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Voltage locking and other interactions in coupled superconducting weak links. II. Experiment

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Experiments have been performed on two superconducting indium microbridges fabricated in close proximity ($\approx 2 \mu\text{m}$) to one another. The observed interactions include the following: (i) the apparent critical current of one bridge depends upon the current and voltage across the other bridge; (ii) gross modifications of the current-voltage characteristics of one bridge occur which are dependent on the current and voltage across the neighboring bridge; and (iii) strong voltage pulling is observed whenever the two bridge voltages are approximately equal, resulting in dc-voltage locking under appropriate conditions. The "voltage-locking" interaction is of particular interest, and data are presented showing its observed temperature and frequency dependence. It is shown that the observed interactions are in qualitative agreement with a model based upon the diffusion of quasiparticles generated during the phase-slip process.

I. INTRODUCTION

The problem of coupling between individual weak-link Josephson junctions and within arrays of weak links has become the focus of increasing effort in recent years.¹⁻⁷ Not only is coupling of intrinsic interest, but it can yield valuable information about the nature of the phase-slip process in particular and nonequilibrium superconductivity in general. Indeed, it has been demonstrated by Jillie *et al.*¹ that a superconducting microbridge can be used as a probe of quasiparticle diffusion currents produced by a phase-slip center (PSC) in a second microbridge. In addition, arrays of weak links have important potential applications as rf detectors and oscillators, in voltage standards, and possibly in Josephson computer logic and memory devices. These applications will not be fully realized, however, until the interaction mechanisms between weak links are understood.

Of the various interactions between coupled weak links, the observation of voltage locking in the dc current-voltage characteristic is of particular interest. This was first observed by Jillie *et al.*⁵ between series pairs of superconducting indium microbridges separated by about two microns. They observed full dc-voltage locking only with opposing current bias and interpreted this as supporting a proposed quasiparticle current-injection link-coupling mechanism. Subsequent experiments by Lindelof *et al.*⁴ showed for the first time that dc-voltage locking of a pair of

series-connected links leads to coherent radiation. In this case, however, the results were not affected by changes in the relative directions of the individual bridge bias current, but their results did depend on such factors as bias levels and bridge spacing. Lindelof also presented a simple model for weak-link coupling based upon diffusion of branch-imbalanced quasiparticle currents. Further experiments by Varmazis *et al.*³ on coupled indium microbridge samples similar to those used by Jillie corroborated the results of Lindelof. They observed coherence independent of the relative directions of individual bridge bias currents; however, the bridges did not display dc-voltage locking when series biased, only when opposing-current biased.

Recently, Sandell *et al.*² and Varmazis *et al.*³ have reported studies of microbridges in which coupling was *forced* by the addition of gold shunt resistors to the samples. Their results are of importance for two reasons. First, the equivalent circuit and resulting equations are easily derived in this case, as shown in Paper I. Second, many of the Sandell *et al.* and the Varmazis *et al.* experimental results are the same as the results in the other experiments described above, in which coupling occurs due to close physical location of each weak link to the other. This suggests that a common mechanism may be responsible for coupling in both resistively shunted and closely spaced microbridge pairs. Indeed, in Sec. II we show that the equations describing two weak links coupled

by a resistive shunt are formally identical to the equations describing two weak links coupled by quasiparticle diffusion currents. The results derived in Paper I are thus expected to apply in both cases. In Sec. III our experimental results for two closely spaced indium microbridges are reported and compared to the theoretical results, with particular emphasis given to the voltage-locking interactions. The published results of other weak-link coupling experiments are also discussed, and appropriate extensions to the theory are suggested. Section IV summarizes our results and indicates potentially profitable directions for further research.

