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Abstract

We consider a Yukawa-type gravitational potential combined with the Poynting-Robertson effect. Dust particles originating within the asteroid belt and moving on circular and elliptic trajectories are studied and expressions for the time rate of change of their orbital radii and semimajor axes, respectively, are obtained. These expressions are written in terms of basic particle parameters, namely their density and diameter. Then, they are applied to produce expressions for the time required by the dust particles to reach the orbit of Earth. For the Yukawa gravitational potential, dust particles of diameter 10^{-3} m in circular orbits require times of the order of 8.557×10^{6} y and for elliptic orbits of eccentricities e =0.1, 0.5 require times of 9.396×10^{6} and 2.129×10^{6} y respectively to reach Earth's orbit. Finally, various cases of the Yukawa potential are studied and the corresponding particle times to reach Earth's are derived per case along with numerical results for circular and various elliptical orbits.

Key words: celestial mechanics; orbital and rotational dynamics; classical field theories;

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1 Introduction

In today's scientific establishment, everybody will agree with the fact that the first known fundamental force in our universe was found to be the gravitation. In spite all that, gravitation now escapes all efforts to be included into the quantum frame work of a more integrated theory of the standard model, that describes the strong and weak interactions. To unify gravitation with all other interaction will be probably one of the greatest achievements of our physics era. On the other hand, general relativity that has been the most successful theory of gravity it is not in the position to explain, for example, the accelerating expansion of the universe, unless a cosmological constant or a dark energy fluid is considered into the scenario. In our days a variety of gravitational theories, such that quantum gravitational theories, $F(R)$ theories of gravity, scalar-tensor theory of gravitation etc, attempt to give an alternative explanation to this accelerating expansion (Piazza. and Marinoni, 2003). Most of these theories, although they use different approaches, in the weak-field limit all predict, a Yukawa-like gravitational potential. In a paper by Chan (2013) the authors put forward the idea that there is observational evidence of a cold dark matter particles interacting through a Yukawa potential could naturally explain the cores in dwarf galaxies therefore, the search for such deviation experimentally and observationally might shed some light and introduce, new physics (Adelberger et al. 2003). The Yukawa correction to the Newtonian gravitational potential can be written as:

$$V(r) = -\frac{GM}{r}\left(1+\alpha e^{-\frac{r}{\lambda}}\right) \tag{1}$$

Equation (1) gives the potential of a point mass m at a distance r, and G is the universal gravitational constant, λ is a constant called the range of the potential. If $\lambda \to \infty$, this potential tends to the Newtonian potential. Note that the parameter λ is the Compton wavelength of the exchange particle, which in the present case is a graviton, and α is the coupling constant of the new acting force. The graviton mass is related to λ through the well known equation $m_g = \hbar / \lambda c$ where $\hbar$ is the Planck constant and c is the speed of light.

In a recent paper by Mukherjee and Sounda (2017) the authors examine the orbit of a single particle moving under the influence of a Yukawa potential and studied the resulting precessing ellipses in relation to various values of the Yukawa coupling constant α. Similarly, in Haranas et al. (2016) the authors study the effect on the mean motion and period of a secondary body around a primary comprised of the sun itself as well as binary/pulsar systems. In celestial mechanics research papers, a Yukawa-type potential is very often proposed to modify the Newtonian potential (Iorio 2002, 2007, Brownstein and Moffat 2006; Haranas and Ragos 2010, Haranas et al. 2010), and its effects on various gravitational, astrophysical, and orbital scenarios are examined. In an older paper by D'Olivo and Ryan (1987) the authors investigate the inclusion of a Yukawa type of interaction in the framework of Newtonian cosmology. In particular when studying an infinite type of universe, they have found that the resulting field equations are similar to those of that general relativity predict a universe that is described by a Robertson-Walker metric. The force corresponding to the potential energy per unit mass given in Eq. (1) is equal to:

$$F = -\frac{\partial V}{\partial r} = -\frac{GMm}{r^2}\left[1+\alpha\left(1+\frac{r}{\lambda}\right)e^{\frac{r}{\lambda}}\right]. \tag{2}$$

In this contribution we deal with the orbits of spherical dust particles moving under the influence of a Yukawa-type potential and the Poynting-Robertson effect. The result of the radiation pressure is assumed to be negligible, since we consider particle of relatively large diameters, i.e. in the order of 10^{-2} to 10^{-3} m. For such particles moving on circular or elliptic orbits, we shall also examine the change of their radius or semimajor axis, respectively.

Section 1 is our introduction to the Yukawa potential in relation to recent gravitational theories. In section 2 we derive an expression for the Yukawa potential orbital energy using the virial theorem, we set up the Poynting - Robertson theory in a Yukawa potential, and we consider the case of circular orbits where expressions for the time rate of change of the orbital semimajor axis and the time that particles take to reach Earth's orbit are derived.

In section 3, we consider the elliptic orbits, and we examine the case $r > \lambda$. In section 4 we examine the case $\lambda > r$, and similar results to section 2 are derived in a first order perturbation treatment

Finally, in section 5, we examine the variable $\lambda(r)$ and $\alpha(r)$ cases and results similar to sections 3 and 4 are derived in first order perturbation treatment

In section 6 we calculate various numerical results using our derived formulae for circular and elliptic orbits, and for dust particles of diameters 10^{-2}, 10^{-3} and 10^{-4} m, of density ρ = 4000 kg m^{-3} respectively, under the influence of the Yukawa potential. Finally, section 7 contains our conclusions.

2 Virial theorem and the Yukawa potential orbital energy

Next, let us now write an expression of the orbital energy by making use of the virial theorem that states that for a bounded system in the unperturbed state using the following relation holds namely (Goldstein, 2002). This equation is only true for the standard Newtonian potential, and therefore we can write that:

$$\langle T \rangle = -\frac{\langle V \rangle}{2}, \tag{3}$$

where T is the time average on the system's kinetic energy, and V the time average of the system's potential energy. Therefore, conservation of energy implies that the total orbital energy is equal to (ibid, 2002):

$$E = \langle T \rangle + \langle V \rangle = \frac{\langle V \rangle}{2} \quad . \tag{4}$$

Anticipating small perturbations, we can add the average value of the perturbing term to the average of the Newtonian potential using the fact that the semimajor axis a is the average value of the radial distance r along the orbit, we can further write that the following expression for the Yukawa orbital energy of the dust particles to be:

$$E = -\frac{GMm}{2r}\left(1 + \alpha e^{-r/\lambda}\right). \tag{5}$$

At this point we must say that equation (5) is an approximate expression for the total energy of the dust particles under the influence of a Newtonian potential corrected by a Yukawa term. To start our treatment, we will present the basic theory in relation to Poynting-Robertson effect following Stacey (1977). For that we assume during a time interval dt

the particle receives an amount $d\varepsilon$ of energy from the solar radiation. This energy increases the particle's mass by an amount $dm = d\varepsilon / c^2$, where c is the speed of light. A particle that is travelling radially outward from the Sun, it will not carry any orbital angular momentum. Therefore, the angular momentum of the particle is conserved. Next, the particle reradiates the absorbed energy $d\varepsilon$ isotropically in its own reference frame, but this does not imply any reaction on the particle. Therefore, the orbital velocity of the particle is conserved in the radiation process. However, when the absorbed energy is emitted, the mass dm is lost. Thus, a resulting modification $v \cdot r \cdot dm$ of the angular momentum occurs**.** This angular momentum is carried away by the reemitted radiation and appears to be Doppler-shifted when viewed in the stationary reference frame of the Sun. The energy and momentum emitted forward from the particle exceed the energy and momentum radiated backward.

