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Neuronal Correlation Parameter in the Idea of Thermodynamic Entropy of an N -Body Gravitationally Bounded System

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Abstract: Understanding how the brain encodes information and performs computation requires statistical and functional analysis. Given the complexity of the human brain, simple methods that facilitate the interpretation of statistical correlations among different brain regions can be very useful. In this report we introduce a numerical correlation measure that may serve the interpretation of correlational neuronal data, and may assist in the evaluation of different brain states. The description of the dynamical brain system, through a global numerical measure may indicate the presence of an action principle which may facilitate a application of physics principles in the study of the human brain and cognition.

Introduction

Theoretical analysis of brain dynamics may provide insights into cognitive, psychiatric and neurological disorders¹ and brain network changes, and may aid the early detection of pathophysiology in patients, and therefore it may have a very relevant significance. Different brain regions have been identified that relate to different functions and there is a reach literature of structural-anatomical and theoretical models that have been proposed to integrate brain areas and functions. It remains however, a very challenging problem to accurately describe the dynamics of the brain due to its high anatomical, physiological and functional complexity. for example, Cerebrovascular alterations include vascular density, vascular plasticity, and vascular reactivity to acute metabolic changes² with important consequences. Moreover, certain anatomical regions such as the Posterior Cingulate Cortex (PCC) that have high rate of metabolism (associated with normal conscious state) may play a role in the tuning of metastability of intra- and inter- network connectivity³. Alterations in functional properties or anatomical disconnection between brain regions may be caused by white matter loss or

demyelination (“disconnection” hypothesis”⁴). Of course, there exist vast literature related to studies that discuss healthy, diseased and aging conditions of the human brain, and modeling of such different states is desirable. In general some examples of the functional and anatomical changes that would require a non-linear and most likely stochastic modeling may be (a) metabolic changes within regions of brain networks, (b) vascular and myelin abnormalities in white matter connections, (c) abnormalities in other brain regions with which they interact⁵, (d) alterations in brain cells such as neuroglial cells⁶, (e) changes in brain volume and neurotransmission, (e) network alterations through influences by agent-independent connections between environment and observer, (f) functional compensation. A characteristic example is the aging brain where dopaminergic receptors decline, structures volumetrically shrink^{7,8,9}) and white matter becomes less dense^{10,11} which points to a less efficient information transmission system. However, the brain continuously engages in functional reorganization and functional repair for self-generated support¹² and to meet extrinsically imposed as well as intrinsic biochemical and cognitive challenge.

The brain may be thought of as a biochemical and bioelectric system with neuronal chemical, and electrical discharges that form the substrate for the encoding and processing of neuronal network information that facilitates sensory as well as motor events. The brain coordinates information crucial to genetic, chemical, and physiological processes largely of a nonlocal “entanglement” character that is likely to be operating in a non-algorithmic way. Therefore, it may be useful as a first approach to describe the coding and processing of information in the brain in terms of a simple parameter. perhaps a single number, a “determinant” of the “matrix” of electrophysiological and biochemical processes, that form a substrate for semantic processing of neuronal information. It seems natural that such a numerical description of this hyper-complex system should include the memory of previous configurations of the system as a dynamic parameter, and from the physics point of view it may be thought of as some kind of an action principle. A selection of attributes describing the brain dynamics may lead to practical and applicable conclusions, however the brain as a collection of neurons and other cells may require a practically intractable amount of degrees of freedom to fully describe its different states, and a global functional numerical measure may be useful as a binding bio index of its state.