II. EQUATIONS DESCRIBING QUASIPARTICLE-COUPLED WEAK LINKS

In a microbridge, the finite-voltage state is the result of periodic collapse of the order parameter and its subsequent recovery to a state supporting 2π less phase difference across the bridge. This process is called a phase slip, with the bridge referred to as the phase-slip center (PSC). Both the pair and normal electron (quasiparticle) densities and currents are out of equilibrium during these oscillations. It has been amply demonstrated that the quasiparticle diffusion current generated during the phase slip is one of the major interaction mechanisms between PSC's located in close proximity to one another.⁸⁻¹⁰ It is thus reasonable to include this current within the framework of the resistively shunted junction (RSJ) model^{11,12} by including a quasiparticle current in the RSJ equations

$$\frac{d\phi_1}{dt} = \frac{2eR_1^*}{\hbar} (i_1 - i_{c1} \sin\phi_1 + i_{q2}) , \quad (1)$$

and

$$\frac{d\phi_2}{dt} = \frac{2eR_2^*}{\hbar} (i_2 - i_{c2} \sin\phi_2 + i_{q1}) , \quad (2)$$

where $i_{q2}(i_{q1})$ is the quasiparticle current generated by bridge 2 (1) that actually flows through bridge 1 (2); R_1^* and R_2^* are the resistances of bridges 1 and 2, respectively; i_{c1} and i_{c2} are their critical currents; and ϕ_1 and ϕ_2 are their phases. The two weak links are biased by currents i_1 and i_2 . The total quasiparticle current generated by bridge 2 is given by

$$i_{q2}^{\text{tot}} = (i_2 - i_{c2} \sin\phi_2 + i_{q1}) . \quad (3)$$

If a fraction α_1 of this current actually flows through bridge 1 we have

$$i_{q2} = \alpha_1 i_{q2}^{\text{tot}} = \alpha_1 (i_2 - i_{c2} \sin\phi_2 + i_{q1}) , \quad (4)$$

and similarly

$$i_{q1} = \alpha_2 i_{q1}^{\text{tot}} = \alpha_2 (i_1 - i_{c1} \sin\phi_1 + i_{q2}) . \quad (5)$$

Substituting the expression for i_{q1} into i_{q2} we find

$$i_{q2} = \alpha_1 i_2 - \alpha_1 i_{c2} \sin\phi_2 + \alpha_1 \alpha_2 i_1 - \alpha_1 \alpha_2 i_{c1} \sin\phi_1 + \alpha_1 \alpha_2 i_{q2} , \quad (6)$$

and similarly for i_{q1} . If the coupling between the bridges is sufficiently weak ($\alpha_1 \ll 1$ and $\alpha_2 \ll 1$), the terms in $\alpha_1 \alpha_2$ and higher order may be ignored, and the resulting equations are

$$\frac{d\phi_1}{dt} = \frac{2eR_1^*}{\hbar} (i_1 - i_{c1} \sin\phi_1 + \alpha_1 i_2 - \alpha_1 i_{c2} \sin\phi_2) \quad (7)$$

and

$$\frac{d\phi_2}{dt} = \frac{2eR_2^*}{\hbar} (i_2 - i_{c2} \sin\phi_2 + \alpha_2 i_1 - \alpha_2 i_{c1} \sin\phi_1) . \quad (8)$$

With the substitution of

$$\alpha_1 = \frac{R_2}{R_s + R_2} = \alpha , \quad (9)$$

$$\alpha_2 = \frac{R_1}{R_s + R_1} = \frac{\alpha}{\delta} , \quad (10)$$

$$R_1^* = \frac{R_1(R_2 + R_s)}{R_1 + R_2 + R_s} , \quad (11)$$

$$R_2^* = \frac{R_2(R_1 + R_s)}{R_1 + R_2 + R_s} , \quad (12)$$

and the normalization of all currents to the average critical current, the equations for two externally shunted junctions [Eqs. (1) and (2) in Paper I] are regained. In the externally shunted junction model R_1 , R_2 , and R_s are the resistances of bridges 1, 2, and the shunt, respectively; and α and δ are the coupling parameters, as defined in Paper I. The results obtained in Paper I for two PSC's coupled by an external shunt resistance are also expected to apply to the case of two PSC's coupled via quasiparticle diffusion currents. In particular, it is expected that dc-voltage locking will be observed, as well as an interaction at harmonics of the dc voltage.

