1993

Hyperradiance from Soliton Oscillators Synchronized by Capacitive or Inducitve Coupling

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(Received 10 May 1993; accepted for publication 28 July 1993)

The output power from coupled Josephson oscillators is investigated when the junctions are operated in their single fluxon mode. We demonstrate that both inductive and capacitive coupling mechanisms can give rise to hyperradiance when the power is coupled out through a boundary resistor. Analytical expressions are derived from adiabatic perturbation theory and excellent agreement is found between the analytical expression and numerical simulations.

Synchronization of nonlinear oscillations is an important subject in physics and engineering since collective motion may have essentially different features when compared to motion of an individual oscillator. An example is the emitted electromagnetic power from an array of Josephson junctions. For small junctions this power has been shown to follow the "superradient" theory and increase with the square of the number of participating oscillators.[1,2] For junctions of finite length, it has been shown that the emitted power can exceed this limit ("hyperradiance")[3–5] if the oscillators are operated in their fluxon modes. This somewhat surprising result arises from the fact that the emitted power from a phase-locked state is limited by the number of degrees of freedom in the system and not the number of oscillators (which equals the degrees of freedom for point oscillators). In order to realize optimal conditions for hyperradiance it is thus important to understand how spatial deformations are determined by the precise nature of the coupling mechanisms. In contrast to superradiance for point oscillators, hyperradiance in extended oscillators depends on details of the coupling mechanism. It is, therefore, crucial to understand the nature of the different possible interaction mechanisms for these types of oscillators. In this communication we will investigate radiation from the boundaries of pairs of inductively or capacitively coupled linear Josephson junctions operated in their single fluxon mode.

The system is governed by the following set of coupled sine-Gordon (SG) equations[6,7]

$$\begin{aligned}\phi_{xx}-\phi_{tt}-\sin\phi&=\alpha\phi_t-\eta_1+\Delta_1\psi_{xx}+\Delta_2\psi_{tt},\\ \psi_{xx}-\psi_{tt}-\sin\psi&=\alpha\psi_t-\eta_2+\Delta_1\phi_{xx}+\Delta_2\phi_{tt},\end{aligned}\tag{1}$$

where ϕ and ψ are the phase variables of the two Josephson junctions. Space and time are normalized here to the characteristic Josephson length (λ_J) and the inverse plasma frequency (ω_p^{-1}), respectively. The parameter α represents the quasi-particle dissipation, and η_i is the normalized bias current density of the ith junction. The parameters, Δ_1 and Δ_2, represent the inductive and the capacitive coupling, respectively. As noted in Ref. 7 these are typically negative for physically relevant systems. However, when only two coupled SG systems are studied, it is easy to see that reversing the signs of Δ_i can be compensated by reversing the signs of, e.g., ψ and η_2 as well. For simplicity, therefore, positive coupling coefficients will be assumed. In order to make some simple analytical considerations Eq. (1) assumes that all system parameters, except for the bias current η_i, are identical. To do so does not affect the principles discussed in this communication.

We are interested in determining the emitted power from the boundary ($x=0$) of the system. The load of the system can be modeled by different coupling mechanisms to the environment. If radiation is emitted through a resistive load, then the following boundary conditions apply[8] (circuit diagrams for the system can be found in Refs. 7 and 8)

$$\begin{aligned}\phi_x(x=0)&=-R^{-1}\phi_t,\phi_x(x=L)=0,\\ \psi_x(x=0)&=-R^{-1}\psi_t,\psi_x(x=L)=0,\end{aligned}\tag{2}$$

where L is the length of the system and R is the boundary resistance, in units of normal resistance of a unit junction length. When operated in single fluxon modes, we will consider the fluxon profile during one collision with the boundary ($x=0$ and $t=0$) to be given by the kink antikink solution to the unperturbed sine-Gordon equation:[9]

$$\phi^{(0)}=\psi^{(0)}=4\tan^{-1}\left(\frac{1}{u}\frac{\sinh u\gamma(u)t}{\cosh\gamma(u)x}\right),\tag{3}$$

where $\gamma(u)=(1-u^2)^{-1/2}$ is the inverse Lorentz contraction of the solution with the asymptotic velocity u. This solution is valid only in the region $0\leqslant x\lesssim L/2$ and $|t|\lesssim T/4$, where T is the period of the fluxon motion. When the fluxon motions in the two junctions are synchronized and, when $\eta_1=\eta_2$, we have the modified solution given by ($\phi=\psi$),

$$\phi^{(l)}=\psi^{(l)}=4\tan^{-1}\left(\frac{1}{\tilde{u}}\frac{\sinh\tilde{u}\gamma(\tilde{u})\tilde{t}}{\cosh\gamma(\tilde{u})\tilde{x}}\right),\tag{4}$$

where

$$\tilde{x}=\frac{x}{\sqrt{1-\Delta_1}},\quad \tilde{t}=\frac{t}{\sqrt{1+\Delta_2}},\quad \tilde{u}=u\frac{\sqrt{1+\Delta_2}}{\sqrt{1-\Delta_1}}.\tag{5}$$

The superscript $^{(l)}$ here denotes the locked state. As concluded in Ref. 4 this rescaling of the fields is enough to explain hyperradiance. We will now consider the amount of energy emitted through the boundary resistor during one reflection of the fluxons at $x=0$. In doing this we will consider the semi-infinite system $(L\to\infty)$ in the time interval $-\infty<t<\infty$.