The time rate of loss of the orbital angular momentum J can be equated to the torque T, and therefore, we can write the following equation:

$$T = \frac{\mathrm{d}J}{\mathrm{d}t} = -vr\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{vr}{c^2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}, \tag{6}$$

so that:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{Tv}{r} = -\frac{v^2}{c^2}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}, \tag{7}$$

The time rate of change of the total energy dE/dt can be calculated to be:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{GMm}{2r^2}\left[1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}\right]\frac{\mathrm{d}r}{\mathrm{d}t}. \tag{8}$$

Furthermore, the time rate $\mathrm{d}\varepsilon/\mathrm{d}t$ at which the particle receives the solar radiation from the sun is given by:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \left(\frac{r_E^2}{r^2}\right)AS_s, \tag{9}$$

where A is the cross-sectional area of the particle, S_s is the flux of the radiation and r_E is the mean Earth-Sun distance (1 AU). At this distance, $S_S \approx 1367\ \mathrm{Wm}^{-2}$ (Montenbruck and Gill 2000). By differentiating Eq. (5) w.r.t. to the time t and using Eqs (7) and (9), we obtain the following differential equation for the time rate of change of the radius of the particle's orbit:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{2AS_s r_E^2}{c^2 mr}\left[1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}\right]^{-1}. \tag{10}$$

For circular orbits $r = a$ where a is the semimajor axis of the particle orbit. For the solar system we can safely assume that $r < \lambda$ (Talmage et al. 1998) and also (Moffat 2006). Approximating $e^{-r/\lambda} \cong 1 - r/\lambda + \ldots$ and then power series expand the result to first order we obtain the following differential equation for the time rate of change of the semimajor axis a to be:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{2AS_s r_E^2}{c^2 mr(1+\alpha)}\left[1+\frac{\alpha}{\lambda^2(1+\alpha)}a^2\right] \quad . \tag{11}$$

Solving Eq. (1) subjected to the initial condition that $a(0) = a_0$ we obtain the following solutions:

$$a(t) = +\left[e^{-Q_0}\left(a_0^2 + \frac{(1+\alpha)\lambda^2}{\alpha}\right) - \frac{(1+\alpha)\lambda^2}{\alpha}\right]^{1/2}, \tag{12}$$

$$a(t) = -\left[e^{-Q_0}\left(a_0^2 + \frac{(1+\alpha)\lambda^2}{\alpha}\right) - \frac{(1+\alpha)\lambda^2}{\alpha}\right]^{1/2}, \tag{13}$$

where:

$$Q_0 = -\frac{4ASr_E^2\alpha}{c^2 m(1+\alpha)\lambda^2}t\,. \tag{14}$$

Expressing the mass and cross-sectional area of the particle in terms of its diameter *d* and density ρ via the relations $m = \frac{4\pi\rho}{3}\left(\frac{d}{2}\right)^3$ and $A = \frac{\pi}{4}d^2$ we have obtained that:

$$a(t) = +\frac{\lambda(1+\alpha)}{\alpha}\left[\left(a_0^2 + \frac{(1+\alpha)\lambda^2}{\alpha}\right)e^{F_1} - \frac{(1+\alpha)\lambda^2}{\alpha}\right]^{1/2}, \tag{15}$$

$$a(t) = -\frac{\lambda(1+\alpha)}{\alpha}\left[\left(a_0^2 + \frac{(1+\alpha)\lambda^2}{\alpha}\right)e^{F_1} - \frac{(1+\alpha)\lambda^2}{\alpha}\right]^{1/2}, \tag{16}$$

where:

$$F_1 = -\frac{6\alpha Sr_E^2}{c^2\rho d\lambda^2(1+\alpha)}t\,. \tag{17}$$

Eq. (15) and (16) can be used to derive the time that a particle, originally moving on an orbit of radius a_0 in the solar system takes to reach the Earth's orbit, i.e. the time elapsed until the dust particle starts orbiting at a distance $a = a_E$ from the Sun, where a_E denotes the semimajor axis of the Earth's orbit. Solving equations (15) and (16) for t, we obtain that the following only real root:

$$t_{E_{yuk}} = \frac{c^2 \rho d \lambda^2}{6 r_E^2 S}\left(2+\alpha+\frac{1}{\alpha}\right)\ln\left[\frac{\left(1+\frac{\alpha}{(1+\alpha)}\left(\frac{a_0}{\lambda}\right)^2\right)}{\left(1+\frac{\alpha}{(1+\alpha)}\left(\frac{a_E}{\lambda}\right)^2\right)}\right], \tag{18}$$

3 Particles moving in elliptic orbits $r > \lambda$ case

Let us now consider a particle moving on an elliptic orbit under the influence of the Yukawa potential. Then, if this potential is calculated on the Keplerian ellipse, we can use the well-known relation:

$$r = \frac{a\left(1-e^2\right)}{(1+e\cos f)}, \tag{19}$$

where a is the semimajor axis, e is the eccentricity and f is the true anomaly of the orbit. Next, write the total energy function i.e. Eq. (5) in the following form:

$$E = -\frac{GMma\left(1-e^2\right)}{(1+e\cos f)}\left[1+\alpha e^{-\frac{a\left(1-e^2\right)}{\lambda(1+e\cos f)}}\right], \tag{20}$$

where the quantity $e^{-\frac{a\left(1-e^2\right)}{\lambda(1+e\cos f)}}$ is the exponential of $e^{-r/\lambda}$ and anywhere else indicates the eccentricity of the orbit. In taking the time derivative of Eq. (20) we assume that the orbital elements a, e, f are in general functions of time t, and therefore we obtain:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{\mathrm{d}E}{\mathrm{d}e}\frac{\mathrm{d}e}{\mathrm{d}t} + \frac{\mathrm{d}E}{\mathrm{d}f}\frac{\mathrm{d}f}{\mathrm{d}t} \quad , \tag{21}$$

and therefore, we obtain:

$$\frac{\mathrm{d}E}{\mathrm{d}t}=-\frac{GMm(1+e\cos f)}{a^2(1-e^2)}\left[1-\alpha e^{-\frac{a(1-e^2)}{\lambda(1+e\cos f)}}\left(1+\frac{a(1-e^2)}{\lambda(1+e\cos f)}\right)\right]\frac{\mathrm{d}a}{\mathrm{d}t}$$

$$-\frac{GMm(2e+(1+e^2)\cos f)}{a(1-e^2)^2}\left[1+\frac{\alpha e^{-\frac{a(1-e^2)}{\lambda(1+e\cos f)}}}{e}\left(1+\frac{a(1-e^2)}{\lambda(1+e\cos f)}\right)\right]\frac{\mathrm{d}e}{\mathrm{d}t} \qquad (22)$$

$$+\frac{GMme}{\lambda(1-e^2)}\left[1+\alpha e^{-\frac{a(1-e^2)}{\lambda(1+e\cos f)}}\left(1+\frac{a(1-e^2)}{\lambda(1+e\cos f)}\right)\right]\frac{\mathrm{d}f}{\mathrm{d}t}.$$