III. THE EXPERIMENT

The geometry of the coupled microbridge samples used in the experiments is indicated in Fig. 1. The microbridges are always $< 1 \mu\text{m}$ in length and width, and usually about 1000 \AA thick. They are fabricated from indium using electron-beam lithography and lift-off techniques, permitting precise fabrication of submicron bridges with separations as small as $1.6 \mu\text{m}$. The details of the fabrication and general characteristics of these microbridges have been described elsewhere.^{6,13} The coupling of the two microbridges in these experiments is due solely to their

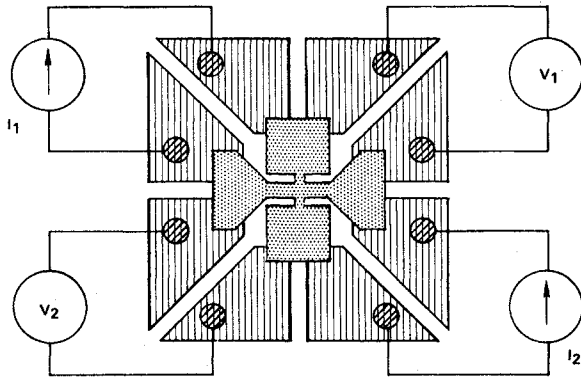


FIG. 1. Experimental arrangement: the thin-film indium microbridge structure is indicated by the dotted region and is drawn in the correct proportions. Eight gold contacts (the hatched regions) are provided for separate four-terminal measurement of each microbridge.

close proximity to one another; no resistive shunts are included.

The measurement circuit is also indicated schematically in Fig. 1. Eight leads are provided, resulting in independent four-terminal measurement of the current-voltage characteristic and differential resistance of each bridge. Measurements take place in a fully shielded Dewar with cryogenic low-pass filters used to protect the microbridges from external shocks and noise. A germanium thermometer is used to measure the temperature, and electronic feedback regulation is used to maintain the temperature to within a few microkelvins.

A. General interaction between coupled microbridges

Results for the general (nonvoltage-locking) interactions between these coupled microbridges have been published elsewhere.¹ These interactions are exemplified by the change in the apparent critical current of one bridge as a function of the current and voltage in the other bridge. This change in critical current can be divided into two parts: a symmetric portion that is independent of whether the current flows in the same direction (series aiding or just series) or in opposite directions (series opposed or simply opposed) through the two bridges, and an antisymmetric portion that does depend on the relative direction of current flow through the two bridges.

The symmetric part of the interaction is consistent with two mechanisms: current-induced order-parameter depression between the bridges, and order-parameter depression due to lattice heating and/or an excess of "hot" quasiparticles in the region between the bridges. Current-induced order-parameter depression in the stationary ($V=0$) state has been investigated in detail by Way, Hsu, and