Understanding information encoding and neuronal processing requires study of correlations between neurons. How the populations of neurons encode information and control human behavior is major point of interest of today's neuroscience. Neurons respond with variable strength to stimulation (Tolhurst et al., 1983) and (Shadlen, and Newsome 1998). This variability may be shared among different neurons, indicating a form of correlation. This responsiveness can result to a substantial effect on the amount of information encoded by a neuronal population. (Averbeck, 2006). In computational and cognitive neuroscience, it is important to determine how correlations are affected by stimulus drive, experience, learning or various changes in behavioral context, as well as how human brain connectivity reflects higher level network organization of the human brain (Sporns et al., 2005). Neuronal interconnections cannot be directly observed, and therefore the construction of brain networks is usually an inference problem. Furthermore, there are various different approaches for the construction of brain networks. These methods are usually based upon image modality and the type of connectivity. The connectome to be the comprehensive map of all these connections Sporns et al. (2005). In a recent paper by Prasad et al. (2013) the authors present a method for studying brain connectivity by simulating a dynamical evolution of the nodes of the network (Prasad et al. 2013). The nodes are treated as particles which are evolved under a simulated force analogous to the gravitational acceleration in a well-known N -body problem, where the particle nodes correspond to regions of the cortex, and where the locations of particles are defined as the centers of respective regions on the cortex and their masses are proportional to each region's volume. Furthermore, the attractive force is modeled on the gravitational force, and is explicitly made proportional to the elements of the connectivity matrix derived from imaging data. The authors also present experimental results of simulations on a population of 110 subjects from the Alzheimer's Disease Neuroimaging Initiative (ADNI), that consists of healthy elderly controls, early mild cognitively impaired (eMCI), late MCI (LMCI), and Alzheimer's disease(AD) patients. In healthy controls, the results demonstrate a significant difference in the dynamical properties of connectivity networks when compared to eMCI as well as AD patients. Inspired by the idea that nodes can be treated as particle in an N -body scenario as in (Prasad et al. 2013), where the particle nodes correspond to regions of the cortex, we attempt to calculate neuronal correlation, by implementing the ideas of thermodynamic galaxy clustering theory. In particular, galactic clustering theory predicts that a two-particle correlation function contains much

information about large scale clustering, for it evolves self consistently with all the higher order correlation functions (Saslaw 1987). The problem is to extract information about higher order correlations from a two-particle function without having to solve the BBGKY (Bogoliubov–Born–Green–Kirkwood–Yvon) hierarchy, that sometimes is called Bogoliubov hierarchy (Saslaw 1987) (This is a system of coupled equations describing the dynamics of interacting particles where a particle distribution function involves with $n+1$ particle distribution function). Assuming that the neurons are non extended structures (REF) the entropy of such a system can be written in terms of the temperature of the system T and the correlation parameter b . Finally, considering that brain has measurable physical parameters (such as energy and radius in a spherical approximation), we will use Bekenstein entropy bound to calculate the upper entropy limit of the brain. Thus, equating the two entropies (Bekenstein and N -Body) we calculate the neuronal correlation parameter b , expressed as a function of the brain parameters, thus providing possible a numerical correlation measure of the entire brain network activity, in the form of a single number.

Galactic Cluster Entropy and Bekenstein Bound

Thermodynamics and statistical mechanics is the tool for the description of the entropy of a system. The entropy of a system is a non conserved state function that is a very important in science and scientific research. Following (Saslaw 1987) we can write the entropy of an N -body system to be:

$$S = S_0 - (3b + T^{3/2} \ln b) \quad (1)$$

where b is the correlation parameter between two particles in the given system, and T is the temperature of the system. Because our system of interest (the brain) has finite dimensions, we use the Bekenstein bound in the estimation of entropy S . This is an upper limit of the entropy S , or information N , contained within a given finite region of space, of finite amount of energy content E that corresponds to a total amount of mass m . The Bekenstein bound can also be thought as the maximum amount of information required to completely describe a given physical system down to the quantum level. Therefore in relation to the brain the Bekenstein bound relation for the entropy S can be written as:

$$S_B \leq \frac{2\pi k_B RE}{\hbar c} \leq \frac{2\pi k_B R_b m_b c}{\hbar} \quad (2)$$

where S is the entropy, k_B is Boltzmann's constant, R is the radius of a sphere that can enclose the given system, $E = mc^2$ is the total mass–energy including any rest masses, which in the case of the brain is equal to $E_b = m_b c^2$, \hbar is the reduced Planck constant, and c is the speed of light. Note that while gravity plays a significant role in its enforcement, the expression for the bound does not contain the gravitational constant G .