Next, let us consider the following cases: Case where $r > \lambda$. In this case Eq. (22) becomes:

$$\frac{\mathrm{d}E}{\mathrm{d}t}=-\frac{GMm(1+e\cos f)}{a^2(1-e^2)}\frac{\mathrm{d}a}{\mathrm{d}t}-\left(\frac{GMm\cos f}{2a(1-e^2)^2}+\frac{GMme(1+e\cos f)}{a(1-e^2)^2}\right)\frac{\mathrm{d}e}{\mathrm{d}t}+\frac{GMme\sin f}{2a(1-e^2)}\frac{\mathrm{d}f}{\mathrm{d}t}. \qquad (23)$$

Next using Eqs. (6) and (9) equating with Eq. (23) and solving for $\dot{a}(t)$ we obtain the following equation:

$$\frac{\mathrm{d}a}{\mathrm{d}t}=-\frac{2ASr_E^2\left(1+e^2+4e\cos f+(5e^2+e^4)\cos^2 f+2e^3\cos f\left(1+\cos^2 f\right)\right)}{mc^2a(1-e^2)^2}$$
$$+\left(\frac{-ea\sin f+2e^3a\sin f-e^5a\sin f}{(1-e^2)^2(1+e\cos f)}\right)\frac{\mathrm{d}f}{\mathrm{d}t}+\left(\frac{a(1-e^2)\cos f+2ae(1-e^2)}{(1-e^2)^2(1+e\cos f)}\right)\frac{\mathrm{d}e}{\mathrm{d}t} \qquad (24)$$

Expressing the mass and cross-sectional area of the particle in terms of its diameter d and density ρ via the relations $m=\frac{4\pi\rho}{3}\left(\frac{d}{2}\right)^3$ and $A=\frac{\pi}{4}d^2$ we have that equation (24) takes the form:

$$\frac{\mathrm{d}a}{\mathrm{d}t}=-\frac{Sr_E^2}{c^2ad(1-e^2)^2(1+e\cos f)}\left[\begin{array}{l}\left(3+12e\left(1+\frac{e^2}{2}\right)\cos f+3e^2\left(1+5\cos^2 f\right)+6e^3\cos f\left(1+\cos^2 f\right)+e^4\cos^2 f\right)\\ +a\left(e(1-2e^2)\sin f+e^5\sin f\right)\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)+a\left(2e^3-2e-\cos f\left(1-e^4\right)\right)\left(\frac{\mathrm{d}e}{\mathrm{d}t}\right)\end{array}\right].$$

(25)

According to Wyatt and Whipple (1950) we find that the time rate of the eccentricity due to Poynting-Robertson effect is given by:

$$\frac{\mathrm{d}e}{\mathrm{d}t}=-\frac{15L_se}{32\pi c^2d\rho a^2\sqrt{1-e^2}}, \qquad (26)$$

where $L_s = 3.826 \times 10^{26}$ W is the luminosity of the Sun (Phillips 1999). Also, the true anomaly f can be expressed as a series of. t (see, e.g., Murray and Dermott, 1999):

$$f \cong nt + 2e\sin(nt) + \frac{5e^2}{4}\sin(2nt) + ..., \tag{27}$$

(see, e.g., Murray and Dermott, 1999), where n is the mean motion of the particle. As before, we approximating this series up to $O(e^2)$. Then, df/dt can be easily evaluated to be:

$$\frac{\mathrm{d}f}{\mathrm{d}t} \cong n + 2ne\cos(nt) + \frac{5ne^2}{2}\cos(2nt) + ... \tag{28}$$

Substituting in Eq.(27) and (28) equation (25) and applying first order perturbation solution $t = t_0 = 0$ during which $a(0)=a_0$, $n(0) = n_0$ and $e(0) = e_0$ as well as $\dot{e}(0) = e_0 = 0$ since the effect on eccentricity has not yet started at $t = t_0 = 0$, we obtain the following first order differential equation :

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{3Sr_E^2\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)\left(1+e_0^2+2e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)}{c^2\rho d a_0\left(1-e_0^2\right)^2} - \frac{n_0e_0a_0(1+2e_0)\sin\left(\frac{5n_0e_0^2}{2}\right)}{\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)}. \tag{29}$$

Solving Eq. (29) with the initial condition that $a(0)=a_0$ we obtain:

$$a(t) = a_0 - \left[\frac{3Sr_E^2\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)\left(1+e_0^2+2e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)}{c^2\rho d a_0\left(1-e_0^2\right)^2} - \frac{n_0e_0a_0(1+2e_0)\sin\left(\frac{5n_0e_0^2}{2}\right)}{\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)}\right]t\,. \tag{30}$$

Next solving for the time t_E for the particles to reach Earth's orbit by equating $a(t) = r_E = a_E$ we obtain:

$$t_E = \frac{c^2 d\rho a_0\left(1-e_0^2\right)^2(a_0-a_E)\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)}{6Sa_E\left(1+e_0^2+2e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)\left(1+e_0\cos\left(\frac{5n_0e_0^2}{2}\right)\right)+2c^2dn_0e_0\rho a_0^2(1+2e_0)\left(1-e_0^2\right)^2\sin\left(\frac{5n_0e_0^2}{2}\right)}. \tag{31}$$

4. The $\lambda > r$ case

Next let us look the case where $\lambda > r$. In this case we can approximate the orbital energy to first order in the following way:

$$E=-\frac{GMm\left(1+e_0\cos f\right)}{2a\left(1-e_0^2\right)}\left(1+\alpha\left(1-\frac{a\left(1-e_0^2\right)}{\lambda\left(1+e_0\cos f\right)}\right)\right). \tag{32}$$

Differentiating w.r.t time we obtain the equation:

$$\frac{\mathrm{d}E}{\mathrm{d}t}=A_0\frac{\mathrm{d}a}{\mathrm{d}t}+B_0\frac{\mathrm{d}f}{\mathrm{d}t}+C_0\frac{\mathrm{d}e}{\mathrm{d}t} \tag{33}$$

$$A_0=\frac{GMm\alpha}{2\lambda a}+\frac{GMme\left(1+e\cos f\right)\left(1+\alpha\left(1-\frac{a\left(1-e^2\right)}{\lambda\left(1+e\cos f\right)}\right)\right)}{2a^2\left(1-e^2\right)}, \tag{34}$$

$$B_0=\frac{GMm\alpha e\sin f}{2\lambda\left(1+e\cos f\right)}+\frac{GMme\left(1+\alpha\left(1-\frac{a\left(1-e^2\right)}{\lambda\left(1+e\cos f\right)}\right)\right)\sin f}{2a\left(1-e^2\right)}, \tag{35}$$

$$\begin{aligned}C_0=&-\frac{GMm\alpha\cos f}{2\lambda\left(1+e_0\cos f\right)}-\frac{GMm\alpha e}{\lambda\left(1-e_0^2\right)}-\frac{GMm\left(1+e_0\cos f\right)}{a\left(1-e_0^2\right)^2}\left(1+\alpha\left(1-\frac{a\left(1-e^2\right)}{\lambda\left(1+e\cos f\right)}\right)\right)\\&-\frac{GMm\cos f}{2a\left(1-e_0^2\right)^2}\left(1+\alpha\left(1-\frac{a\left(1-e^2\right)}{\lambda\left(1+e\cos f\right)}\right)\right).\end{aligned} \tag{36}$$

