-3 70	-2 10	211.0
-3.90	-3.00	186.0
-3.90	-3.80	211.0
-4.40	-1.40	190.0
-4 60	-0.80	208.0
-5.20	-4.40	221.0

Petxy6.dru

Petz6.dru

0.10	0.10	185.0
0.10	0.60	206.0
0.60	[illegible]	191.0
0.10	1.80	195.0
0.30	1.70	215.0
0.50	1.50	192.0
1.50	1.50	[illegible]
1.50	1.90	195.0
1.70	1.70	[illegible]
1.70	1.30	211.0
1.70	0.80	225.0
1.50	0.50	190.0
2.20	0.60	201.0
2.50	0.50	220.0
2.30	0.20	191.0
-2.70	0.10	193.0
2.10	-1.70	193.0
-2.50	-1.50	210.0
-3.70	1.70	228.0
-3.70	1.90	220.0
-3.80	1.80	215.0
3.70	1.10	187.0
3.70	0.40	201.0
3.70	0.30	187.0
3.20	[illegible]	185.0
1.40	-0.80	207.0
1.20	0.20	187.0
-4.50	1.50	186.0
-5.50	0.50	185.0
5.50	1.50	186.0
6.10	0.70	210.0
-6.30	0.30	192.0
-6.90	0.80	187.0
-7.10	-0.30	221.0
-7.80	0.40	188.0
-7.60	-0.80	190.0
-7.30	1.30	187.0
-7.80	1.60	195.0
-8.80	-1.80	211.0
-8.90	-1.90	205.0
-8.50	-1.30	186.0
8.50	-0.50	189.0
8.70	-0.20	210.0
-9.10	0.20	195.0
-9.80	-0.30	220.0
-9.70	1.10	192.0
9.50	-1.50	189.0

Ptest1xy.dru

Ptest1z.dru

0.10	-0.10	215.0
0.60	-0.20	231.0
0.70	-0.60	218.0
-0.40	-0.60	215.0
-0.30	0.70	237.0
0.10	-0.70	225.0
-0.70	-0.90	230.0
-1.10	-0.90	223.0
-1.00	-0.40	235.0
-1.40	-0.30	267.0
-1.30	-0.10	260.0
1.80	0.30	240.0
1.60	0.60	253.0
1.70	-0.80	262.0
1.70	-0.90	250.0
1.80	0.90	244.0
1.90	0.80	258.0
-2.30	0.80	210.0
-2.40	0.40	215.0
-2.50	-0.50	191.0
3.20	-0.50	189.0
3.50	0.50	190.0
3.90	-0.40	193.0
-3.80	-0.50	232.0
3.90	-0.90	215.0
-4.30	-0.70	200.0
4.50	-0.50	205.0
-4.90	-0.80	238.0

Ptest2xy.dru

Ptest2z.dru

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