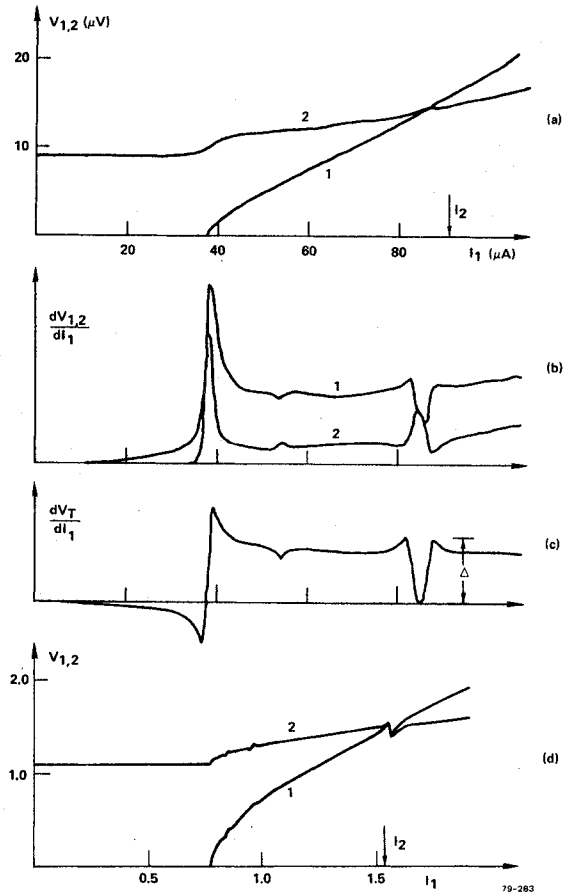


FIG. 2. (a) Experimental I - V curves for a coupled indium microbridge sample with series-opposing current bias. I_2 is held constant at $91 \mu\text{A}$, while I_1 is swept. The noninteracting critical currents are $I_{c1} = 61 \mu\text{A}$ and $I_{c2} = 58 \mu\text{A}$. (b) The differential resistance of the two curves shown in (a). (c) The total differential resistance $dV_T/dI_1 = dV_1/dI_1 - dV_2/dI_1$. During voltage locking $dV_T/dI_1 = 0$. $\Delta(dV_T/dI_1)$ in the locking region is a measure of the strength of the interaction. (d) Numerical simulation using the correct parameters for the coupled microbridges: $i_{c1} = 1.025$, $i_{c2} = 0.975$, $i_2 = 1.529$, $\alpha = 0.21$, and $\delta = 1.0$. Although the detailed fit of the curves is poor, the qualitative agreement is good, including the prediction of voltage locking over a small region.

Kao¹⁴, and heating has been thoroughly explored by Skocpol *et al.*¹⁵ These mechanisms pose a formidable two-dimensional computation problem when applied in detail to our coupled microbridges; however, they are well understood and can be solved in principle.

The antisymmetric interaction has been found to be consistent with the diffusion of dc quasiparticle current.¹ In the neighborhood of a PSC, the current flows a distance corresponding to the quasiparticle branch-relaxation time τ_Q . In particular, for our in-

dium microbridges in the temperature range of interest ($0.95 < T/T_c < 0.999$), this distance is about $5 \mu\text{m}$.¹³ Thus, quasiparticle diffusion plays an important role in the interactions between the two coupled bridges.

Since the quasiparticle coupling between the two bridges is included in Eqs. (7) and (8) in such an obvious way, it is no surprise that the equations correctly describe the asymmetric nonvoltage-locking interactions. If $d\phi_1/dt$ in Eq. (7) is set to zero and the equation is solved for i_1 , the result is $i_1 = i_{c1} \pm \alpha_1 V_{1rsj}/R_1^*$, where (+) is for series-aiding and (-) is for series opposed and $V_{1rsj} = R_1(i_1^2 - i_{c1}^2)^{1/2}$ is the time-averaged RSJ-voltage solution for a single weak link. Thus a plot of the change in critical current versus V/R should yield a straight line with a slope equal to the quasiparticle coupling parameter α_1 . This is indeed the case,¹ and the resultant value of $\alpha_1 = 0.22$ justifies dropping the higher-order terms in Eq. (6). This value of α_1 has been shown to agree with values derived from the normal state resistances of the two microbridges and the connecting strip between them.¹ This constitutes the one area of quantitative agreement between the theory and experiment. Equations (7) and (8) also describe the general increase in voltages in the series-opposing bias case, and the general decrease in voltages in the series-aiding bias instance. This is illustrated in the series-opposing instance by