Substituting again for f, and $\dot{f}$ and again using a first order perturbation treatment we assume we obtain the following first order differential equation:

$$\frac{\mathrm{d}a}{\mathrm{d}t}=-\frac{\pi d^2\left(1+e_0\right)^4a_E^2S}{4c^2a_0^3\left(1-e_0^2\right)^3\left[\frac{\pi\alpha\rho d^3}{12a_0\lambda}+\frac{\pi d^3\rho\left(1+e_0\right)}{12a_0^2\left(1-e_0^2\right)}\left(1+\alpha\left(1-\frac{a_0\left(1-e_0^2\right)}{\lambda\left(1+e_0\right)}\right)\right)\right]}, \tag{37}$$

which simplifies to

$$\frac{\mathrm{d}a}{\mathrm{d}t}=-\frac{3\left(1+e_0\right)Sr_E^2}{c^2a_0d\rho\left(1-e_0\right)^2\left(1+\alpha\right)}, \tag{38}$$

and has a first order solution of the form:

$$a(t)=a_0-\frac{3\left(1+e_0\right)r_E^2S}{c^2d\rho a_0\left(1-e_0\right)^2\left(1+\alpha\right)}t\,. \tag{39}$$

Equating Eq. (40) to the earth distance $r_E = a_E$ we solve for the time t_E that the particles take to reach Earth distance starting from a_0 =2.7 AU. Solving we obtain:

$$t_E = \frac{c^2 d\rho a_0 (1-e_0)^2 (a_0 - a_E)(1+\alpha)}{3(1+e_0) r_E^2 S}. \tag{40}$$

5 The variable λ and α case

With reference to Brownstein and Moffat (2006), we adopt the following parametric representations of the "running" of $\alpha(r)$ and $\lambda(r)$ and we write down the following expressions of the variability of the range $\boldsymbol{\lambda}$ as well as the coupling constant $\boldsymbol{\alpha}$ to be:

$$\alpha(r) = \alpha_\infty \left(1 - e^{-\left(\frac{r}{\bar{r}}\right)}\right)^{b/2} \tag{41}$$

$$\lambda(r) = \lambda_\infty \left(1 - e^{-\left(\frac{r}{\bar{r}}\right)}\right)^{-b} \tag{42}$$

where $\alpha_\infty = (1.00 \pm 0.02) \times 10^{-3}$, $\bar{r} = 4.6 \pm 0.2$ AU, λ_∞ = 47± 1 AU, $b = 4.0$ (Brownstein and Moffat 2006), and r represents the orbital distance away from the sun into our solar system. Therefore, we write Eqs. (41) and (42) in the following form:

$$\alpha(r) = \alpha_\infty \left(1 - e^{-\Gamma_0}\right)^2, \tag{43}$$

$$\lambda(r) = \lambda_\infty \left(1 - e^{-\Gamma_0}\right)^{-4}. \tag{44}$$

Considering only the $\lambda > r$, the equation for the total orbital energy becomes:

$$E = -\frac{GMm(1+e\cos f)}{2a\left(1-e^2\right)}\left[1 + \alpha_\infty \left(1 - e^{\Gamma_0}\right)^2 \left(1 - \frac{a\left(1-e^2\right)\left(1-e^{\Gamma_0}\right)^4}{\lambda_\infty (1+e\cos f)}\right)\right], \tag{45}$$

where

$$\Gamma_0 = \frac{a\left(1-e^2\right)}{\bar{r}(1+e\cos f)}. \tag{46}$$

Using Eqs. (7) and (9) equating with the time derivative of Eq. (46), solving for $\frac{\mathrm{d}a}{\mathrm{d}t}$ and substituting at t = 0 the initial values of the orbital elements as above we obtain the following first order equation:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{3(1+e_0)^4 S r_E^2}{c^2 a_0^3 d\rho (1-e_0)^2 \Lambda_0}, \tag{47}$$

$$\Lambda_0=\left(\begin{array}{l}\frac{(1+e_0)}{a_0^2(1-e_0^2)}\left(1+\alpha_\infty\left(e^{\frac{a_0(1-e_0)}{\bar r}}-1\right)^2\left(1-\frac{a_0(1-e_0)}{\lambda_\infty}\left(e^{\frac{a_0(1-e_0)}{\bar r}}-1\right)^4\right)\right)-\frac{\alpha_\infty}{\lambda_\infty a_0}\left(e^{\frac{a_0(1-e_0)}{\bar r}}-1\right)^6\\ -\frac{2\alpha_\infty e^{\frac{a_0(1-e_0)}{\bar r}}}{a_0\bar r}\left(1-e^{\frac{a_0(1-e_0)}{\bar r}}\right)\left(1-\frac{a_0(1-e_0)}{\lambda_\infty}\left(e^{\frac{a_0(1-e_0)}{\bar r}}+1\right)^4\right)-\frac{4\alpha_\infty(1-e_0)e^{\frac{a_0(1-e_0)}{\bar r}}}{\lambda_\infty\bar r}\left(e^{\frac{a_0(1-e_0)}{\bar r}}-1\right)^5\end{array}\right).$$

(48)

Equation (49) has the first order solution:

$$a(t)=a_0-\frac{3(1+e_0)\lambda_\infty e^{-\frac{a_0(6-7e_0)}{\bar r}}Sa_E^2\bar r}{c^2a_0(1-e_0)^2\Phi_0}t \tag{49}$$

$$\Phi_0=\left(\begin{array}{l}6\alpha_\infty a_0^2(1-e_0^2)^2e^{\frac{a_0e_0}{\bar r}}-30\alpha_\infty(1-e_0)^2a_0^2(1-e_0^2)e^{-\frac{a_0(1-2e_0)}{\bar r}}+60\alpha_\infty a_0^2(1-e_0^2)e^{-\frac{a_0(2-3e_0)}{\bar r}}-60\alpha_\infty a_0^2(1-e_0^2)e^{-\frac{a_0(2-4e_0)}{\bar r}}\\ +\lambda_\infty\bar r(1+\alpha_\infty)e^{-\frac{a_0(6-7e_0)}{\bar r}}-2\alpha_\infty e^{-\frac{a_0(5-6e_0)}{\bar r}}\left(a_0(1-e_0)(3(1-e_0)a_0+\lambda_\infty)+\lambda_\infty\bar r\right)\\ +\alpha_\infty e^{-\frac{a_0(4-5e_0)}{\bar r}}\left(2a_0(1-e_0)(15a_0(1-e_0)+\lambda_\infty)+\lambda_\infty\bar r\right)\end{array}\right).$$

(50)

Next, solving for the time t_E that the dust particles require to reach Earth we obtain:

$$t_E=\frac{c^2d\rho a_0(a_0-a_E)(1-e_0)^2\Phi_0}{3(1-e_0)a_E^2S\lambda_\infty\bar r}e^{\frac{(6-7e_0)}{\bar r}}. \tag{51}$$