The total energy delivered through the boundaries is given by

$$E=R^{-1}\int_{-\infty}^{\infty}(\phi_t+\psi_t)^2\,dt. \tag{6}$$

For the case where $\eta_1=\eta_2$, with ϕ and ψ as given by Eq. (4) we then obtain

$$E^{(12)}=R^{-1}4\int_{-\infty}^{\infty}(\phi_t^{(l)})^2\,dt =\frac{64}{\sqrt{1+\Delta_2}}R^{-1}\gamma(\tilde{u})\left(\tilde{u}+\frac{\sin^{-1}[\gamma^{-1}(\tilde{u})]}{\gamma^{-1}(\tilde{u})}\right). \tag{7}$$

The corresponding energy emitted from just one junction is given by[8]

$$E^{(1)}=R^{-1}\int_{-\infty}^{\infty}(\phi_t^{(0)})^2\,dt =16R^{-1}\gamma(u)\left(u+\frac{\sin^{-1}[\gamma^{-1}(u)]}{\gamma^{-1}(u)}\right). \tag{8}$$

In comparing the emitted power from a single oscillator to the power from a coupled pair of synchronized oscillators (of finite length), the asymptotic velocities must be related to a common frequency. As shown in Ref. 10, the relation between the system length L, the soliton frequency ω_s, and the soliton velocity $(\tilde{u})$ is given by

$$\cosh\left(\frac{L}{2}\frac{\gamma(\tilde{u})}{\sqrt{1-\Delta_1}}\right)=\frac{1}{\tilde{u}}\sinh\left(\frac{\tilde{u}\gamma(\tilde{u})\pi}{2\omega_s\sqrt{1+\Delta_2}}\right). \tag{9}$$

This expression, taking the phase shift during reflections into account, is derived over half the soliton period. The average output power of the system is then given by

$$P=\frac{\omega_s E}{2\pi}, \tag{10}$$

and the ratio between the power from the synchronized system $P^{(12)}$ and the power from the single junction system $P^{(1)}$ is then found to be

$$\kappa^*\equiv P^{(12)}/P^{(1)} =\frac{4}{\sqrt{1+\Delta_2}}\frac{\gamma(\tilde{u})}{\gamma(u)}\frac{\tilde{u}+\gamma(\tilde{u})\sin^{-1}[\gamma^{-1}(\tilde{u})]}{u+\gamma(u)\sin^{-1}[\gamma^{-1}(u)]}, \tag{11}$$

where $\tilde{u}$ is given implicitly by Eq. (9) and u can be found from Eq. (9) by setting $\Delta_1=\Delta_2=0$. In the low frequency limit the power ratio κ^* approaches the ratio,

$$\kappa^*\approx\frac{4}{\sqrt{1+\Delta_2}},\quad \text{for } u,\tilde{u}\to 0. \tag{12}$$

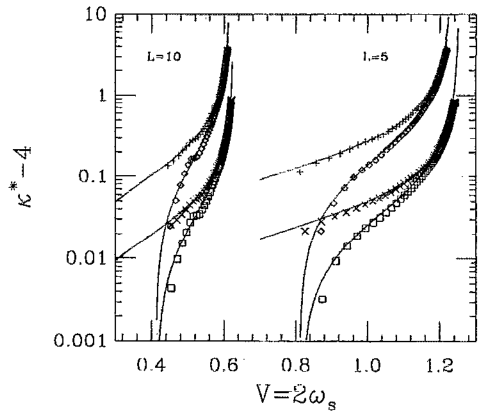

FIG. 1. The power ratio κ^* between the output from a coupled system and a single system as a function of the soliton frequency $\omega_s\equiv 0.5$ V. Parameters are $\alpha=0.1$ and $R^{-1}=0.01$. The markers represent the numerical data: $\Delta_1=0.01$, $\Delta_2=0$ (×). $\Delta_1=0.05$, $\Delta_2=0$ (+). $\Delta_1=0$, $\Delta_2=0.01$ (□). $\Delta_1=0$, $\Delta_2=0.05$ (◇). The solid lines represent the analytical expression of Eq. (11).

However, in the high frequency limit, $u\to\sqrt{1-\Delta_1}/\sqrt{1+\Delta_2}$, we find that κ^* is, in principle, unbounded:

$$\kappa^*\approx\frac{4}{\sqrt{1-\Delta_1}}\frac{\gamma(\tilde{u})}{\gamma[(\sqrt{1-\Delta_1})/(\sqrt{1+\Delta_2})]},\quad \text{for } \tilde{u}\to 1. \tag{13}$$

From Eqs. (11) and (13) it is now clear that hyperradiance can be expected from synchronized fluxon oscillators for inductive as well as capacitive coupling mechanisms. Here too, as in a previous analysis,[4] the power ratio is largest in the high frequency limit.