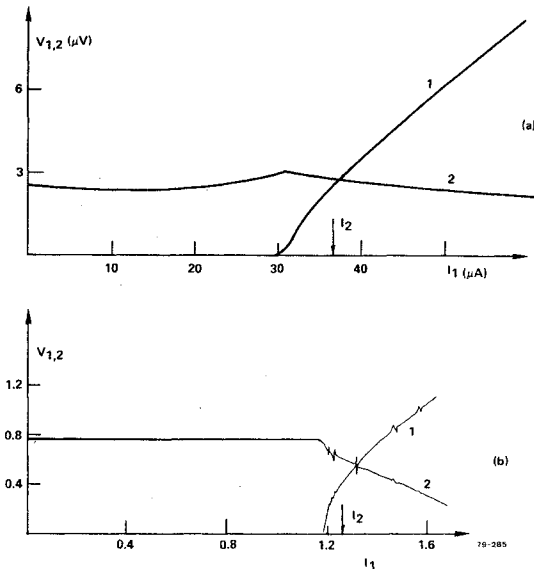


FIG. 3. Experimental I - V curves for same sample, as in Fig. 2, using series-aiding current bias. I_2 is held constant at $37 \mu\text{A}$. The noninteracting critical currents are $I_{c1} = 30 \mu\text{A}$ and $I_{c2} = 28 \mu\text{A}$. (b) Numerical simulation of (a), the parameters are $i_{c1} = 1.025$, $i_{c2} = 0.975$, $i_2 = 1.267$, $\alpha = 0.21$, and $\delta = 1.0$. Locking of the dc voltages is predicted but not observed.

comparing Figs. 2(a) and 2(d): As V_1 increases V_2 is seen to increase proportionally. In the series-aiding case, shown in Figs. 3(a) and 3(b), as V_1 increases V_2 decreases. Due to the presence of the symmetric interactions mentioned previously, the quantitative agreement between the experimental and theoretical curves is not good, and no attempt has been made to do a direct comparison. However, the qualitative agreement of the general forms for the I - V curves is reasonable and is interpreted as supporting the quasiparticle diffusion interaction mechanism.

B. Voltage-locking interaction

The three interactions noted above: current-induced order-parameter depression, heating, and the dc component of the quasiparticle diffusion current, are consistent with the observed nonvoltage-locking interactions between the coupled microbridges. Of much more interest is the voltage-locking interaction, which presents a very rich experimental phenomenology. We wish to present this phenomenology in some detail, and also further justify the application of the coupled Eqs. (7) and (8) to this problem.

Possible causes of the voltage-locking interaction, in addition to quasiparticle diffusion currents, are the symmetric interaction mechanisms described previously: order-parameter depression, and general heating effects. However, these are unlikely candidates for voltage locking. Locking has been observed at bridge separations of 1.6 – $3.0 \mu\text{m}$, much longer than the coherence length in indium. Simple heating, meaning a nonequilibrium distribution of phonons, has a relaxation time that is dependent upon film thickness, thermal conductivity, and transmissivity out of the thin film.^{15,16} For our samples this time is estimated to be on the order of 10^{-9} sec, too slow to account for locking interactions at frequencies as high as 40 GHz. In addition, a nonequilibrium phonon distribution does not have the required "polarity" to account for the differences observed in the interactions with opposing and aiding current bias. This leaves quasiparticle diffusion as the most likely cause of voltage locking. The quasiparticle branch relaxation length is consistent with the observed locking range, and the theoretical model embodied in Eqs. (7) and (8), which describes coupling via quasiparticle diffusion, does indeed predict locking. Unfortunately, the detailed shapes of the I - V curves in the locking region do not agree very well, as seen in Figs. 2 and 3. Nonetheless, this model appears to be the most promising basis available for a theory of coupled microbridge weak links.

One of the fundamental observations on these coupled samples is that actual dc-voltage locking is only observed in the opposed-bias situation. This is illustrated in detail in Fig. 4(a). In the case of series-