Setting the eccentricity $e_0=0$ we obtain the time required for the dust particles in circular orbits to reach Earth's orbits therefore Eq. (51) becomes:

$$t_E=\frac{c^2d\rho a_0(a_0-a_E)\Phi_0'}{3r_E^2S\lambda_\infty\bar r}e^{\frac{6}{\bar r}}, \tag{52}$$

where Φ_0' is equal too:

$$\Phi_0'=\left(\begin{array}{l}6\alpha_\infty a_0^2-30\alpha_\infty a_0^2e^{-\frac{a_0}{\bar r}}+60\alpha_\infty a_0^2e^{-\frac{2a_0}{\bar r}}-60\alpha_\infty a_0^2e^{-\frac{2a_0}{\bar r}}+\lambda_\infty\bar r(1+\alpha_\infty)e^{-\frac{6a_0}{\bar r}}-2\alpha_\infty e^{-\frac{5a_0}{\bar r}}\left(a_0(3a_0+\lambda_\infty)+\lambda_\infty\bar r\right)\\ +\alpha_\infty e^{-\frac{4a_0}{\bar r}}\left(2a_0(15a_0+\lambda_\infty)+\lambda_\infty\bar r\right)\end{array}\right).$$

(53)

6 Numerical results and discussion: Yukawa circular and elliptical orbits

For all our calculations, we assume that a particle of diameter d originates in the asteroid belt at $a_0 = 2.7$ AU 4.05×10^{11} m, with density $\rho = 4000$ kg/m^3 (Stacey, 1997) and mean motion $n_0 = \left(GM_{sun}/r_0^3\right)^{1/2} = 4.4\times10^{-8}$ s^{-1}. At this distance away from the sun and using Eqs. (41) and (42) we calculate that $\lambda = 6.00324\times10^{13}$m and $\alpha = 0.00034269$. Let first consider circular orbits i.e. $e_0 = 0$. Thus, using equation (18) we write an equation for the time it takes to reach Earth's orbit as a function of its diameter only to be:

$$t_E = 8.760\times10^9 d \text{ [y]}. \tag{54}$$

Similarly, for circular orbits using Eq. (18) we derive an equation in terms of the particle's density ρ, diameter d, as well as the coupling constant α and the range λ of the Yukawa potential to be:

$$t_E = 4.877\times10^{-10}\rho d\lambda^2\left(2+\alpha+\frac{1}{\alpha}\right)\ln\left[\frac{1.640\times10^{22}\alpha+\lambda^2(1+\alpha)}{2.250\times10^{22}\alpha+\lambda^2(1+\alpha)}\right]\text{[y]}. \tag{55}$$

In table 1 and for circular orbits we tabulate the times to reach Earth for particles of various diameters d and density $\rho = 4000$ kg/m^3.

Table 1 Time to reach Earth's orbit for particles of various diameters in circular orbits of eccentricity $e = 0$

Particle diameter d [m]	Time to reach Earth's Orbit t_E [y]
10^{-4}	8.757×10^5
10^{-3}	8.757×10^6
10^{-2}	8.757×10^7

We can easily see that larger particles require longer times to reach Earth's orbit, where small ones will be blown away by the Poynting-Robertson effect and therefore shorter times are required. For constant density the final equation for the aforementioned time t_E for a particle of a given diameters d depends linearly on the particle's diameter and density ρ. For example, for the diameters $d = 10^{-4}$m, 10^{-3}m, 10^{-2} m used in table 1 we find that the time scales linearly with density according to the equations $t_E = 219.935\rho$, 2189.35ρ, $1.751\times10^7\rho$ respectively. Furthermore, we find that in circular orbits the time that dust particles require to reach Earth also depends on the Yukawa parameters α, λ. Therefore, for the diameters used in our calculation the time to reach Earth is within the range $8.760\times10^5 \le t_E \le 8.760\times10^7$ years. From Eq. (18) and in the case where $\alpha -> 0$ i.e. No Yukawa effect exists, the limit of equation 18 tends to the expression $t_{E_N} = c^2 d\rho \left(r_0^2 - r_E^2\right) / 6 r_E^2 S$. Next, we write Eq. (18) in the following form:

$$t_{Yuk_E} = t_{E_{Yuk_0}} \left(1 + \frac{\alpha}{2}\left(1 + \frac{1}{a^2}\right) \ln \left[\frac{1 + \frac{\alpha}{(1+\alpha)}\left(\frac{a_0}{\lambda}\right)^2}{1 + \frac{\alpha}{(1+\alpha)}\left(\frac{r_E}{\lambda}\right)^2} \right] \right). \tag{56}$$

where $t_{Yuk_{E_0}} = c^2 d\rho \lambda^2 / 3 r_E^2 S$ is a basic Yukawa time to reach Earth that scales as λ^2. In the special case where $a_0 = \lambda$, equation (56) takes the form:

$$t_{Yuk_E} = t_{E_{Yuk_0}} \left[1 + \frac{\alpha}{2}\left(1 + \frac{1}{\alpha^2}\right) \ln \left(\frac{\left(1 + \frac{\alpha}{(1+\alpha)}\right)}{\left(1 + \frac{\alpha}{(1+\alpha)}\left(\frac{r_E}{a_0}\right)^2\right)} \right) \right]. \tag{57}$$

In the limit where $\alpha -> 0$ i.e. Newtonian potential Eq. (57) becomes:

$$t_{Yuk_E} = \frac{c^2 d\rho}{6 r_E^2 S}\left(a_0^2 - r_E^2\right). \tag{58}$$

As a check we say that this is the same result that can be obtained from Stacey's result for circular orbits (Stacey, 1997) in a Newtonian potential. In figure 1 and for circular orbits we plot the time t_E that it takes for particles of density $\rho = 4000$ kg/m^3 (Stacey, 1997) to reach Earth's orbit. For circular orbits the numerical contribution of the Yukawa effect i.e. the large bracket in Eq. (56) has a numerical value that has an order of magnitude of approximately 3.930×10^{-5} which is 0.1147 times the Yukawa potential coupling constant α. The Yukawa effect has also an identical in order of magnitude contribution for the time to reach Earth's orbit in the case when $a_0 = \lambda$. Looking at Stacey's numerical equation (ibid 1997) for circular orbits we see that the author obtains that $t = 8.8 \times 10^9 d$ years. In our case considering the Yukawa correction to the potential we find that $t_{Yuk_E} / t_{Stacey} = 0.99516$. In figure 2 we plot the time for particles of two different densities and various diameters in circular orbits. Blue corresponds to density $\rho = 2500$ kg/m^3, red corresponds to $\rho = 3000$ kg/m^3, brown corresponds to $\rho = 3500$ kg/m^3 and green corresponds to $\rho = 4000$ kg/m^3.

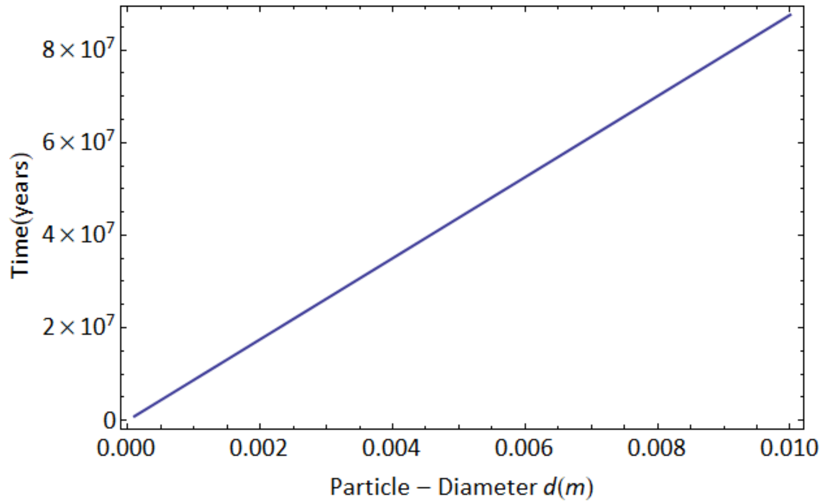

Fig. 1 Time to reach Earth's orbit for particles of density $\rho = 4000$ kg/m^3 and various diameters in circular orbits.