In order to test the analytical prediction in Eq. (11), we have performed numerical simulations of the system, as defined by Eqs. (1) and (2), for several choices of system parameters. All of the simulations were carried out for single soliton motion, with the dissipation parameter, $\alpha=0.1$, and for $\eta_1=\eta_2$. In calculating the maximum power ratio κ^*, we have measured the average power output through the boundary resistor R. This was done by simulating both the coupled Eq. (1) and a reference system consisting of a single junction. The ratio between the powers of these two systems was then evaluated as a function of the soliton frequency ω_s (it is necessary to use slightly different bias currents to produce the same ω_s for the single junction and the coupled junctions).

In Fig. 1 power ratios are plotted as functions of the individual junction voltage, $V\equiv 2\omega_s$, for various choices of Δ_1 and Δ_2. As is evident in the figure, there is excellent agreement between the numerical data and the analytical expression for all the parameter sets. In the high frequency limit, $\omega_s\to\pi/L$, we find that either capacitive or inductive coupling results in the same power ratio. However, for low frequencies, capacitive coupling cannot yield hyperradiance. In fact, as seen from Eq. (12), when $\Delta_2\neq 0$, the resulting power cannot even reach the superradiant limit. This is confirmed by the numerical simulations. We have

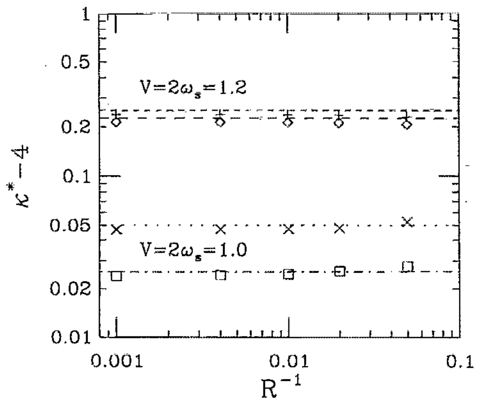

FIG. 2. The power ratio κ^* between the output from a coupled system and a single system as a function of the boundary load R^{-1}. Parameters are: $\alpha=0.1$ and $L=5$. The markers represent the numerical data and the lines represent the analytical expression of Eq. (11). $\Delta_1=0.01$, $\Delta_2=0$, $V=1.0$ ($\times$ and dotted). $\Delta_1=0.01$, $\Delta_2=0$, $V=1.2$ (+ and short dash). $\Delta_1=0$, $\Delta_2=0.01$, $V=1.0$ (□ and dash dot). $\Delta_1=0$, $\Delta_2=0.01$, $V=1.2$ (◇ and long dash).

elected to show only data for $\kappa^*>4$, but the comparison is just as good for the low frequency limit, where $\kappa^*<4$ for capacitive coupling. All the data in Fig. 1 are for a boundary resistance given by $R^{-1}=0.01$.

Figure 2 displays the results of a comparison between the analytical treatment and numerical solutions for different values of the boundary resistor. Here the comparison is made for $L=5$ and for coupling parameters, $\Delta_1=0.01$, $\Delta_2=0$ or $\Delta_1=0$, $\Delta_2=0.01$ (see figure caption for parameters). This figure demonstrates that the analytical prediction is almost unaffected by the particular value of the boundary resistor that models the coupling of power to the environment. For small perturbations (R large) we find that the numerical data are slightly smaller than predicted. This deviation has its origin in the perturbation treatment, where we view a reflection of the fluxon as an event taking place in the interval, $-\infty<t<\infty$. For short junction lengths this is not a very good approximation and the power ratio may be overestimated.

In conclusion, we have investigated the possibilities for hyperradiance from coupled systems of Josephson junctions. It has been demonstrated that both inductive and capacitive coupling between these soliton oscillators can give rise to hyperradiance. A general analytical expression has been obtained for the radiated power, and comparisons between the perturbation result and the numerical simulations show excellent agreement for all the parameter sets considered. We note that many other coupling mechanisms may also give rise to hyperradiance from synchronized fluxon oscillators, but we have here decided to limit this analysis to a study of two well-established types of interaction. A common feature of these two coupling mechanisms, which both result in hyperradiance, is that they stretch the characteristic time and length scales in the system and thereby compress the phase-locked modes accordingly [see Eqs. (4) and (5)]. Due to the slight shift in the asymptotic velocity, the junctions then experience a dramatic change in dynamics when operated in their fluxon modes close to the asymptotic velocity. We believe that hyperradiance could be produced by other coupling mechanisms, provided those mechanisms lead to modifications in either the length or the time scales of the phase-locked system.

This work was performed under the auspices of the U.S. Department of Energy and the Natural Sciences and Engineering Research Council of Canada.

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