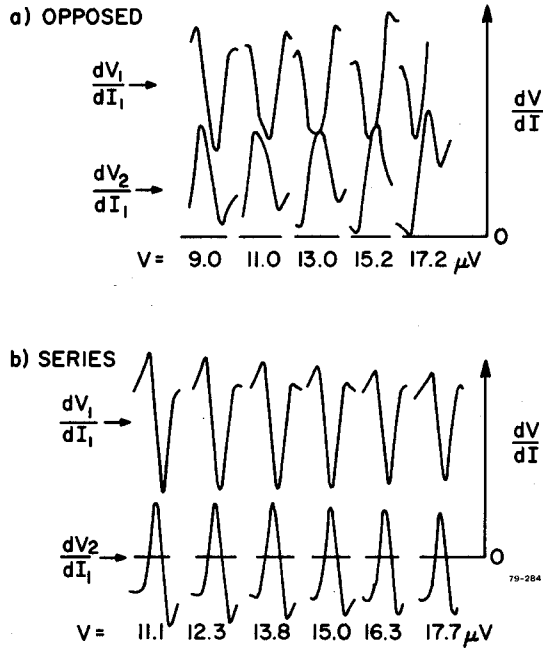


FIG. 4. (a) dV_1/dI_1 and dV_2/dI_1 vs I_2 in the interaction region for series-opposing current bias at several different voltage crossings. Full locking is only observed at 13.0 and 15.2 μV . (b) dV_1/dI_1 and dV_2/dI_1 vs I_2 in the interaction region for series-aiding current bias at several different voltage crossings. True dc voltage locking is not observed, although the interaction is just as strong as in the opposing case.

aiding bias, the differential resistances are pulled more closely together in the interaction region, as illustrated in Fig. 4(b), but no locking results. Recently, Varmazis *et al.*³ have observed the microwave radiation emitted by coupled indium microbridge samples similar to those used in this work. They found coherent emission existed in both the series-opposing and series-aiding bias cases, even when the dc voltages were not locked to a common value. This is consistent with the behavior of the model in the near-locking region. The voltages of the two bridges oscillate in phase for several cycles, and then one of the bridges skips a cycle. The emitted radiation is mostly coherent, even though the average frequencies (and hence average voltages) differ by several percent. Further evidence is provided in the work of Lindelof and Hansen.⁴ They observed coherence in coupled microbridges fabricated from both indium and tin, independent of aiding or opposing bias. In tin, coherence is seen at 12.0 μm and less, and in indium at separations of 3.5 μm and less. This is consistent with their computed values of the quasiparticle branch imbalance diffusion length of $\lambda_Q = 20 \mu\text{m}$ and 4.0 μm for tin and indium, respectively. Our model does not specifically include the effect of the finite

length of λ_Q ; however, this could be included by appropriate choice of α , since α is simply the proportion of quasiparticles generated by one PSC that actually diffuse through the other PSC.

It is interesting to note that the strength of the interaction as measured by the change in differential resistance in the series-aiding and series-opposed situations is about the same (see Fig. 4). However, the "background" differential resistances of the two bridges are much further apart in the series case due to the dc effects of quasiparticle diffusion. Thus, an interaction strength resulting in values of $\Delta(dV_1/dI_1)$ and $\Delta(dV_2/dI_1)$ just sufficient to lock the voltages in the opposing case will not induce locking in the series case.

In Fig. 5 the voltage and temperature dependence of the locking interaction is shown. The measure of the locking interaction is the change in the total differential resistance across both bridges through the interaction region, as shown in Fig. 2(c). Alternately, one could measure the range of current during which the interaction exists.² For our samples we have found that $\Delta(dV_T/dI_1)$ correlates better with the observation of full dc-voltage locking and thus is the preferred measure. Of particular note is the oscillatory character of the locking strength with voltage (also observed in the series-aiding bias situation). This is characteristic of a frequency-dependent phase shift. Lindelof and Hansen⁴ treat the area between the coupled junctions as a resonant cavity excited by a damped propagating quasiparticle branch imbalance oscillation. The wave originating from one bridge is reflected and phase shifted by the other, and, at appropriate frequencies in relation to the bridge spacing, injection locking of one bridge to the other is produced. These qualitative arguments appear to explain the observed oscillations in locking strength;

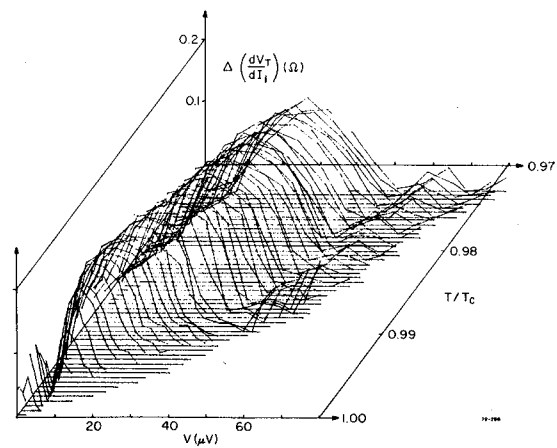


FIG. 5. Interaction strength in the series-opposing current bias case $\Delta(dV_T/dI_1)$, as a function of temperature and voltage.