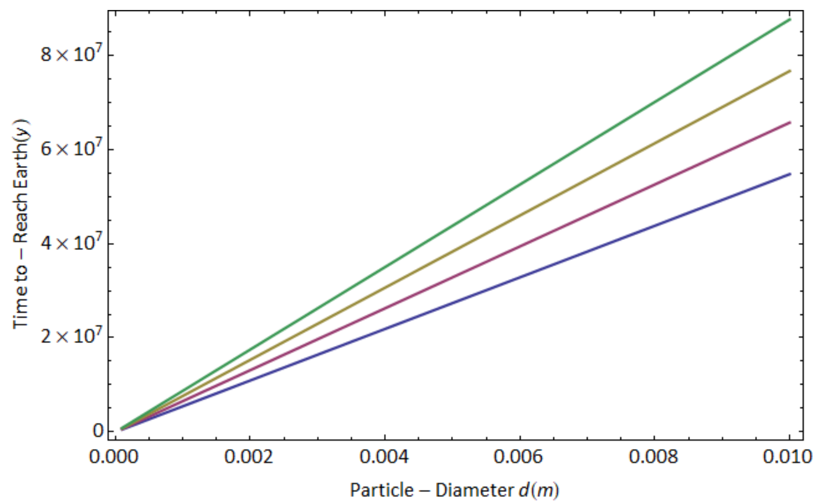

Fig. 2 Time to reach Earth for particles of two different densities and various diameters in circular orbits. Blue corresponds to density $\rho = 2500$ kg/m^3, red corresponds to $\rho = 3000$ kg/m^3, brown corresponds to $\rho = 3500$ kg/m^3 and green corresponds to $\rho = 4000$ kg/m^3.

In the case where $r > \lambda$ and for the same density, using the eccentricities $e_0 = 0.001,\ 0.01,\ 0.1,\ 0.2$ we write a first order solution for the time taken for particles of diameter d to reach Earth's orbit to be:

$$t_E = \frac{7.443 \times 10^{43} d}{\left(1.852 \times 10^{26} + 5.727 \times 10^{20} d\right)_{e_0 = 0.001}}, \tag{59}$$

$$t_E = \frac{7.508 \times 10^{43} d}{\left(1.920 \times 10^{26} + 5.830 \times 10^{23} d\right)_{e_0 = 0.01}}, \tag{60}$$

$$t_E = \frac{8.016\times10^{43}d}{\left(2.702\times10^{26}+6.723\times10^{26}d\right)_{e_0=0.1}}, \tag{61}$$

$$t_E = \frac{8.223\times10^{43}d}{\left(3.827\times10^{26}+5.900\times10^{27}d\right)_{e_0=0.2}}, \tag{62}$$

$$t_E = \frac{8.005\times10^{43}d}{\left(5.271\times10^{26}+2.045\times10^{28}d\right)_{e_0=0.3}}. \tag{63}$$

In this case to check the equation derived for the time taken to reach Earth's orbit i.e. eq. (31), we consider circular orbits and obtain the following equation in terms of all variables to be $t_E = \frac{c^2 a_0 \rho d\left(a_0 - r_E\right)}{r_E^2 S}$. This equation numerically agrees with that given in Stacey (1997). This would imply that Stacey's result can be obtained as a special case i.e. $r > \lambda$ of the Yukawa potential treatment. Next in table 2 we tabulate elliptical orbit results for particles of various diameters, density $\rho = 4000$ kg/m^3, and eccentricity e = 0.001.

Table 2 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity e =0.001

Particle diameter d [m]	Time to reach Earth's Orbit t [y]
10^{-4}	1.274×10^{6}
10^{-3}	1.274×10^{7}
10^{-2}	1.274×10^{8}

Similarly, in table 3 for particles of various diameters and the same density we tabulate elliptical orbits of eccentricity e = 0.1 and their times to reach Earth's orbit.

Table 3 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity e =0.1 case $r > \lambda$

Particle diameter d [m]	Time to reach Earth's Orbit t [y]
10^{-4}	9.406×10^{5}
10^{-3}	9.385×10^{6}
10^{-2}	9.180×10^{7}

Furthermore, in figure 3 we plot the time taken for particles of various diameters, same density to reach Earth's orbit in elliptical orbits of eccentricities e = 0.0001 and 0.01. Similarly, in figure 4 we plot the time for particles of various diameters and density $\rho = 4000$ kg/m^3 in elliptical orbits of eccentricities e = 0.1 to 0.5, and in figure 5 we only consider particles in elliptical orbits of eccentricities e = 0.6 to 0.9.

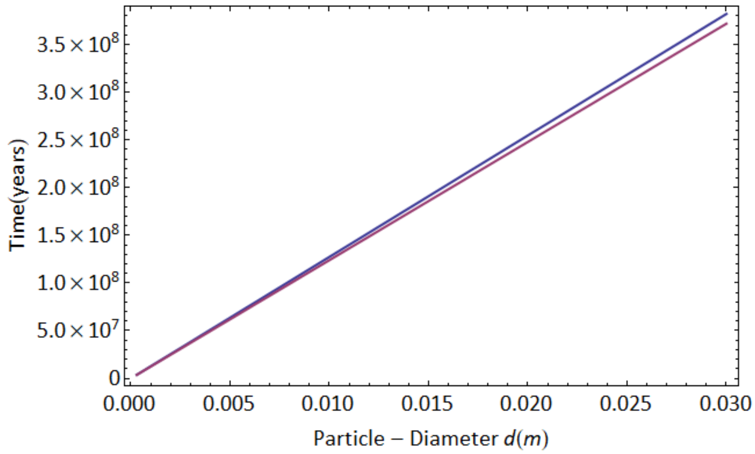

Fig. 3 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3.
and various diameters in elliptical orbits of eccentricities e_0 = blue (0.001), red (0.01).

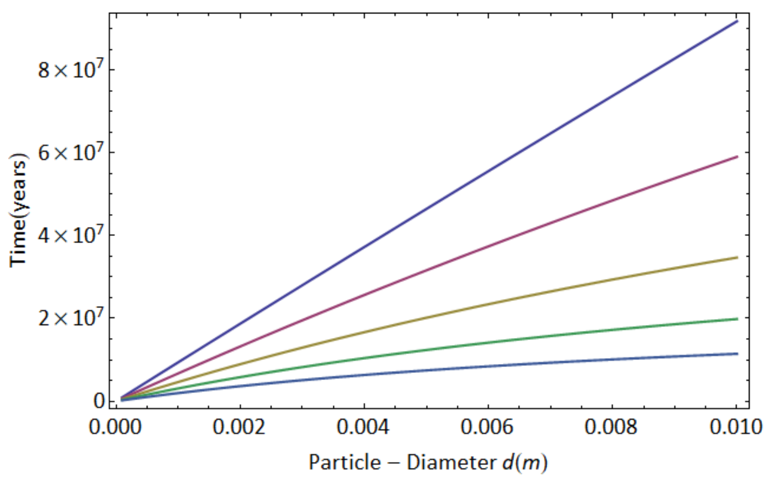

Fig. 4 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3.
and various diameters in elliptical orbits of eccentricities e_0 = blue (0.1), red(0.2), light brown (0.3), green(0.4), light blue (0.5).