To clarify our approach, we simply state that in this paper we will treat neurons as an N -body particle scenario in which clustering capabilities are possible. Thus we can say that correlation coefficient b , measures the influence of the "gravitational correlation energy W " between neurons. In an N -body scenario the correlation coefficient b depends in principle on the form of the two-particle correlation function ξ as it is given in Saslaw (1987). Moreover, and in relation to neurons we can say that the correlation coefficient b is a parameter that contains information about their clustering on all scales through the dependence of ξ . If we treat the brain as theoretically infinite (involving various length scales and large brain size relative to the neurons), as well as a thermodynamic homogeneous system, the only characteristic length scale that enters the potential energy is the average neuron separation given by the relation $\bar{r} = n^{-1/3}$. So b should depend just on the ratio of the potential and kinetic energies of two typical neurons. Our main assumption is that b is the same for each level of any possible clustering hierarchy on scales larger than the two-particle correlation at that level (Our understanding is that this implies more randomness in speed and direction of information transmission among clusters of low hierarchies. And b may be a binding parameter that may provide a link between neurophysiology and cognition.). We also assume that the entropy of the neuron system in the brain is in equilibrium and does not depend on the path via which the system reaches it's state. Equating equations (1) and (2) and solving for the neuron correlation coefficient b we find that:

$$b = \frac{1}{3} T_b^{3/2} W \left[\frac{3}{T_b^{3/2}} e^{-\frac{2\pi k_B c R_b m_b}{\hbar T_b^{3/2}}} \right]. \quad (3)$$

where W is the Lambert function of the indicated argument. So we have obtained an expression for the correlation coefficient between two neurons in an N -body scenario equating the upper entropy limit as it is calculated via the Bekenstein relation to that predicted by N -body correlation scenario as given in Shaslaw (1987) and also (Iqbal et al. 2011). We have found that the correlation coefficient b can be written as the Lambert function of three measurable brain

parameters indicated namely: temperature T_b , radius R_b and mass m_b . Similarly, we calculate that the rate of change of the correlation coefficient w.r.t the brain temperature T_b , mass m_b and radius R_b , are given by:

$$\frac{db}{dT_b} = \frac{T_b^{3/2}W(\Phi) + \frac{2}{\hbar} \left[\pi k_B c m_b R_b - \hbar T_b^{3/2} + \frac{(\hbar T_b^{3/2} - \pi k_B m_b R_b)}{1+W(\Phi)} \right]}{2T_b}, \quad (4)$$

$$\frac{db}{dm_b} = -\frac{2\pi k_B R_b \left[1 - \frac{1}{1+W(\Phi)} \right]}{3\hbar}, \quad (5)$$

$$\frac{db}{dR_b} = -\frac{2\pi k_B m_b \left[1 - \frac{1}{1+W(\Phi)} \right]}{3\hbar}, \quad (6)$$

and where Φ is given by:

$$\Phi = e^{-\frac{2\pi k_B c R_b m_b}{\hbar T_b^{3/2}}}. \quad (7)$$

Discussion and Numerical Results

Before we numerically evaluate our results let us look at the at the exponential term within the Lambert function $W(\Phi)$. The exponential involves the terms:

$$e^{-\frac{2\pi k_B c R_b m_b}{\hbar T_b^{3/2}}}. \quad (8)$$

Looking at the numerator, the term $cm_b R_b$ has units of $\text{kg m}^2 \text{ s}^{-1} = \text{J.s}$. Therefore, we conclude that this term represents some form of action, that reads:

$$\hbar_b = cm_b R_b, \quad (9)$$

as a *brain quantum of action* \hbar_b . Using the following values for the mass of a male and female brains i.e. $m_b = 1.3$ kg, female $m_b = 1.5$ kg, (Shoshani, 2006) to a spherical approximation and using a volume brain $V_b = 1350 \text{ cm}^3$ (Cosgrove, et al., 2007) we find that $R_b = 0.0686$ m. Substituting in Eq. (13) we obtain a first estimate for the brain quantum of action \hbar_n to be:

$$\hbar_{b_{\text{female}}} = cm_b R_b = 2.675 \times 10^7 \text{ Js}, \quad (10)$$

$$\hbar_{b_{\text{male}}} = cm_b R_b = 3.087 \times 10^7 \text{ Js}. \quad (11)$$

Therefore Eq. (7) can be written as follows:

$$b = \frac{1}{3} T_b^{3/2} W \left[\frac{3}{T_b^{3/2}} e^{-\frac{2\pi k_B}{T_b^{3/2}} \left(\frac{\hbar_b}{\hbar} \right)} \right].$$

(12)

We now find that the neuron correlation coefficient b appears to be related to the brain temperature $T_b^{3/2}$ and the ratio of the brain defined action \hbar_b over \hbar i.e. the one defined by quantum mechanics. Numerically the correlation coefficient between neurons falls in the range $0 \leq b \leq 1$. If $b=1$ neuron sub-clusters formed are in virial equilibrium, and $b_n=0$ no correlation exists. As $\hbar_b \rightarrow \hbar$ then $b \rightarrow \alpha$ where α is a number less than 1. In the case where the correlation coefficient \hbar_b is equal to \hbar the correlation coefficient obtained is the limiting value given by the equation:

$$b_n = \frac{1}{3} T_b^{3/2} W \left[\frac{3}{T_b^{3/2}} e^{-\frac{2\pi k_B}{T_b^{3/2}}} \right]. \quad (13)$$

In this case the rate of the neuron correlation coefficient b w.r.t the brain temperature T_b

$$\frac{db_b}{dT_b} = \frac{2\pi k_B W(\Phi') + T_b^{3/2} W^2(\Phi')}{2T_b(1 + W(\Phi'))} \quad (14)$$

Where $\Phi' = e^{-\frac{2\pi k_B}{T_b^{3/2}}}$. Using equation (17) and a brain temperature $T_b = 36.9$ C (Wang et al., 2014) and converting it to an absolute temperature we find the following numerical value of the correlation coefficient to be $b = 0.999450$. In the table 1 below we give correlation coefficient values for a range of brain temperatures

Absolute Brain Temperature T_b [K]	Neuron Correlation Coefficient b_n
308.25	0.999446
308.65	0.999447
309.05	0.999448
309.15	0.999449
309.75	0.999450
310.05	0.999451
310.85	0.999453

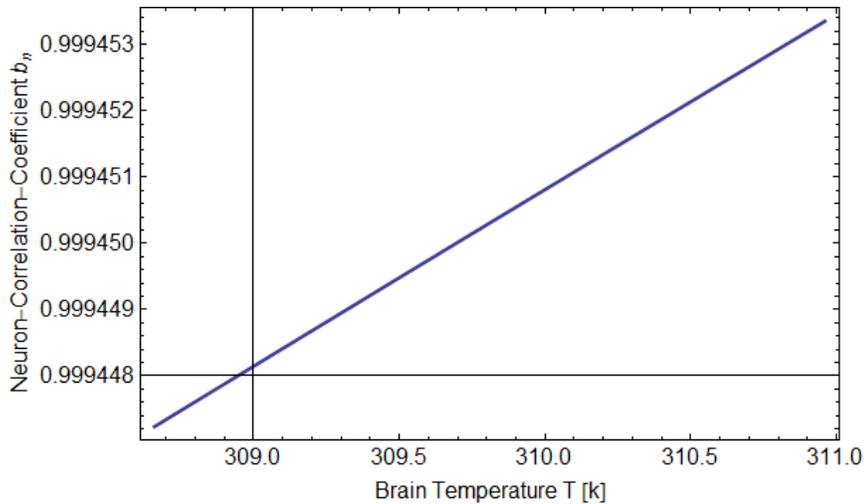


Fig. 1 Plot of brain correlation coefficient as a function of brain temperature in the case where the brain quantum of action is equal to the Planck's \hbar .