however, their predicted temperature dependence does not agree with our data. They give the locking frequency as appropriate submultiples of a branch-imbalance wavelength (which they call λ_Q , not to be confused with the branch-imbalance diffusion length) which scales as $(1 - T/T_c)^{-1/4}$ with temperature. In Fig. 6 we show a plot of the minima and maxima of the locking strength as a function of temperature. The solid lines are proportional to $(1 - T/T_c)^{1/2}$, the dotted line is proportional to $(1 - T/T_c)^{1/4}$. The square-root dependence is clearly a better fit to the data. However, if the possible effect of simple heating, which has been found to be about $1\text{ K}/\mu\text{W}$ in similar bridges,¹³ is considered, the data points could be shifted by a maximum amount indicated by the arrows on two typical points. This temperature rise is at bridge itself—the connecting strip would be somewhat cooler. Depending on the magnitude and distribution of heating that actually occurs, the data points might move closer to a linear dependence. Thus, although the square-root dependence is favored, the linear dependence cannot be ruled out. It is interest-

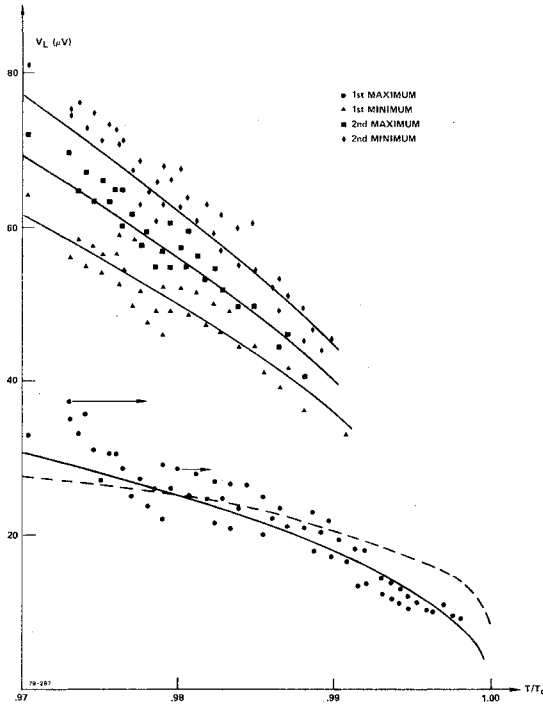


FIG. 6. Minima and maxima of the locking strength (from Fig. 5) plotted as a function of temperature. The solid lines are proportional to $(1 - T/T_c)^{1/2}$; the dotted line proportional to $(1 - T/T_c)^{1/4}$. The two arrowed points indicate the estimated maximum effect due to heating. The solid curves for the first minimum, second maximum, and second minimum are, respectively, 2 , $\frac{9}{4}$, and $\frac{5}{2}$ times the curve for the first maximum, which is suggestive of a resonance phenomenon.

ing to note that in experiments with junctions coupled with an external shunt resistance² these oscillations in locking strength have not been reported.

In addition to the oscillations noted above, there is a general falloff in the locking strength as the locking voltage increases. This was also noted by Sandell *et al.*² for coupling with external shunts. They attribute this partially to the decrease in coupling current through the shunt resistance R_s due to the loop inductance formed by R_s and the bridges. In addition, it is expected that the interaction will die at frequencies much greater than $1/\tau_Q$, since the Josephson oscillation will be much faster than the branch relaxation time,⁴ thus building up an excess distribution of "hot" quasiparticles, but no net branch imbalance. At a temperature of $T/T_c = 0.980$, this voltage (frequency) is $10.3\ \mu\text{V}$. The locking interaction is observed to $\sim 70\ \mu\text{V}$; however, it is extremely weak at voltages greater than $\sim 35\ \mu\text{V}$, or a few times $(1/\tau_Q)$. Since microwave-induced steps have been observed in this sample at voltages in excess of $200\ \mu\text{V}$, the $70\ \mu\text{V}$ limit to the locking interaction can reasonably be interpreted as a limit due to the locking mechanism, rather than a limit in the high-frequency response of the microbridges themselves. This mechanism is also relevant to the externally shunted bridges; since the normal shunt is a finite distance away from the bridges, it is sensitive to the quasiparticle electrochemical potential μ_q at that point.