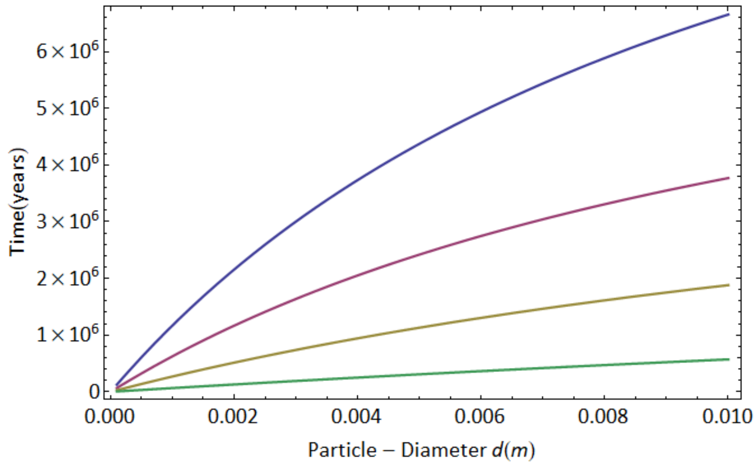

Fig. 5 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3. and various diameters in elliptical orbits of eccentricities e = blue (0.6), red (0.7), light brown(0.8), green(0.9).

Next for the case when $r < \lambda$ we derive numerical results. In tables 4 and 5, we tabulate the times to reach Earth for particles of various diameters but the same density and for elliptical orbits of eccentricities e = 0.001 and 0.1 We also derive the equations of the time in years in terms of the particle diameter for eccentricities e = 0.1 to 0.5 to be:

$$t_E = 9.411 \times 10^9 d \ . \qquad (64)$$

$$t_E = 6.816 \times 10^9 d \ , \qquad (65)$$

$$t_E = 4.817 \times 10^9 d \ , \qquad (66)$$

$$t_E = 3.286 \times 10^9 d \ , \qquad (67)$$

$$t_E = 2.130 \times 10^9 d \ . \qquad (68)$$

Furthermore, using Eq. (40) we derive the same equations as a function of the density, diameter the coupling constant α and the range λ of the Yukawa potential to be:

$$t_E = 2.352 \times 10^6 (1+\alpha)\rho d \quad , \qquad (69)$$

$$t_E = 1.703 \times 10^6 (1+\alpha)\rho d \quad , \qquad (70)$$

$$t_E = 1.204 \times 10^6 (1+\alpha) d\rho \quad , \qquad (71)$$

$$t_E = 8.213 \times 10^5 (1+\alpha) d\rho \quad , \qquad (72)$$

$$t_E = 5.324 \times 10^9 (1+\alpha) d\rho \quad . \tag{73}$$

In the case $r < \lambda$ for elliptical orbits the numerical contribution of the Yukawa effect is of the order of magnitude of $(1+\alpha)$ which approximately equal to 1.0003427 which is approximately 3081times the Yukawa potential coupling constant α. In the case where $\alpha = 0$ there is no Yukawa Poynting Robertson effect acting on the dust particles. Looking at equations (65) to (68) we find that in the case where the range of the Yukawa potential is larger than the orbital radius of the particle, the time to reach the Earth's orbit to a first order approximation depends only on the Yukawa potential coupling constant α and it's independent of the range lambda λ.

Table 4 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity e =0.001, case $r < \lambda$.

Particle diameter d [m]	Time to reach Earth's Orbit t [y]
10^{-4}	1.274×10^{6}
10^{-3}	1.274×10^{7}
10^{-2}	1.274×10^{8}

Table 5 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity e =0.1, case $r < \lambda$.

Particle diameter d [m]	Time to reach Earth's Orbit t [y]
10^{-4}	9.411×10^{5}
10^{-3}	9.411×10^{6}
10^{-2}	9.411×10^{7}

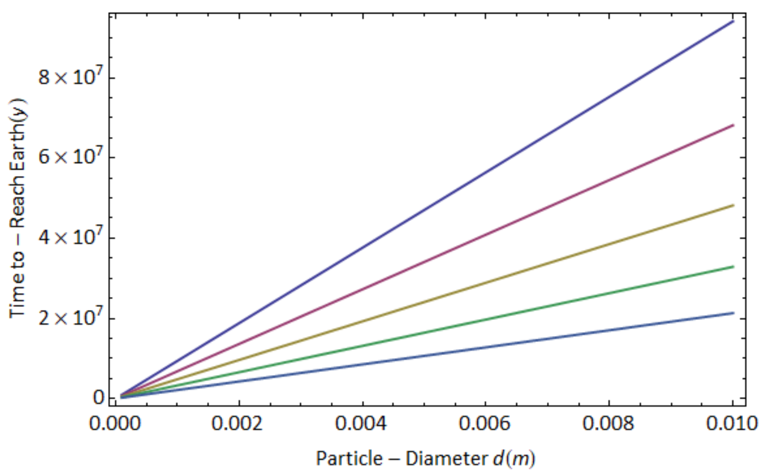

Fig. 5 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3. and various diameters in elliptical orbits of eccentricities e = blue (0.1), red (0.2), light brown(0.3), green(0.4), light blue(0.5).

Finally, we examine the case where λ and α are varying with the distance *r* (Brownstein and Moffat, 2006). First, we consider particles with eccentricities *e* = 0.001, 0.01, and 0.1 and then we derive the time to reach Earth's orbit as a function of the particle's density, and therefore we have:

$$t_E = 1.272 \times 10^{10} d \,, \tag{74}$$

$$t_E = 1.238 \times 10^{10} d \,, \tag{75}$$

$$t_E = 9.396 \times 10^{9} d \,. \tag{76}$$

Next we derive a equations (74) - (76) in terms of the basic dust parameter namely $d, \rho,$ as well as the Yukawa parameters $\alpha_\infty, \lambda_\infty$ to be:

$$t_E = 3.184 \times 10^6 d\rho - 4.922 \times 10^6 d\rho\alpha_\infty + \frac{7.452 \times 10^{18}}{\lambda_\infty} d\rho\alpha_\infty \text{ [y]}, \tag{77}$$

$$t_E = 3.099 \times 10^6 d\rho - 6.654 \times 10^6 d\rho\alpha_\infty + \frac{6.656 \times 10^{18}}{\lambda_\infty} d\rho\alpha_\infty \text{ [y]}. \tag{78}$$

$$t_E = 2.352 \times 10^6 d\rho - 2.618 \times 10^6 d\rho\alpha_\infty + \frac{2.069 \times 10^{18}}{\lambda_\infty} d\rho\alpha_\infty \text{ [y]} \tag{79}$$

Again in equations (77) - (79) if $\alpha = 0$ there is no Yukawa Poynting Robertson effect acting on the dust particles. In the same equations the Yukawa effect contributes two terms, one of which scales as $\alpha_\infty = 1 \times 10^{-3}$ and the other as $\alpha_\infty / \lambda = 1.418 \times 10^{-16}$. In table 6 we consider elliptical orbits of eccentricities *e* = 0.001, 0.01 and 0.1 and we tabulate the time to reach Earth for particles of various diameters and density $\rho = 4000$ kg/m^3.