Conclusions

Network estimation through functional connectivity between network nodes is believed to have profound clinical implications, and the fusion of network analytical and statistical methods may revolutionize the understanding of brain function¹. Multiple network metrics have been used in

¹ Sean L. Simpson, F. DuBois Bowman, Paul J. Laurienti, Analyzing complex functional brain networks: fusing statistics and network science to understand the brain, *Statist. Surv.* Volume 7 (2013), 1-36. DOI: 10.1214/13-SS103

an effort to understand network structure mainly describing complex topologies, multiple variables of interest (disease status, age, race) and local network features (nodal clustering, nodal centrality, etc.). The application of statistical and network science tools for analyzing brain network data leads to the development of complexity theory. Network estimation proceeds through linear association measures (Sean et al., 2013) that include correlation and coherence, nonlinear measures including mutual information, generalized synchronization, functional segregation and integration is estimated by measures such as clustering coefficient and transitivity, characteristic path length, global efficiency, etc. However, the development of informative descriptive metrics and the resolution of computational issues due to dimensionality remain important problems requiring statistical input in network analysis, and propagation of error from network estimation may lead to divergence between the brain activity and statistical interpretation.

At the same time, computational neuroscience converges to an imitation of theoretical physics, to discover mathematical laws that capture the fundamental laws that govern the operation of neural systems² and to understand the brain to a similar degree as we now understand the material world. An ultimate goal of such an approach is to quantitatively predict complicated cognitive behaviours and provide insights into cognitive and affective, psychiatric and neurological disorders, network changes that may relate to early detection of pathophysiology. However, higher level brain functions involve processing of information by a variety of specialized brain areas. Simple elements and complex architectures used to investigate the richness of the brain, results in an incredible complexity in the modeling brain neuronal hardware due to neuronal phenomena such as neuromodulation, synaptic adaptation on various spatial and time scales, diversity of neuron types, and the role of glia cells, etc³. Therefore, the existing challenges may be either resolved or avoided by the use of simple methods. The highly complex structure of the brain involves many distinct neurotransmitters and receptors, cell types, and a variety of wiring patterns⁴, based on small-scale dynamics that may not be successfully mathematically modeled and should also not be ignored. Ideally, one would look for one

² Tsodyks M. Computational Neuroscience Grand Challenges - A Humble Attempt at Future Forecast. *Frontiers in Neuroscience*. 2008;2(1):17-18. doi:10.3389/neuro.01.021.2008.

³ same

⁴ John Beggs, *Phys. Rev. Lett.* 114, 220001 – Published 1 June 2015, Can There Be a Physics of the Brain? *Phys. Rev. Lett.* 114, 220001, <http://dx.doi.org/10.1103/PhysRevLett.114.220001>

variable, a control parameter that would govern the macroscopic phase of the system⁵. This seems to be compatible and in parallel with the anatomical and functional brain approach that large numbers of neurons collectively interact to produce emergent properties like cognition and consciousness. Our work is an effort to describe the dynamics of the human brain through a single parameter such as the entropy of the brain or the correlation parameter that encodes information about the brain as a dynamical system. The clinical significance of this theoretical approach would require an elaborate experimentation that would classify the different numerical values of these parameters with brain states and health conditions of human subjects. However, the approach in this report indicates a simple unifying binding principle of all scales of events in the brain that we conjecture that maybe a link between neurobiology and cognition

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