Full voltage locking ($dV_T/dI_1 = 0$) is observed from just below T_c to about $0.98 T_c$ in our coupled microbridges. As the temperature is lowered, the interaction gradually fades, although it can still be observed at $0.90 T_c$. This is consistent with the observation of coherence by Lindelof *et al.*⁴ only near T_c . Sandell *et al.*² also observe a falloff in the interaction strength at lower temperatures, which they attribute to heating. We would like to point out that at $T/T_c = 0.90$, $\lambda_Q \sim 2\ \mu\text{m}$,¹³ which is comparable to our bridge spacing. This is also comparable to the distance from the bridge to the external shunt in the sample of Sandell *et al.* Thus, even with no heating, the locking interaction would be expected to fall off as the temperature is lowered.

Finally, there is an interaction observed when one bridge is biased at n and $1/n$ times the voltage of the other, where $n = 2$ and 3 . This is observed most easily as a small wiggle in the differential resistance of the two bridges, such as is seen at $I_1 = 55\ \mu\text{A}$ in Fig. 2(b). This was also observed by Sandell *et al.* for external resistively shunted bridges. Based upon the locking model in Paper I, it is expected that this harmonic "locking" interaction will be observed.

IV. DISCUSSION

The equations for two RSJ Josephson weak links coupled by an external shunt resistance have been

solved in Paper I. We have shown these equations to be formally identical to the equations describing two weak links coupled by quasiparticle diffusion currents. The equations predict the following coupling phenomena. First, changes in the critical current of one link that are linearly dependent upon the voltage of the other link are expected. Second, we expect gross changes in the voltage of each link in a coupled pair. Third, locking of dc voltages is predicted, including coherence in the radiated output. Finally, subharmonic locking interactions are predicted.

Qualitatively, all of the predictions above are realized in coupled weak-link experiments described here and elsewhere,^{1,3,4} and also in experiments with weak links coupled by an external resistive shunt.^{2,3} It is interesting to note the remarkable similarity between the data on our coupled microbridges and the externally shunted microbridges of Sandell *et al.*² (Compare our Fig. 4 with Figs. 3 and 4 of Ref. 2.) In addition, quantitative agreement with the change in critical current for quasiparticle coupled weak links has been realized.¹ However, the experimental observations include a number of phenomena that are not directly included in the equations. The temperature dependence of the voltage locking can be explained by the known temperature dependence of τ_Q , as can the general falloff of locking strength with increasing voltage. The oscillation in locking strength with voltage can be explained by considering the area between the weak links as a resonant cavity excited by a propagating quasiparticle branch-imbalance oscillation. This last concept will require further work before a convincing connection between theory and experiment is realized.

Experimentally, a multitude of possibilities for further work present themselves. In order to reduce the effects of heating and order-parameter depression, the use of variable-thickness microbridges fabricated from superconductors in which the ratio $\tau_Q/\xi_0 \gg 1$ is suggested. To improve quasiparticle coupling, it is suggested that the superconducting pad separating the bridges be contacted by a narrow strip of a high- T_c superconducting alloy. This will present a high resistance to the flow of quasiparticles, yet allow monitoring of the pair electrochemical potential and permit appropriate current biasing. Composite devices, including a microbridge coupled to a tunnel junction, are also potentially useful possibilities. Dolan and Jackel¹⁰ have convincingly demonstrated the existence of *dc-quasiparticle-diffusion currents* around a phase-slip center. It is our hope that other investigators will be stimulated to look at *ac-quasiparticle-diffusion currents* around a phase-slip center, thereby gaining a clearer understanding of the nonequilibrium phase-slip process.

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