Table 6 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity *e* =0.001, case *r* and λ variable.

Particle diameter *d* [m]	Time to reach Earth's Orbit *t* [y]
10^{-4}	1.272×10^{8}
10^{-3}	1.272×10^{7}
10^{-2}	1.272×10^{8}

Table 7 Time to reach Earth's orbit for particles of various diameters in elliptical orbits of eccentricity e =0.1, case r and λ variable.

Particle diameter d [m]	Time to reach Earth's Orbit t [y]
10^{-4}	9.396×10^{5}
10^{-3}	9.396×10^{6}
10^{-2}	9.396×10^{7}

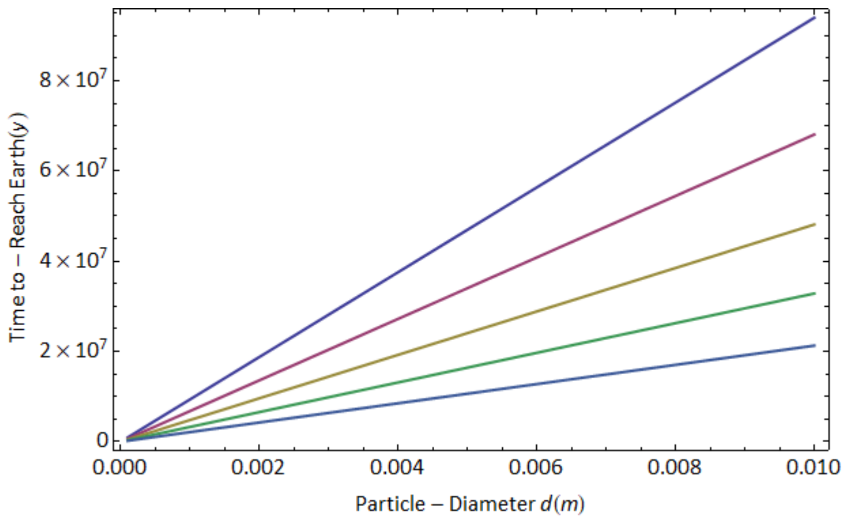

Fig. 6 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3. and various diameters in elliptical orbits of eccentricities e = blue (0.1), red (0.2), light brown(0.3), green(0.4), light blue(0.5) and for variable α and λ calculated at r = 2.7 AU.

Furthermore, we consider particles in circular orbits and we derive the time taken to reach Earth's orbit in years as a function of the particle's parameters namely density ρ, diameter d as well as the Yukawa parameters $\alpha(r)$ and $\lambda(r)$ calculated at r = 2.7 AU. The derived equation is:

$$t_E = 3.194\times10^{6} d\rho - 4.953\times10^{6} d\rho\alpha_\infty + \frac{7.546\times10^{18}}{\lambda_\infty} d\rho\alpha_\infty \,[\text{y}]. \quad (80)$$

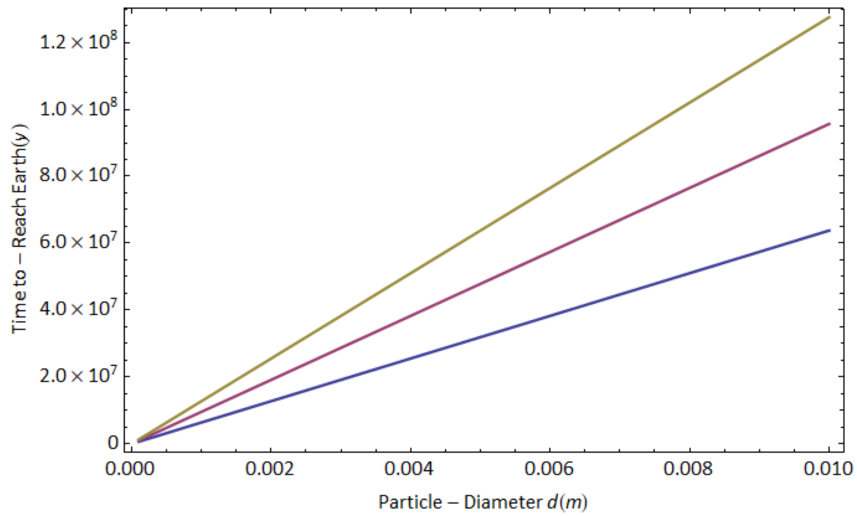

Fig. 7 Time to reach Earth for particles of density $\rho = 4000$ kg/m^3. and various diameters in elliptical orbits of eccentricities *e* = blue (0.1), red (0.2), light brown(0.3), green(0.4), light blue(0.5) and for variable α and λ calculated at *r* = 2.7 AU.

7 Conclusions

Spherical particles originating in the asteroid belt at a distance of $2.7a_E$ away from the Sun are considered to move in circular and elliptic orbits under the influence of a Yukawa-type correction to the gravitational potential as well as the influence of the Poynting-Robertson effect. We first consider circular orbits, and a semi analytical solution for the time that dust particles of various diameters take to reach Earth's orbit are obtained. For circular orbits we have only considered the $\lambda > r$ case. Similarly, for elliptical orbits and for particles of similar diameters but of various eccentricities, we have obtained a first order perturbation solution. In this case we have examined the following sub cases: $r > \lambda$, $\lambda > r$ and the case of variable lambda and alpha i.e. the case of $\lambda(r)$ and $\alpha(r)$. All our calculations have been carried out for the particle density of $\rho = 4000$ kg m^{-3}, and the diameters *d* of $10^{-4}, 10^{-3}, 10^{-2}$ m respectively. For circular orbits the time taken for the dust particles to reach Earth's orbit has been found to depend on the Yukawa parameters according to the term $\left(2+\alpha+\alpha^{-1}\right)\lambda^2 \ln\left(\alpha/(1+\alpha), \lambda^{-2}\right)$. For constant density ρ under the action of the Yukawa potential, circular orbits result to longer times, with the times to scale linearly with the diameter of the dust particles. Similarly, for particles constant diameter *d* the time scales linearly with the dust density ρ resulting to longer times at higher densities. In the case where $r > \lambda$ the dust particle times result to an expression that is a rational function of particle diameter *d* of the form $t_E = (c_1 + c_2 d)/(c_3 + c_4 d)$. Furthermore, for constant density longer times result for higher values of particle diameters and eccentricities. In the case where $\lambda > r$ we find that the time to reach Earth's orbit has been found to be λ independent to a first order perturbation treatment. Larger densities and larger particle diameters imply longer times for particular value of eccentricity. Moreover, at constant values of density and diameter, higher eccentricities result in shorter times. Lastly in the case of variable $\lambda(r)$ and $\alpha(r)$ we find that for eccentricities lying in the range $0.001 \le e_0 \le 0.01$, and for the same particle diameters the particle times are in the range of $10^6 \le t_{Yuk_E} \le 10^8$ years. Similarly, for particles with eccentricities lying in the range $0.1 \le e_0 \le 0.5$ we obtain $10^6 \le t_{Yuk_E} \le 10^8$ years, both differing by two orders of magnitude.

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